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Traveling Wave Solution for Sharma–Tasso–Olver–Burgers（STOB） Equation by the (G’/G)-Expansion Method

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Abstract **Review Article**

In this paper, we use the (G'/G)-expansion method to construct the traveling wave solution of the Sharma–Tasso–Olver–Burgers (STOB) equation based on the idea of homogeneous equilibrium. At the same time, it is verified that the (G'/G) -expansion method has wider applicability for dealing with nonlinear evolution equations.
Keywords: Nonlinear evolution equation (G’/G)-expansion method traveling wave solution.

INTRODUCTION

In essence, phenomenon in nature is complex and nonlinear, and do not satisfy the principle of linear superposition. With the in-depth study of various natural phenomenons by researchers, nonlinear science has gradually entered people's field of vision, and it has a pivotal position in modern science. The nonlinear evolution equation is an important research object in nonlinear science, and its exact solution is of great significance to help researchers understand some important physical phenomenon.

Solitary waves are one of the three branches of nonlinear science and are widely used in mathematics, physics and other fields. When the British scientist Russell observed the wooden boat, he found that the water wave formed after the wooden boat stopped would maintain a stable shape and continue to move forward for a certain distance, and later called such a wave as a solitary wave. It has been further discovered that, in addition to water waves, solitary waves can also appear in other substances, and the existence of solitary waves has been found in solid-state physics, plasma physics, and optical experiments. Soliton molecules have been experimentally discovered in optics and theoretically investigated for coupled systems. Recently, Lou introduced a new velocity resonant mechanism to create soliton molecules [1].

In reference [2], Yan *et al*. proposed the Sharma-Tasso-Olver-Burgers (STOB) equation when they focused on the formation of soliton molecules through the resonance mechanism of uncoupled systems. By introducing velocity resonance conditions, they derived the soliton (kink) molecule, half-period kink (HPK) molecule, and respiratory soliton branches of the STOB equation. At the same time, they also revealed the phenomenon of fission and fusion between kinks, kink molecules, HPKs and HPK molecules. In 2021, Miao *et al*. studied the interaction solutions of multiple solitary waves, solitary waves and triangular periodic waves of Sharma-Tasso-Olver-Burgers (STOB) equation through Cole-Hopf transformation, and analyzed some specific interaction phenomena through limiting behavior [3]. Meanwhile, in 2021, Hu *et al*. studied soliton (kink) molecules, half-period kink (HPK) and HPK molecules of (2+1) dimensional Sharma-Tasso-Olver-Burgers (STOB) equation. Then the lump solution was obtained, and they discussed the interactions between lump and kink molecule [4].

This paper focuses on the exact solution of Sharma-Tasso- Olver -Burgers (STOB) equation. So far, there are many methods to calculate the exact solution of the nonlinear evolution equation. In this paper, the (G'/G) expansion method is mainly used, and the symbolic calculation software maple is used to calculate the STOB equation. Finally, we have carried out numerical simulation on the obtained results, and obtained the numerical simulation figure.

The (G'/G)-expansion method

The (G'/G)-expansion method was formally proposed by Wang Ming liang *et al*. in 2007[5]. The main idea of this method is that the exact solution of the nonlinear partial differential equation can be represented by a polynomial of (G'/G), where $G = G(\xi)$ satisfies the second-order ordinary linear differential equation (LODE) $G''(\xi) + \lambda G'(\xi) + \mu G(\xi) = 0$, λ and μ are two constants, $G' = \frac{dG(\xi)}{d\xi}$, $\xi = x - Vt$, The degree of the polynomial can be determined by considering the homogeneous balance between the highest order derivative occurring in a given nonlinear partial differential equation and the nonlinear term. Next are the detailed steps of the method.

