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Research Article

Degree Distance and Eccentric Distance Sum of Certain Special Molecular Graphs

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Abstract: In this paper, we determine the degree distance and eccentric distance sum of *r*-corona graphs of fan graph, wheel graph, gear fan graph, and gear wheel graph.
Keywords: Chemical graph theory, Organic molecules, Degree distance, Eccentric distance sum, Fan graph, Wheel graph, Gear fan graph, Gear wheel graph, *r*-corona graph

INTRODUCTION

Wiener index, edge Wiener index, Hyper-wiener index, degree distance and eccentric distance sum are introduced to reflect certain structural features of organic molecules. Several papers contributed to determine the Wiener index or Hyper-wiener index of special graphs (See Yan et al., [1], Gao and Shi [2] and [3] for more detail). LetP_n and $C_n$be path and cycle with *n* vertices. The graph $F_n=\{v\}\vee P_n$is called a fan graph and the graph $W_n=\{v\}\vee C_n$is called a wheel graph. Graph $I_r(G)$ is called *r*- crown graph of *G* which splicing *r* hang edges for every vertex in *G*. By adding one vertex in every two adjacent vertices of the fan path $P_n$of fan graph F_n, the resulting graph is a subdivision graph called gear fan graph, denote as $\tilde{F}_n$. By adding one vertex in every two adjacent vertices of the wheel cycle $C_n$of wheel graph W_n, The resulting graph is a subdivision graph, called gear wheel graph, denoted as $\tilde{W}_n$.

The graphs considered in this paper are simple and connected. Let $\deg(v)$ be the degree of vertex *v*, and $D_G(v)$ (or $D(v)$) be the sum of all distances from *v*.Namely, $D(v)=\sum_{u\in V(G)} d(u,v)$. The eccentricity $ec(u)$ of vertex $u\in V(G)$ is the maximum distance between *u* and any other vertex in *G*. The parameter $DD(G)$ is called the degree distance of molecular *G* and it was introduced by Dobrynin and Kochetova [4] and Gutman [5],

$$DD(G)=\sum_{\{u,v\}\subseteq V(G)} (\deg(u)+\deg(v))d(u,v)=\sum_{v\in V(G)} \deg(v)D(v).$$

The eccentric distance sum (EDS) $\xi^d(G)$ of a molecular graph *G* is defined as

$$\xi^d(G)=\sum_{\{u,v\}\subseteq V(G)} (ec(v)+ec(u))d(u,v)=\sum_{v\in V(G)} ec(v)D(v).$$

Several papers contributed to determine the eccentric distance sum of special molecular graphs. Ilicet. al., [6] characterized the extremal trees and graphs with maximal eccentric distance sum. Azari and Iranmanesh [7] presented explicit formulas for computing the eccentric-distance sum of the most important graph operations such as the Cartesian product, join, composition, disjunction, symmetric difference, cluster and corona product of graphs. Genget. al., [8] presented the sharp upper and lower bounds on the eccentric distance sums among the *n*-vertex trees with k leaves. Qu and Yu [9] characterized the chain hexagonal cactus with the minimal and the maximal eccentric distance sum among all chain hexagonal cacti of length n, respectively. Moreover, they presented exact formulas for eccentric distance sum of two types of hexagonal cacti. Rodriguez [10] proposed formulas for the eccentric distance sum of distance-regular hypergraphs in terms of its intersection array. Hua et. al., [11] obtained some further results on eccentric distance sum.

In this paper, we present the degree distance of $I_r(F_n)$, $I_r(W_n)$, $I_r(\tilde{F}_n)$ and $I_r(\tilde{W}_n)$. Also, the eccentric distance sum of $I_r(F_n)$, $I_r(W_n)$, $I_r(\tilde{F}_n)$ and $I_r(\tilde{W}_n)$ are derived.

DEGREE DISTANCE

Theorem1. $DD(I_r(F_n))=[r+n+2rn](r+n)+2[(5r+2)+(2+3r)(n-2)](2+r)$
$+(n-2)[(7r+3)+(2+3r)(n-3)](3+r)+[1+2(r-1)+2n+3nr]r+[2r[1+2(r-1)+2(2+3r)$
$+(3+4r)(n-2)]+(n-2)r[1+2(r-1)+3(2+3r)+(3+4r)(n-3)]]$.

