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Nonlinear Analysis of SSSS Thin Isotropic Rectangular Plate Using Polynomial as Shape Function

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Abstract **Review Article**

This paper presents non¬linear analysis of isotropic rectangular plate with all edges simply supported (SSSS) under a uniform distributed load using polynomial series as shape function, and under the application of Ritz method. There are several analytical tools available for analyzing isotropic rectangular plate. Among all the available analytical methods, polynomial offers a better approach. Earlier researchers on SSSS plate were, however, centered on the use trigonometric series and approximate method in form of numerical and energy method. General expressions for displacement and stress functions for large deflection of isotropic thin rectangular plate under uniformly distributed transverse loading were obtained by direct integration of Von Karman's non- linear governing differential compatibility and equilibrium equations. Total potential energy functional was formulated based on the derived displacement and stress functions. Subsequently, the formulated total potential energy functional was minimized and resulted to a general amplitude equation of the form K1Δ3+K2Δ+K3. Where K1, K2 and K3 are coefficients of amplitude equation and Δ is the deflection coefficient (factor). Newton-Raphson method was used to evaluate the deflection coefficient. Values of Δ from Timoshenko and that from present study were compared with an aspect ratios ranging from 1.0 to 1.5 with an increment of 0.1. From results obtained, the average percentage difference for the pervious and present studies is 4.0198%. The percentage difference for the plate was within acceptable limit of 0.05 or 5% level of significance in statistics. From the comparison, an excellent agreement exist between the present and previous works. Thus, this indicate applicability and validity of the polynomial function for solving exact plate bending problem.

Keywords: Nonlinear Analysis, Rectangular Thin Plates, Ritz Methods, von Karman's Equation, Variational Principles, Large deflection, Total potential energy functional.

1. INTRODUCTION

Thin plate is an indispensable structural element, which has been in use for ages. It is initially flat structural element with the thickness dimension being much smaller than its planer dimension hence yielding a "thin walled" type of structure [1]. Fundamentally, the lightweight nature of thin plate has endured its wide usage today. Included among many other applications are in aeronautical, mechanical, marine, and civil engineering [2]. Generally, plates may be classified into two main groups: Thin plates with small and large deflection, and Thick plates. This paper centered only on the thin plates with large deflection. Realistically, analyses of thin plates subjected to lateral loads are easily achieved by using a linear theory in which one assumes that the lateral displacements or deflections due to the loads are small. However, this linear analysis will not be valid if deflection of the plate is large [3]. As the deflections of thin plate gains magnitude beyond a certain level ($\frac{w}{h} > 0.3$) compared to its thickness, the Kirchhoff's theory of stiff plate (linear theory) ceases to be valid, hence the plate equation can no longer be used to analyze the deflection. Thus, the nonlinear plate theory developed by von Karman becomes inevitable for the analysis of thin plates. There are varied approaches, which can range from numerical to energy method. Earlier studies on the large deflection of thin rectangular plates were, however, centered on the use of trigonometric series and approximate methods in the form of numerical and energy methods. The present study used polynomial series to evaluate large deflection of SSSS thin plate. The previous researcher had dealt with large deflection of thin rectangular plate with all four edges simply supported only that used other approaches other than polynomial. Okodi, Yasin and Jackson, [4] carried out exact analysis of large deflection of thin rectangular plates under distributed lateral line-load, which were based on Von Karman's equations. They used MATLAB function solver, (fsolve) to get deflections caused by different loads. Vanam, Rajyalakshmi, and Inala, [5] studied Static analysis of an isotropic rectangular plate using finite element analysis (FEA). Debabrata, Prasanta, and Kashinath [6], carried out Large deflection analysis of skew plates under uniformly distributed load for mixed boundary condition. Chi-Kyung and Myung-Hwan [7] worked on Nonlinear analysis of skew plate by finite difference method. Osama [8] worked on Validation of Dynamic Relaxation (DR) method in rectangular laminates using large deflection theory. Sidda, Suresh and Vijaya [9] worked on Prediction of deflection and stresses of laminated composite plate with an artificial neural network aid. Derras, Abdelhakim, Kada, and Abdelouahed [10] studied the Non-linear analysis of functionally graded plates in cylindrical bending based on a new refined shear deformation theory. Jeom, Ju, and Bong [11] carried out Analysis of the elastic large deflection behavior for metal plates under non-uniformly distributed lateral pressure with in-plane loads. Evidently, one thing stands out in solutions of the previous researchers. They adopted trigonometric series as their shape function. Obviously, they chose appromate approach to the analyses in other to avoid the rigorous processes that are involved in the direct integration of governing differential equation. Bridging this gap, this present work has used polynomial function in Ritz method to obtain the solution for large deflection of thin rectangular SSSS thin plates.