Assume a nonlinear partial differential equation

$$P(u, u_t, u_x, u_{tt}, u_{xt}, \cdots) = 0, \qquad (1)$$

Where x and t are two independent variables, $u = u(x,t)$ is an unknown function, P is a polynomial with respect to $u = u(x,t)$ and various partial derivatives of $u = u(x,t)$. Take the following traveling wave transform

$$\xi = x - Vt, \quad u(x,t) = u(\xi). \qquad (2)$$

At this point, (1) is transformed into the following equivalent form

$$P(u, -Vu', u', V^2u'', -Vu'', \cdots) = 0. \qquad (3)$$

It is assumed that the solution of ordinary differential equation (3) can be expressed by the polynomial of $(\frac{G'}{G})$, the specific form is as follows

$$u(\xi) = \delta_m (\frac{G'}{G})^m + \cdots, \qquad (4)$$

Where $G = G(\xi)$ satisfies the second order LODE in the form

$$G''(\xi) + \lambda G'(\xi) + \mu G(\xi) = 0, \qquad (5)$$

The specific value of $\delta_m, \cdots, \lambda$ and μ will be calculated in the subsequent process, $\delta_m \neq 0$. The omitted part of (4) is also a polynomial of $(\frac{G'}{G})$, the value of the positive integer m can be determined by considering the homogeneous balance between the highest order derivative and the nonlinear term appearing in ordinary differential equation (3).

Bring (4) into equation (3) while considering second order linear ordinary differential equation (5). The left-hand side of equation (3) is now transformed into another polynomial, and merge similar items of $(\frac{G'}{G})$ with the same order, and the coefficients of each term of the polynomial are set to 0, this produces a system of algebraic equations for $\delta_m, \cdots, V, \lambda, \mu$.

The value of the constant $\delta_m, \cdots, V, \lambda, \mu$ can be obtained by solving the above system of algebraic equations, since the general solution of the second-order linear ordinary differential equation (5) is well known to us. Bringing the value of $\delta_m, \cdots, V, \lambda, \mu$ and the solution of equation (5) into (4) we can derive the exact solution of (1).

Applying the (G’/G)-expansion method to solve the Sharma–Tasso–Olver–Burgers（STOB）equation

In this paper, we mainly use the (G’/G)-expansion method to construct the traveling wave solution of the STOB equation. When Lou *et al*. are concerned with the formation of soliton molecules by the resonant mechanism for a noncoupled system; they find the STOB equation [2]. The specific form of STOB equation is as follows.

$$u_t + \alpha(3u_x^2 + 3u^2u_x + 3uu_{xx} + u_{xxx}) + \beta(2uu_x + u_{xx}) = 0, \qquad (6)$$

Where α and β are two constants, when the value of α is 0, STOB equation is the famous Burgers equation, when the value of β is 0, STOB equation is the famous STO equation. Take the following traveling wave transform.

$$\xi = x - Vt, \quad u(x,t) = u(\xi). \qquad (7)$$

Bring (7) into (1), we have

$$-\mathrm{V}u' + \alpha(3(u')^2 + 3u^2u' + 3uu'' + u''') + \beta(2uu' + u'') = 0, \qquad (8)$$

Integrating it with respect to ξ once yields

$$C - Vu + \alpha(\frac{3}{2}(u^2)' + u^3 + u'') + \beta(u^2 + u') = 0, \qquad (9)$$

C is a constant of integration, which could be calculated in the subsequent process. Solution of (9) can be expressed as a polynomial of $(\frac{G'}{G})$, the form below

$$u(\xi)=\delta_n(\frac{G'}{G})^n+\cdots.$$

Considering u^3 and u'' in (9), using the homogeneous equilibrium method, the value of n can be obtained as 1. So, the specific form of the solution of (9) is as follows.

$$u(\xi)=\delta_0+\delta_1(\frac{G'}{G})^1. \qquad (10)$$

Where $G=G(\xi)$ satisfies the second-order linear ordinary differential equation

$$G''(\xi)+\lambda G'(\xi)+\mu G(\xi)=0, \qquad (11)$$

λ and μ are two constants. Thus, we can express each item in (9) with a formula including $(\frac{G'}{G})$, the results are as follows.

$$-Vu=-\delta_0 V-\delta_1 V(\frac{G'}{G}),$$
$$3\alpha uu'=-3\delta_0\alpha\delta_1-(3\delta_0\alpha\delta_1\lambda+3\alpha\delta_1^2\mu)(\frac{G'}{G})-(3\alpha\delta_1^2\lambda+3\delta_0\delta_1)(\frac{G'}{G})^2-3\alpha\delta_1^2(\frac{G'}{G})^3,$$
$$\alpha u^3=\alpha\delta_0^3+3\delta_0^2\delta_1(\frac{G'}{G})+3\delta_0\alpha\delta_1^2(\frac{G'}{G})^2+\alpha\delta_1^3(\frac{G'}{G})^3,$$
$$\alpha u''=\alpha\delta_1\lambda\mu+(\alpha\delta_1\lambda^2+2\alpha\delta_1\mu)(\frac{G'}{G})+3\delta_1\lambda(\frac{G'}{G})^2+2\delta_1(\frac{G'}{G})^3,$$
$$\beta u^2=\beta\delta_0^2+2\beta\delta_0\delta_1(\frac{G'}{G})+\beta\delta_1^2(\frac{G'}{G})^2,$$
$$\beta u'=\beta\delta_1\mu-\beta\delta_1\lambda(\frac{G'}{G})-\beta\delta_1(\frac{G'}{G})^2.$$