Proof. Let $P_n=v_1v_2\ldots v_n$ and the r hanging vertices of v_i be v_i^1, v_i^2,…, v_i^r ($1\le i\le n$). Let v be a vertex in F_n beside P_n, and the r hanging vertices of v be v^1, v^2, …, v^r. By the definition of degree distance, we have

$$DD(I_r(F_n))=D(v)\deg(v)+\sum_{i=1}^{n}D(v_i)\deg(v_i)+\sum_{i=1}^{r}D(v^i)\deg(v^i)+\sum_{i=1}^{n}\sum_{j=1}^{r}D(v_i^j)\deg(v_i^j)$$
$$=[r+n+2rn](r+n)+2[(5r+2)+(2+3r)(n-2)](2+r)+(n-2)[(7r+3)+(2+3r)(n-3)](3+r)$$
$$+[1+2(r-1)+2n+3nr]r+[2r[1+2(r-1)+2(2+3r)+(3+4r)(n-2)]+(n-2)r[1+2(r-1)$$
$$+3(2+3r)+(3+4r)(n-3)]].$$

Theorem2. $DD(I_r(W_n))=[r+n+2rn](r+n)+n[(7r+3)+(2+3r)(n-3)](3+r)+[1+2(r-1)+2n+3nr]r+$
$nr[1+2(r-1)+3(2+3r)+(3+4r)(n-3)]$.

Proof. Let $C_n=v_1v_2\ldots v_n$ and v_i^1, v_i^2,…, v_i^r be the r hanging vertices of v_i($1\le i\le n$). Let v be a vertex in W_n beside C_n, and v^1, v^2, …, v^r be the r hanging vertices of v. By the definition of degree distance, we have

$$DD(I_r(W_n))=D(v)\deg(v)+\sum_{i=1}^{n}D(v_i)\deg(v_i)+\sum_{i=1}^{r}D(v^i)\deg(v^i)+\sum_{i=1}^{n}\sum_{j=1}^{r}D(v_i^j)\deg(v_i^j)$$
$$=[r+n+2rn](r+n)+n[(7r+3)+(2+3r)(n-3)](3+r)+[1+2(r-1)+2n+3nr]r+$$
$$nr[1+2(r-1)+3(2+3r)+(3+4r)(n-3)].$$

Theorem3. $DD(I_r(\tilde{F}_n))=[r+n+2rn+(n-1)(2+3r)](r+n)+$
$2[(3r+1)+(2+3r)(n-1)+(1+2r)+(n-2)(3+4r)](2+r)$
$+(n-2)[(3r+1)+(2+3r)(n-1)+2(1+2r)+(n-3)(3+4r)](3+r)+[(2r-1)+2n+3nr$
$+(n-1)(4r+3)]r+[2r[1+2(r-1)+(2+3r)+(3+4r)(n-1)+(2+3r)+(n-2)(4+5r)]$
$+(n-2)r[1+2(r-1)+(2+3r)+(3+4r)(n-1)+2(2+3r)+(n-3)(4+5r)]]+[r+2(1+2r)$
$+(2+3r)+(n-2)(3+4r)](2+r)+(n-1)r[1+2(r-1)+2(2+3r)+(3+4r)+(4+5r)(n-2)]$.

Proof. Let $P_n=v_1v_2\ldots v_n$ and $v_{i,i+1}$ be the adding vertex between v_i and v_{i+1}. Let v_i^1, v_i^2,…, v_i^r be the r hanging vertices of v_i($1\le i\le n$). Let $v_{i,i+1}^1$, $v_{i,i+1}^2$,…, $v_{i,i+1}^r$ be the r hanging vertices of $v_{i,i+1}$ ($1\le i\le n$-1). Let v be a vertex in F_n beside P_n, and the r hanging vertices of v be v^1, v^2, …, v^r. By virtue of he degree distance, we get

$$DD(I_r(\tilde{F}_n))=D(v)\deg(v)+\sum_{i=1}^{n}D(v_i)\deg(v_i)+\sum_{i=1}^{r}D(v^i)\deg(v^i)+\sum_{i=1}^{n}\sum_{j=1}^{r}D(v_i^j)\deg(v_i^j)+$$
$$\sum_{i=1}^{n-1}D(v_{i,i+1})\deg(v_{i,i+1})+\sum_{i=1}^{n-1}\sum_{j=1}^{r}D(v_{i,i+1}^j)\deg(v_{i,i+1}^j)$$
$$=[r+n+2rn+(n-1)(2+3r)](r+n)+2[(3r+1)+(2+3r)(n-1)+(1+2r)+(n-2)(3+4r)](2+r)$$
$$+(n-2)[(3r+1)+(2+3r)(n-1)+2(1+2r)+(n-3)(3+4r)](3+r)+[(2r-1)+2n+3nr$$