2. NONLINEAR PLATE THEORY

$$\frac{\partial^4\phi}{\partial x^4}+\frac{2\partial^4\phi}{\partial x^2\partial y^2}+\frac{\partial^4\phi}{\partial y^4}=E\left[\left(\frac{\partial^2 w}{\partial x\partial y}\right)^2-\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2}\right]\ \ldots\ldots\ldots\ldots\ldots\ldots\ 1$$

$$\frac{\partial^4 w}{\partial x^4}+\frac{2\partial^4 w}{\partial x^2\partial y^2}+\frac{\partial^4 w}{\partial y^4}=\frac{1}{D}\left[q+h\left(\frac{\partial^2\phi}{\partial y^2}\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2\phi}{\partial x^2}\frac{\partial^2 w}{\partial y^2}-\frac{2\partial^2\phi}{\partial x\partial y}\frac{\partial^2 w}{\partial x\partial y}\right)\right]\ \ldots\ldots\ldots\ 2$$

Equations 1 and 2 define a system of nonlinear, partial differential equations, and they are referred to as the governing differential equations for large deflections theory of plates. The first equation can be described as compatibility equation and, describing the second equation in the same tone as equilibrium equation.

$$\frac{\partial^4\phi}{\propto^4\partial R^4}+\frac{2\partial^4\phi}{\propto^2\partial R^2\partial Q^2}+\frac{\partial^4\phi}{\partial Q^4}=\frac{E}{\propto^2}\left(\left(\frac{\partial^2 w}{\partial R\,\partial Q}\right)^2-\frac{\partial^2 w}{\partial R^2}\frac{\partial^2 w}{\partial Q^2}\right)\ \ldots\ldots\ldots\ldots\ldots\ 3$$

$$\frac{\partial^4 w}{\propto^4\partial R^4}+\frac{2\partial^4 w}{\propto^2\partial R^2\partial Q^2}+\frac{\partial^4 w}{\partial Q^4}=\frac{qb^4}{D}+\frac{h}{\propto^2 D}\left(\frac{\partial^2\phi}{\partial Q^2}\frac{\partial^2 w}{\partial R^2}+\frac{\partial^2\phi}{\partial R^2}\frac{\partial^2 w}{\partial Q^2}-\frac{2\partial^2\phi}{\partial R\,\partial Q}\frac{\partial^2 w}{\partial R\,\partial Q}\right)\ \ldots\ldots\ 4$$

Equations 3 and 4 are nonlinear differential equation for large deflection of plate under normal load represented in non-dimensional axes.