Combining the same items in the above equation, and setting the coefficient of each item to 0, the following equations can be obtained.

$$C-\delta_0 V-3\delta_0\alpha\delta_1+\delta_0^3\alpha+\alpha\delta_1\lambda\mu+\beta\delta_0^2+\beta\delta_1\mu=0,$$
$$-\delta_1 V-3\delta_0\alpha\delta_1\lambda-3\alpha\delta_1^2\mu+3\delta_0^2\alpha\delta_1+\alpha\delta_1\lambda^2+2\alpha\delta_1\mu+2\beta\delta_0\delta_1-\beta\delta_1\lambda=0,$$
$$-3\alpha\delta_1^2\lambda-3\delta_0\alpha\delta_1+3\delta_0\alpha\delta_1^2+3\delta_1\lambda+\beta\delta_1^2-\beta\delta_1=0,$$
$$-3\alpha\delta_1^2+\alpha\delta_1^3+2\delta_1=0.$$

Solving the above system of equations yields the following results

$$\delta_0=\frac{9\alpha\lambda+3\lambda\sqrt{9\alpha^2-8\alpha}-4\beta}{12\alpha},\ \frac{9\alpha\lambda-3\lambda\sqrt{9\alpha^2-8\alpha}-4\beta}{12\alpha},$$
$$\delta_1=\frac{3\alpha+\sqrt{9\alpha^2-8\alpha}}{2\alpha},\ \frac{3\alpha-\sqrt{9\alpha^2-8\alpha}}{2\alpha},$$
$$V=\frac{((63\lambda^2-36\mu)\alpha-24\beta\lambda)\sqrt{9\alpha^2-8\alpha}+(213\lambda^2-60\mu)\alpha^2-72\lambda(\beta+\frac{3\lambda}{2})\alpha+8\beta^2}{24\alpha},$$
$$\frac{((-63\lambda^2+36\mu)\alpha+24\beta\lambda)\sqrt{9\alpha^2-8\alpha}+(213\lambda^2-60\mu)\alpha^2-72\lambda(\beta+\frac{3\lambda}{2})\alpha+8\beta^2}{24\alpha}.$$

In addition, the following results can be obtained from $G''(\xi)+\lambda G'(\xi)+\mu G(\xi)=0$

$$\frac{G'(\xi)}{G(\xi)}=\begin{cases}\frac{\sqrt{\lambda^2-4\mu}}{2}\left(\frac{c_1\sinh(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi)+c_2\cosh(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi)}{c_1\cosh(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi)+c_2\sinh(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi)}\right)-\frac{\lambda}{2}, & \lambda^2-4\mu>0\\ \frac{c_1}{c_1\xi+c_2}, & \lambda^2-4\mu=0\\ \frac{\sqrt{4\mu-\lambda^2}}{2}\left(\frac{-c_1\sinh(\frac{\sqrt{4\mu-\lambda^2}}{2}\xi)+c_2\cosh(\frac{\sqrt{4\mu-\lambda^2}}{2}\xi)}{c_1\cosh(\frac{\sqrt{4\mu-\lambda^2}}{2}\xi)+c_2\sinh(\frac{\sqrt{4\mu-\lambda^2}}{2}\xi)}\right)-\frac{\lambda}{2}, & \lambda^2-4\mu<0\end{cases}$$

