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On Accounting for Evaporation or Infiltration Free Surface in Some Problems of Filtration Theory

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Abstract **Review Article**

The following filtration flows with unknown free boundaries are investigated: In case of a flow past the Zhukovsky groove in the case when the soil layer is underlain along its entire length with an impermeable base and evaporation from the free surface occurs; In case of a flow past the Zhukovsky groove in the case when the underlying layer is a completely well-permeable aquifer and infiltration occurs on the free surface; When groundwater moves in a rectangular bridge with a partially impenetrable vertical wall in the presence of evaporation from the free surface; When groundwater moves to an imperfect gallery in the presence of evaporation from the free surface.

Keywords: Zhukovsky groove, soil layer, groundwater, filtration.

INTRODUCTION

Within the theory of the flat established filtering of an incompressible fluid under Darci's law in homogeneous and isotropic soil some tasks connected with currents in the presence of evaporation or infiltration on a free surface of subsoil waters are considered.

1. Currents at flow of a groove Zhukovsky

The task about flow of a groove was for the first time studied by N.E. Zhukovsky [1] where Kirchhoff's method altered by it in the theory of streams was used for a solution of tasks with a free surface and special analytic function which is widely used in the theory of filtering is entered. Since function, and a task and a groove bear a name of Zhukovsky [2-6]. Work [1] opened a possibility of mathematical modeling of the movement of subsoil waters under Zhukovsky's groove and laid the foundation for researches of the specified class of filtrational currents (see, for example, reviews [2-6]).

It should be noted that in tasks about flow of a groove of Zhukovsky application of function of Zhukovsky only then results in effective results when in addition to a free surface the border of area of a current contains only horizontal lines of equal potential and vertical lines of current (V.V. Vedernikov, F.B. Nelson Furriers, S. N. Numerov, V. I. Aravin, etc.). However in actual practice hydrotechnical construction [2-5], the irrigated agriculture [2, 4, 7], etc. directly under integumentary deposits along with horizontal pressure head water-bearing layers more high-permeability [7] also horizontal waterproof inclusions often meet that radically affects the nature of filtrational currents [8-12].

At the same time so far there are no works devoted to a special research of impact of evaporation or infiltration on filtrational processes. Accounting of these important physical factors for the present did not become broad property of exact analytical solutions.

In the presented work on the example of two limit filtrational schemes which arise at flow of a groove of Zhukovsky, the impact of evaporation or infiltration on a current picture is studied.

The first limit scheme corresponds to a case when the layer of earth on all the extent is spread by the impenetrable horizontal basis and from a free surface there is a uniform evaporation of intensity ε ($0 < \varepsilon < 1$). The current is provided with water inflow from the left part of a band of flooding with a liquid layer, invariable on time. As the right edge of a band of flooding serves the impenetrable vertical screen in the form of a groove of Zhukovsky which basis is located in layer, at the same time the static height of a capillary raising of a subsoil water can be considered (Fig 1a).

In the second limit scheme the layer of earth is spread by well permeable pressure head aquifer in which pressure has constant H_0 value, and on a free surface there is a uniform infiltration of intensity ε. Far from a groove (at $x \to \infty$) the curve of a depression is horizontal and located at H_0 height over an aquifer (Fig 1b).

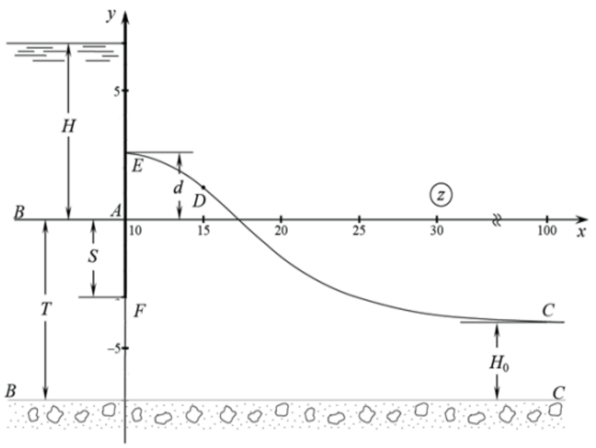

Fig 1a: The current picture calculated at ε = 0.6, h_c = 0.5, T = 7, S = 3, H = 5