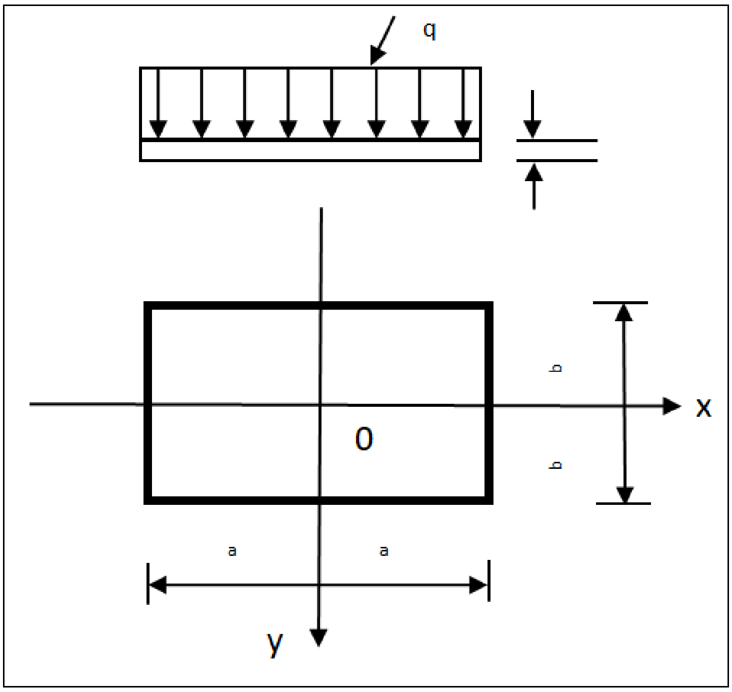

Figure 1: Typical of Rectangular Plate Uniformly Loaded and Simply Supported At the Four Edges (SSSS)

3. SHAPE FUNCTION

Assuming a displacement function of:
$W = w(x,y)\sum_{m=0}^{\infty}\sum_{n=0}^{\infty} P_m B_n x^m y^n$ ……………………… 5

Displacement functions in Equation 5 was expressed in terms of non-dimensional parameters (Q and R).

Recall that.
X = aR, and y = Bq …………………………………….….. 6

Substituting Equation 6 into Equation 5 and terminating the series at m = n = 4 gave
$W = W(R,Q) = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} P_m B_n a^m R^m b^n Q^n$ ………….... 7
$Let\ a_m = P_m a^m$ and $b_n = B_n b^n$ ……………………..….... 8

Thus:
$W = W(R,Q) = \sum_{m=0}^{4}\sum_{n=0}^{4} a_m R^m b_n Q^n$ ………………… 9

Expanding Equation 9 to 4th series over two bases, one in the x-direction and the other in y-direction and normalizing the function by makes the denominator equal to one gave:
$W(a_iR, b_iQ) = (a_0 + a_1R + a_2R^2 + a_3R^3 + a_4R^4)(b_0 + b_1Q + b_2Q^2 + b_3Q^3 + b_4Q^4)$ ……………………………………….. 10

Equation 10 gives polynomial approximation of the shape function. With the proper use of the boundary conditions of the plate, the deflection of the plate will be adequately defined.

4. BOUNDARY CONDITION

W(R) = 0; W (Q) = 0 when R = 0 and R = 1;
$W''(R) = 0$; W (Q) = 0 when R = 0 and R = 1

Applying these boundary conditions on Equation 10 gave the displacement functions:
For SS:$a_4(R - 2R^3 + R^4)$ (R-direction)
For SS:$b_4(Q - 2Q^3 + Q^4)$ (Q-direction)

Multiplying these displacement functions together gave:
$SSSS = a_4(R - 2R^3 + R^4)\, b_4(Q - 2Q^3 + Q^4)$ …………………….. 11

Factorizing Equation 11 and letting a_4b_4 be equal to Δ gave:
$w = \Delta(R - 2R^3 + R^4)(Q - 2Q^3 + Q^4)$ …………………………..... 12

Enem (2018) established particular stress function for SSSS thin rectangular isotropic plate by substituting shape function into von Karman compatibility equation and the resultant is given as:

$$\emptyset_{SSSS} = \frac{\beta\Delta^2}{6350400}[(105R^4 - 84R^6 + 24R^7 + 54R^8 - 40R^9 + 8R^{10})(105Q^4 - 84Q^6 + 24Q^7 + 54Q^8 - 40Q^9 + 8Q^{10}) - (-84R^6 + 36R^7 + 36R^8 - 30R^9 + 6R^{10})(-84Q^6 + 36Q^7 + 36Q^8 - 30Q^9 + 6Q^{10})] \quad ………..…….. 13$$

β in Equation 13 represents $\frac{E}{\alpha^2\left(\frac{1}{\alpha^4}+\frac{2}{\alpha^2}+1\right)}$