$+(n-1)(4r+3)]r\ [2r[1+2(r-1)+(2+3r)+(3+4r)(n-1)+(2+3r)+(n-2)(4+5r)]+$
$+(n-2)r[1+2(r-1)+(2+3r)+(3+4r)(n-1)+2(2+3r)+(n-3)(4+5r)]]+[r+2(1+2r)$
$+(2+3r)+(n-2)(3+4r)](2+r)+(n-1)r[1+2(r-1)+2(2+3r)+(3+4r)+(4+5r)(n-2)]$.

Theorem4. $DD(I_r(\tilde{W}_n))=[r+n+2rn+n(2+3r)](r+n)+$
$n[(3r+1)+(2+3r)(n-1)+2(1+2r)+(n-2)(3+4r)](3+r)+[1+2(r-1)+2n+3nr+n(4r+3)]r+$
$nr[1+2(r-1)+(2+3r)+(3+4r)(n-1)++2(2+3r)+(n-2)(4+5r)]]+[(2+5r)+(2+3r)$
$+(n-1)(3+4r)](2+r)+nr[1+2(r-1)+2(2+3r)+2(3+4r)+(4+5r)(n-2)]$.

Proof. Let $C_n=v_1v_2\ldots v_n$ and v be a vertex in W_n beside C_n. $v_{i,i+1}$ be the adding vertex between v_i and v_{i+1}. Let v^1, v^2, …, v^r be the r hanging vertices of v and v_i^1, v_i^2,…, v_i^r be the r hanging vertices of $v_i(1\le i\le n)$. Let $v_{n,n+1}=v_{1,n}$ and $v_{i,i+1}^1$, $v_{i,i+1}^2$,…, $v_{i,i+1}^r$ be the r hanging vertices of $v_{i,i+1}$ $(1\le i\le n)$. In view of the definition of degree distance, we deduce

$$DD(I_r(\tilde{W}_n))=D(v)\deg(v)+\sum_{i=1}^{n}D(v_i)\deg(v_i)+\sum_{i=1}^{r}D(v^i)\deg(v^i)+\sum_{i=1}^{n}\sum_{j=1}^{r}D(v_i^j)\deg(v_i^j)+$$
$$\sum_{i=1}^{n}D(v_{i,i+1})\deg(v_{i,i+1})+\sum_{i=1}^{n}\sum_{j=1}^{r}D(v_{i,i+1}^j)\deg(v_{i,i+1}^j)$$
$$=[r+n+2rn+n(2+3r)](r+n)+n[(3r+1)+(2+3r)(n-1)+2(1+2r)+(n-2)(3+4r)](3+r)+$$
$$[1+2(r-1)+2n+3nr+n(4r+3)]r+nr[1+2(r-1)+(2+3r)+(3+4r)(n-1)+$$
$$+2(2+3r)+(n-2)(4+5r)]]+[r+2(1+2r)+(2+3r)+(n-1)(3+4r)](2+r)+$$
$$nr[1+2(r-1)+2(2+3r)+2(3+4r)+(4+5r)(n-2)].$$

ECCENTRIC DISTANCE SUM

Theorem5. $\xi^d(I_r(F_n))=2[r+n+2rn]+3r[(2r-1)+2n+3nr]+$
$6[(5r+2)+(2+3r)(n-2)]+3(n-2)[(7r+3)+(2+3r)(n-3)]+[8r[1+2(r-1)+2(2+3r)\ +(3+4r)(n-2)]+$
$4(n-2)r[1+2(r-1)+3(2+3r)+(3+4r)(n-3)]]$.

Proof. By the definition of eccentric distance sum, we have

$$\xi^d(I_r(F_n))=D(v)ec(v)+\sum_{i=1}^{n}D(v_i)ec(v_i)+\sum_{i=1}^{r}D(v^i)ec(v^i)+\sum_{i=1}^{n}\sum_{j=1}^{r}D(v_i^j)ec(v_i^j)$$
$$=2[r+n+2rn]+6[(5r+2)+(2+3r)(n-2)]+3(n-2)[(7r+3)+(2+3r)(n-3)]+3r[(2r-1)+2n+3nr]+$$
$$[2r\times4[1+2(r-1)+2(2+3r)+(3+4r)(n-2)]+$$
$$4(n-2)r[1+2(r-1)+3(2+3r)+(3+4r)(n-3)]].$$

Theorem6. $\xi^d(I_r(W_n))=[r+n+2rn]2+3n[(7r+3)+(2+3r)(n-3)]+3r[1+2(r-1)+2n+3nr]+$
$4nr[1+2(r-1)+3(2+3r)+(3+4r)(n-3)]$.