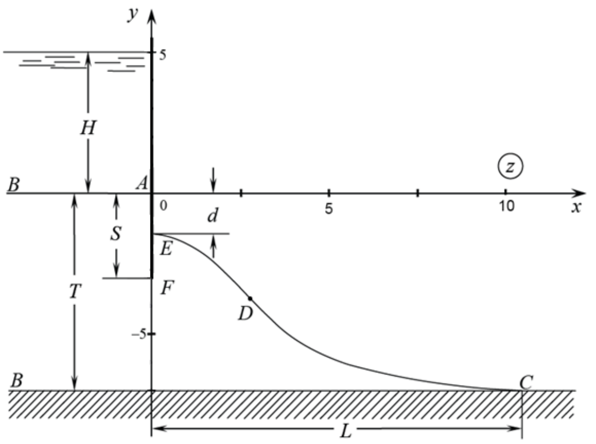

Fig 1b: The current picture calculated at ε = 0.6, T = 7, S = 3, H = 7, H_0 = 3 и x_C = 100

2. Currents in a rectangular jumper with partially impenetrable vertical wall and to imperfect gallery

The exact solution of a task on a fluid influx to the imperfect well with the flooded filter (i.e. an axisymmetric task) or the tubular well representing an impenetrable pipe with the filter in some (usually lower) its part is connected with great mathematical difficulties and so far is not found. Therefore in due time as first approximation to a solution of similar tasks by P.Ya. Polubarinova-Kochina, V. G. Pryazhinska, V. A. Postnov and V. N. Emikh [2, 6, 7, 17, 18] considered some corresponding flat task analogs about filtering in a rectangular jumper with partially impenetrable vertical wall and to imperfect gallery. It should be noted that areas of values of complex speed in the specified cases allow to apply by means of inversion at a solution Christoffel-Schwartz's formula.

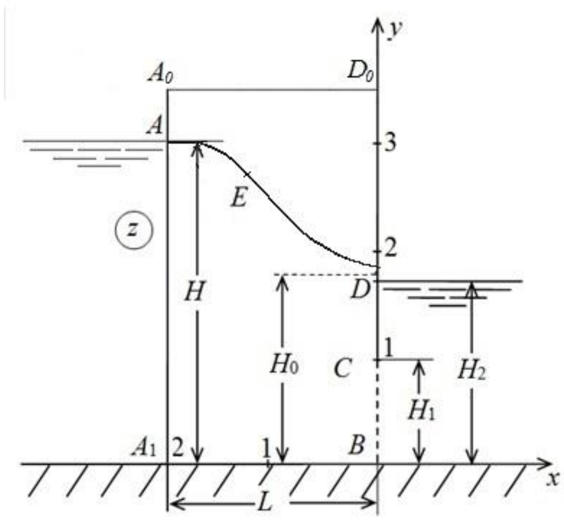

Fig 2: The current picture in a rectangular jumper calculated at ε = 0.5, H = 3, L = 2, H_1 = 1, H_2 = 1.4

In work the exact analytical solution of a task on a current of subsoil waters in a rectangular jumper with slopes of A_0A_1 and D_0B, width of L located on the impenetrable horizontal basis of length of L is given. Water height is equal in an upper reach to H, lower reach with water level of H_2, having partially impenetrable vertical wall CD (screen), adjoins a basis sole. The upper bound of area of the movement is free pover khnost AD which is coming out with which there is a uniform evaporation to intensity ε (Fig 2). In the considered area of complex speed, unlike [2, 6, 7, 17, 18], there are not rectilinear, but circular polygons that does not give the chance to use classical integral of Christoffel-Schwartz.

The task solution on a current to the imperfect well formally turns out from a task solution on filtering in a rectangular jumper with partially impenetrable vertical wall in case of its infinite width, i.e. at $L = \infty$ [19, 20].

3. Technique of solutions

For studying of the specified currents in the presence of evaporation or infiltration on a free surface the mixed multiple parameter boundary value problems of the theory of analytic functions which solution is carried out with use of the method of P. Ya. Polubarinova-Kochina [2-7] based on application of the analytical theory of linear differential equations of a class of Fuchs are formulated. And also [21-23] ways of conformal mapping of a special type of circular polygons [24] developed for areas which are very typical for tasks of the theory of filtering. Accounting of characteristics of the considered classes of areas of the hodograph of speed allowed to present solutions of tasks in the closed form through elementary functions that does their use the simplest and convenient in practice.

CONCLUSION

On the basis of the studied models calculation algorithms are developed:

- Heights of a raising of a subsoil water behind Zhukovsky's groove at it required, width of capillary spreading of liquid on a water emphasis (in scheme 1) and also values of a filtrational expense;
- Ordinates of exit point of a curve depression on the screen, a filtrational expense and coordinates of points of a free surface when filtering in a rectangular jumper and to imperfect gallery. The received results give an idea (at least qualitatively) of possible dependence of characteristics of a current by filtering consideration already to the imperfect well or a tubular well.

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