5. NON-LINEAR TOTAL POTENTIAL ENERGY

Equation 4 was considered as a functional expressing total potential energy, π of a deformed elastic body and load acting on it. Equation 4 therefore, consists of potential energy of internal forces and potential energy of external forces. However, potential energy of a body is a measure of work done by external and internal forces in moving the body from its initial position to a final one and it will be observed that all the terms in Equation 4 are in form of forces. Equation 4 was therefore converted to full potential energy by multiplying all the terms in it by displacement, *w*, hence:

$$\pi = \frac{1}{2}\int_0^1\int_0^1\left(\frac{\partial^4 w}{\alpha^4\partial R^4}.w + \frac{2\partial^4 w}{\alpha^2\partial R^2\partial Q^2}.w + \frac{\partial^4 w}{\partial Q^4}.w\right)\partial R\partial Q - \frac{1}{D}\int_0^1\int_0^1\left[qb^4.w + \frac{h}{2\alpha^2}\left(\frac{\partial^2\phi}{\partial Q^2}\frac{\partial^2 w}{\partial R^2}.w + \frac{\partial^2\phi}{\partial R^2}\frac{2\partial^2 w}{\partial Q^2}.w - \frac{2\partial^2\phi}{\partial R\partial Q}.\frac{\partial^2 w}{\partial R\partial Q}.w\right)\right]\partial R\partial Q \quad ……………………………………..…….. 14$$

Letting $w = \Delta H_1$ and $\emptyset = \Delta^2 H_2$

Where Δ is the deflection coefficient of the plate. H_1 and H_2 are the profiles of the deflection and stress function respectively. Substituting for *w* and ∅ into Equation 14 gave:

$$\pi = \frac{1}{2}\int_0^1\int_0^1\left(\frac{\partial^4\Delta H_1}{\propto^4\partial R^4}.\Delta H_1 + \frac{2\partial^4\Delta H_1}{\propto^2\partial R^2\partial Q^2}.\Delta H_1 + \frac{\partial^4\Delta H_1}{\partial Q^4}.\Delta H_1\right)\partial R\partial Q - \frac{1}{D}\int_0^1\int_0^1\left[qb^4.\Delta H_1 + \frac{h}{2\propto^2}\left(\frac{\partial^2\Delta^2 H_2}{\partial Q^2}\frac{\partial^2\Delta H_1}{\partial R^2}.\Delta H_1 + \frac{\partial^2\Delta^2 H_2}{\partial R^2}\frac{\partial^2\Delta H_1}{\partial Q^2}.\Delta H_1 - \frac{2\partial^2\Delta^2 H_2}{\partial R\partial Q}.\frac{\partial^2\Delta H_1}{\partial R\partial Q}.\Delta H_1\right)\right]\partial R\partial Q \text{ ……………………. 15}$$

Factorizing coefficient factor Δ out reduced Equation 15 to

$$\pi = \frac{\Delta^2}{2}\int_0^1\int_0^1\left(\frac{\partial^4 H_1}{\propto^4\partial R^4}.H_1 + \frac{2\partial^4 H_1}{\propto^2\partial R^2\partial Q^2}.H_1 + \frac{\partial^4 H_1}{\partial Q^4}.H_1\right)\partial R\partial Q - \frac{1}{D}\int_0^1\int_0^1\left[\Delta.qb^4.H_1 + \frac{\Delta^4 h}{2\propto^2}\left(\frac{\partial^2 H_2}{\partial Q^2}\frac{\partial^2 H_1}{\partial R^2}.H_1 + \frac{\partial^2 H_2}{\partial R^2}\frac{\partial^2 H_1}{\partial Q^2}.H_1 - \frac{2\partial^2 H_2}{\partial R\partial Q}.\frac{\partial^2 H_1}{\partial R\partial Q}.H_1\right)\right]\partial R\partial Q \text{ ………………………………………..…. 16}$$