Where c_1 and c_2 are arbitrary constants. Bringing the above results into $u(\xi)=\delta_0+\delta_1(\frac{G'}{G})$, the traveling wave solution of the original equation can be obtained, and the results are as follows

case 1: $\lambda^2-4\mu>0$

$$u(\xi)_1=\frac{3\alpha+\sqrt{9\alpha^2-8\alpha}}{2\alpha}\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\left(\frac{c_1\sinh(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi)+c_2\cosh(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi)}{c_1\cosh(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi)+c_2\sinh(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi)}\right)-\frac{\lambda}{2}\right)$$
$$+\frac{9\alpha\lambda+3\lambda\sqrt{9\alpha^2-8\alpha}-4\beta}{12\alpha},$$

$$u(\xi)_2=\frac{3\alpha-\sqrt{9\alpha^2-8\alpha}}{2\alpha}\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\left(\frac{c_1\sinh(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi)+c_2\cosh(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi)}{c_1\cosh(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi)+c_2\sinh(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi)}\right)-\frac{\lambda}{2}\right)$$
$$+\frac{9\alpha\lambda-3\lambda\sqrt{9\alpha^2-8\alpha}-4\beta}{12\alpha},$$

case 2: $\lambda^2-4\mu=0$

$$u(\xi)_3=\frac{3\alpha+\sqrt{9\alpha^2-8\alpha}}{2\alpha}\frac{c_1}{c_1\xi+c_2}+\frac{9\alpha\lambda+3\lambda\sqrt{9\alpha^2-8\alpha}-4\beta}{12\alpha},$$

$$u(\xi)_4=\frac{3\alpha-\sqrt{9\alpha^2-8\alpha}}{2\alpha}\frac{c_1}{c_1\xi+c_2}+\frac{9\alpha\lambda-3\lambda\sqrt{9\alpha^2-8\alpha}-4\beta}{12\alpha},$$

case 4: $\lambda^2-4\mu<0$

$$u(\xi)_5=\frac{3\alpha+\sqrt{9\alpha^2-8\alpha}}{2\alpha}\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\left(\frac{-c_1\sinh(\frac{\sqrt{4\mu-\lambda^2}}{2}\xi)+c_2\cosh(\frac{\sqrt{4\mu-\lambda^2}}{2}\xi)}{c_1\cosh(\frac{\sqrt{4\mu-\lambda^2}}{2}\xi)+c_2\sinh(\frac{\sqrt{4\mu-\lambda^2}}{2}\xi)}\right)-\frac{\lambda}{2}\right)$$
$$+\frac{9\alpha\lambda+3\lambda\sqrt{9\alpha^2-8\alpha}-4\beta}{12\alpha},$$

$$u(\xi)_6=\frac{3\alpha-\sqrt{9\alpha^2-8\alpha}}{2\alpha}\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\left(\frac{-c_1\sinh(\frac{\sqrt{4\mu-\lambda^2}}{2}\xi)+c_2\cosh(\frac{\sqrt{4\mu-\lambda^2}}{2}\xi)}{c_1\cosh(\frac{\sqrt{4\mu-\lambda^2}}{2}\xi)+c_2\sinh(\frac{\sqrt{4\mu-\lambda^2}}{2}\xi)}\right)-\frac{\lambda}{2}\right)$$

$$+\frac{9\alpha\lambda-3\lambda\sqrt{9\alpha^2-8\alpha}-4\beta}{12\alpha},$$

Numerical Simulation

According to the conditions to be satisfied by the equation, at the same time, in order to facilitate the calculation, when $\lambda^2-4\mu>0$, for $u(\xi)_1$, let $\alpha=1$, $\lambda=4$, $\mu=1$, $\beta=1$, $c_1=1$, $c_2=2$, We can conclude that the value of V is $\frac{277}{3}$, the numerical simulation results are shown in Fig 1. when $\lambda^2-4\mu=0$, for $u(\xi)_3$, let $\alpha=1$, $\lambda=4$, $\mu=1$, $\beta=1$, $c_1=1$, $c_2=2$, We can conclude that the value of V is $\frac{277}{3}$, the numerical simulation results are shown in Fig 2. when $\lambda^2-4\mu<0$, for $u(\xi)_5$, let $\alpha=1$, $\lambda=1$, $\mu=1$, $\beta=1$, $c_1=1$, $c_2=2$, We can conclude that the value of V is $\frac{277}{3}$, the numerical simulation results are shown in Fig 3. Other cases are similar and will be omitted here.