6. MINIMIZATION OF TOTAL POTENTIAL ENERGY

Minimization of total potential energy is a very vital aspect of variational principle in which the functional was decomposed. Equation 16 was therefore minimized by differentiating total potential energy partially with respect to deflection coefficient, thereby reducing the power of deflection coefficient from four to three. The minimized total potential energy gave:

$$\frac{\partial\pi}{\partial\Delta} = \Delta\int_0^1\int_0^1\left(\frac{\partial^4 H_1}{\propto^4\partial R^4}.H_1 + \frac{2\partial^4 H_1}{\propto^2\partial R^2\partial Q^2}.H_1 + \frac{\partial^4 H_1}{\partial Q^4}.H_1\right)\partial R\partial Q - \frac{1}{D}\int_0^1\int_0^1 qb^4.H_1\partial R\partial Q - \frac{2\Delta^3 h}{D\propto^2}\int_0^1\int_0^1\left(\frac{\partial^2 H_2}{\partial Q^2}\frac{\partial^2 H_1}{\partial R^2}.H_1 + \frac{\partial^2 H_2}{\partial R^2}\frac{\partial^2 H_1}{\partial Q^2}.H_1 - \frac{2\partial^2 H_2}{\partial R\partial Q}.\frac{\partial^2 H_1}{\partial R\partial Q}.H_1\right)\partial R\partial Q \text{ ……………………….………. 17}$$

7. AMPLITUDE EQUATION FOR SSSS PLATE

Amplitude equation determines the extent of plate deflection, the larger the amplitude the larger the deflection and vice versa. In determining this amplitude, reference was made to the displacement functions in Equation 12 and stress functions in 13, hence:

$$H_1 = (R - 2R^3 + R^4)(Q - 2Q^3 + Q^4) \text{ ……………………………..……. 18}$$

$$H_2 = \frac{\beta}{6350400}[(105R^4 - 84R^6 + 24R^7 + 54R^8 - 40R^9 + 8R^{10})(105Q^4 - 84Q^6 + 24Q^7 + 54Q^8 - 40Q^9 + 8Q^{10}) - (-84R^6 + 36R^7 + 36R^8 - 30R^9 + 6R^{10})(-84Q^6 + 36Q^7 + 36Q^8 - 30Q^9 + 6Q^{10})] \text{ …………………..………. 19}$$

Substituting Equations 18 and 19 into Equation 17 and carrying out the respective differentiation and integration accordingly (it was done in parts).

The first term in the Equation after differentiation and integration gave:

$$\int_0^1\int_0^1\frac{\partial^4 H_1}{\propto^4\partial R^4}.H_1\,\partial R\partial Q = \frac{2.36190476}{\propto^4}e^{-1} \text{ ……………………………. 20}$$

The second term in the Equation after differentiation and integration gave:

$$\int_0^1\int_0^1 2\frac{\partial^4 H_1}{\propto^2\partial R^2\partial Q^2}.H_i\,\partial R\partial Q = \frac{4.71836734}{\propto^2}e^{-1} \text{……………………….. 21}$$

The third term in the Equation after differentiation and integration gave:

$$\int_0^1\int_0^1\frac{\partial^4 H_1}{\partial Q^4}.H\,\partial R\partial Q = 2.36190476e^{-1} \text{………………………….. 22}$$

The fourth term in the Equation after differentiation and integration gave:

$$\int_0^1\int_0^1 H_1\,\partial R\partial Q = 0.04 \text{ …………………………………....…….. 23}$$

The fifth term in the Equation after differentiation and integration gave:

$$\int_0^1\int_0^1\frac{\partial^4 H_2}{\partial Q^2}\frac{\partial^2 H_1}{\partial R^2}.H_1\,\partial R\partial Q = -6.386936332\beta e^{-6} \text{ …………….... 24}$$

The sixth term in the Equation after differentiation and integration gave:

$$\int_0^1\int_0^1\frac{\partial^4 H_2}{\partial R^2}\frac{\partial^2 H_1}{\partial Q^2}.H_1\,\partial R\partial Q = -6.386936332\beta e^{-6} \text{ …………….... 25}$$

The seventh term in the Equation after differentiation and integration gave:

$$\int_0^1\int_0^1 2\frac{\partial^2 H_2}{\partial R\partial Q}\frac{\partial^2 H_1}{\partial R\partial Q}.H_1\,\partial R\partial Q = 6.160747701\beta e^{-6} \text{ ……………... 26}$$

Substituting Equations 20, 21, 22, 23, 24, 25, and 26 into Equation 17 gave:

$$\left(\frac{2.36190476e^{-1}}{\propto^4}+\frac{4.71836734e^{-1}}{\propto^2}+2.36190476e^{-1}\right)\Delta-0.04\frac{qb^4}{D}-\left[(-6.38693633e^{-6})+(-6.38693633e^{-6})-(6.16074770e^{-6})\right]\frac{2\Delta^3\beta h}{\propto^2 D}=0 \quad \ldots\ldots 27$$

Recall that $\beta=\frac{E}{\propto^2\left(\frac{1}{\propto^4}+\frac{2}{\propto^2}+1\right)}$ and …………………………... 28

$$D=\frac{Eh^3}{12(1-\mu^2)} \quad \ldots\ldots 29$$

Combining Equations 28 and 29 together gave:

$$\beta=\frac{12(1-\mu^2)D}{h^3\propto^2\left(\frac{1}{\propto^4}+\frac{2}{\propto^2}+1\right)} \quad \ldots\ldots 30$$

Substituting Equation 30 into Equation 27 gave:

$$(4.544308886e^{-4})\frac{(1-\mu^2)\Delta^3}{h^2(1+2\propto^2+\propto^4)}+\left(\frac{2.36190476e^{-1}}{\propto^4}+\frac{4.71836735e^{-1}}{\propto^2}+2.36190476e^{-1}\right)\Delta-0.04\frac{qb^4}{D}=0 \quad \ldots\ldots 31$$

Recall that $w = \Delta H_1 = \Delta(R - 2R^3 + R^4)(Q - 2Q^3 + Q^4)$ …..... 32

Equation 31 gives the amplitude equation for SSSS thin rectangular plate, which is in the form of a cubic equation: $K_1\Delta^3 + K_2\Delta + K_3 = 0$, where K_1, K_2, K_3 are constants. This cubic equation was solved and deflection coefficient obtained. Newton-Raphson method was used in solving for this deflection coefficient.

8. RESULTS AND DISCUSSION

Table 1: Coefficient of Deflection for SSSS Plate with $v = 0.3$

Aspect ratio ($\propto=\frac{a}{b}$)	$\frac{W(0,0)}{qb^4/D}$			Difference in the results	
	Present	**Timoshenko**	**Cui shuang**	**Difference**	**% Difference**
1	0.0041	0.00406	0.00406235	0.00004	0.9852
1.1	0.0049	0.00485	-	0.00005	1.0309
1.2	0.0056	0.00564	0.00565053	0.00004	0.7092
1.3	0.0061	0.00638	-	0.00028	4.3887
1.4	0.0066	0.00705	-	0.00045	6.3829
1.5	0.0069	0.00772	0.00772402	0.00082	10.6218

Table 1 contains the non-dimensional deflection coefficient of SSSS plate at its center. This boundary condition has well-established exact solution for comparison. The second column in the table contains the results from the present study, the third and fourth columns contain the results from Timoshenko and Woinowsky-Krieger, 1959 and Shuang, 2007 respectively.

The results from the present study are obtained by using polynomial as shape function whereas others made use of trigonometry as their shape function. Notwithstanding the differences in approach used, the results from the present study compare favorably with the previous researchers. The fifth and sixth columns show the difference and, percentage difference obtained from the present and previous studies. It is observed that the percentage differences are within the accepted limit in statistics. The difference in the above results could be attributed to the approximation made in both methods. The results are all approximates and are quite close.

9. Discussion on the Deflection, *w* for SSSS plate

The numerical studies of deflection under different loads were carried out to determine how this plate behaves under these loads. The nonlinear analysis of SSSS thin rectangular plate will be easily evaluated when the physical and geometric properties of the plate is defined (that is knowing the values of breath, b; length, a; thickness, h; deflection coefficient, Δ and young's modulus of elasticity E). With all these values, the corresponding deflection, *w* could be determined.