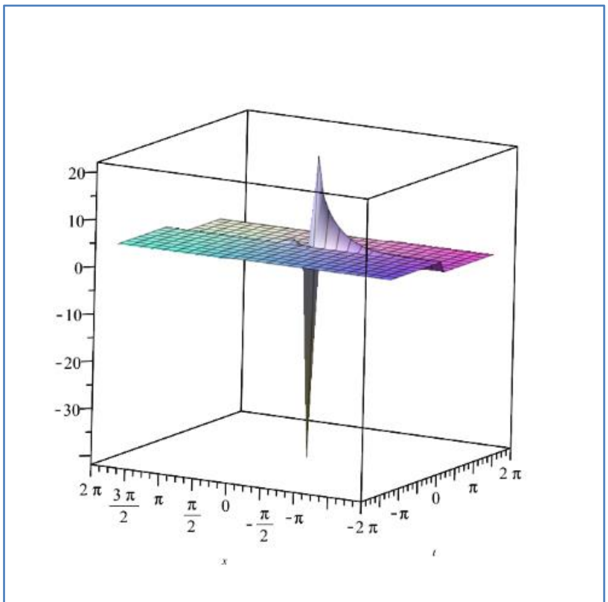

Fig-1: $\lambda^2-4\mu>0$, $\alpha=1$, $\lambda=4$, $\mu=1$, $\beta=1$, $c_1=1$, $c_2=2$

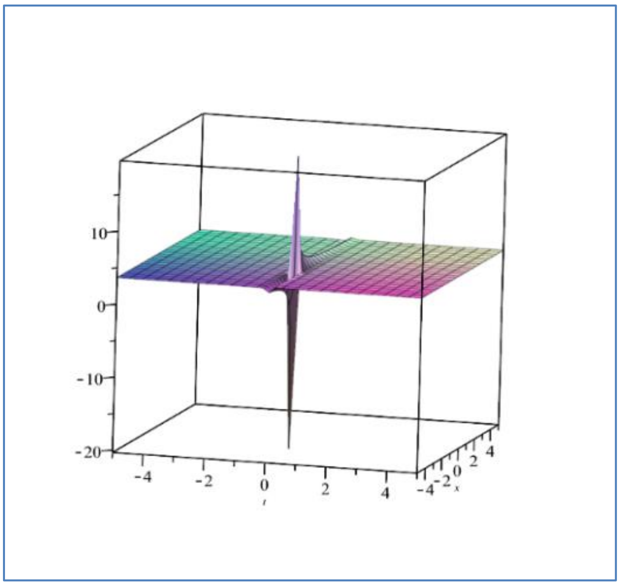

Fig-2: $\lambda^2-4\mu=0$, $\alpha=1$, $\lambda=4$, $\mu=1$, $\beta=1$, $c_1=1$, $c_2=2$

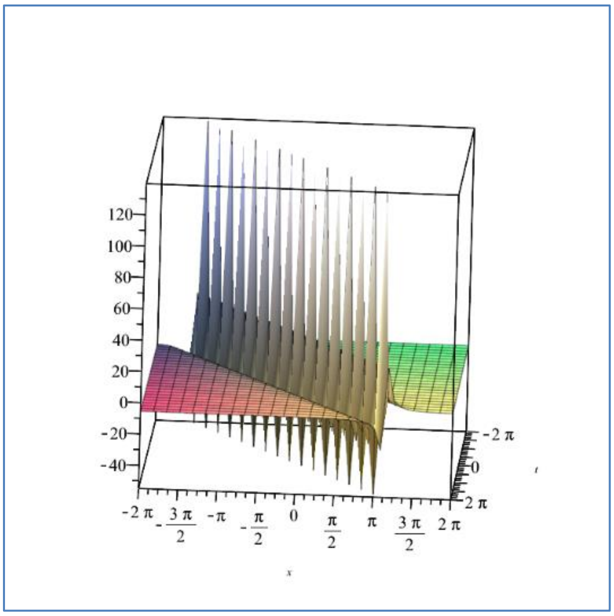

Fig-3: $\lambda^2-4\mu>0$, $\alpha=1$, $\lambda=1$, $\mu=1$, $\beta=1$, $c_1=1$, $c_2=2$

Summarize

In this paper we use the (G'/G) expansion method to find the traveling wave solution of the STOB equation, at the same time, we give numerical simulations of the solutions of the equations. In addition, the (G'/G) expansion method can solve the exact solution of nonlinear partial differential equations very quickly.

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