Values of the parameter were substituted for each of the load and evaluated using Equations 31 and 32. The deflections obtained by these analyses for each load were plotted against the aspect ratio, which is change in its linear dimensions (a, b).

Table 1 summarizes the results obtained by geometric nonlinear analysis of SSSS thin rectangular plate under the action of three different loads. All other parameters were kept constant all through the processes except for the loads (q) and the linear dimensions (a, b) that were varied. At the first aspect ratio, the linear dimensions (a, b) were the same. Subsequently, the linear dimension (b) was stepped down gradually whereas the linear dimension (a) remains the same. These processes were repeated for each of the loads under investigation.

Observing Table 1 closely, it could be seen that the deflection reduces with an increase in the aspect ratio. This reduction was informed by the reduction in the dimensions, because at aspect ratio 1.0, the linear dimensions were 1000mm by 1000mm but from 1.1 down the linear dimension, b started reducing proportionally. The implication of this is that the deflection is a function of the linear dimension, and that a refined deflection is obtained when one of the linear dimension is relatively smaller than the other.

In collaboration, Figure 2 strengthens the results in Table 1. For clarity purposes, different load cases were separated by colours. The red line depicts load case with highest load. Looking at the graph closely, it could be seen that deflection reduces as the aspect ratio is increasing as explained earlier.

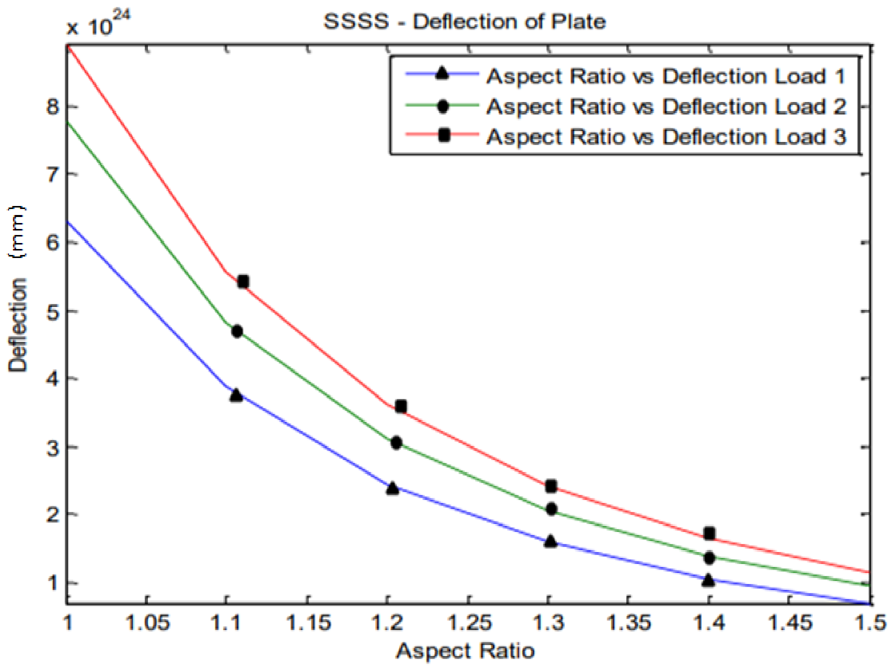

Figure 2: Relationship between the deflection and aspect ratio of SSS plate

10. CONCLUSION

Polynomial series has been used effectively in the paper as a shape function to determine the large deflection of thin rectangular isotropic plate against the traditional approach of using trigonometric series. With this polynomial, particular stress and displacement function SSSS thin plate were established. In all sincerity, this approach (polynomial) to the solution of large deflection of thin plate has satisfied all necessaries boundary conditions of the plate and the results obtained with it are in agreement with the results obtained by the previous researcher. Therefore, it can be concluded that this present approach can the handle the analyses of large deflection of plates and its application could be easier.

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