Proof. By the definition of eccentric distance sum, we have

$$\xi^A(I_r(W_n))=D(v)ec(v)+\sum_{i=1}^{n}D(v_i)ec(v_i)+\sum_{i=1}^{r}D(v^i)ec(v^i)+\sum_{i=1}^{n}\sum_{j=1}^{r}D(v_i^j)ec(v_i^j)$$
$$=[r+n+2rn]2+n[(7r+3)+(2+3r)(n-3)]\times3+[1+2(r-1)+2n+3nr]r\times3+$$
$$4nr[1+2(r-1)+3(2+3r)+(3+4r)(n-3)].$$

Theorem 7. $\xi^d(I_r(\tilde{F}_n))=[r+n+2rn+(n-1)(2+3r)]3+8[(3r+1)+(2+3r)(n-1)+(1+2r)$

$$+(n-2)(3+4r)] +4(n-2)[(3r+1)+(2+3r)(n-1)+2(1+2r)+(n-3)(3+4r)]+[(2r-1)+2n+3nr$$
$$+(n-1)(4r+3)]4r +[10r[1+2(r-1)+(2+3r)+(3+4r)(n-1)$$
$$+(2+3r)+(n-2)(4+5r)] +5(n-2)r[1+2(r-1) +(2+3r)+(3+4r)(n-1)$$
$$+2(2+3r)+(n-3)(4+5r)]]+[r+2(1+2r) +(2+3r)+(n-2)(3+4r)]5+$$
$$6(n-1)r[1+2(r-1)+2(2+3r)+(3+4r)+(4+5r)(n-2)].$$

Proof. By virtue of the definition ofeccentric distance sum, we get

$$\xi^d(I_r(\tilde{F}_n)) = D(v)ec(v) + \sum_{i=1}^{n} D(v_i)ec(v_i) + \sum_{i=1}^{r} D(v^i)ec(v^i) + \sum_{i=1}^{n}\sum_{j=1}^{r} D(v_i^j)ec(v_i^j) + \sum_{i=1}^{n-1} D(v_{i,i+1})ec(v_{i,i+1}) +$$
$$\sum_{i=1}^{n-1}\sum_{j=1}^{r} D(v_{i,i+1}^j)ec(v_{i,i+1}^j)$$
$$=[r+n+2rn+(n-1)(2+3r)]3+8[(3r+1)+(2+3r)(n-1)+(1+2r)+(n-2)(3+4r)]$$
$$+4(n-2)[(3r+1)+(2+3r)(n-1)+2(1+2r)+(n-3)(3+4r)]+[(2r-1)+2n+3nr +(n-1)(4r+3)]4r+$$
$$[10r[1+2(r-1)+(2+3r)+(3+4r)(n-1)+(2+3r)+(n-2)(4+5r)] +5(n-2)r[1+2(r-1)$$
$$+(2+3r)+(3+4r)(n-1)+2(2+3r)+(n-3)(4+5r)]]+[r+2(1+2r) +(2+3r)+(n-2)(3+4r)]5+$$
$$6(n-1)r[1+2(r-1)+2(2+3r)+(3+4r)+(4+5r)(n-2)].$$

Theorem8. $\xi^d(I_r(\tilde{W}_n)) = [r+n+2rn+n(2+3r)]3+4n[(3r+1)+(2+3r)(n-1) +2(1+2r)+(n-2)(3+4r)]+$

$$[1+2(r-1)+2n+3nr+n(4r+3)]4r+5nr[1+2(r-1)+(2+3r)+(3+4r)(n-1)$$
$$+2(2+3r)+(n-2)(4+5r)]]+5[r+2(1+2r)$$
$$+(2+3r)+(n-1)(3+4r)]+6nr[1+2(r-1)+2(2+3r)+2(3+4r)+(4+5r)(n-2)].$$

Proof. In view of the definition of eccentric distance sum, we deduce

$$\xi^d(I_r(\tilde{W}_n)) = D(v)ec(v) + \sum_{i=1}^{n} D(v_i)ec(v_i) + \sum_{i=1}^{r} D(v^i)ec(v^i) + \sum_{i=1}^{n}\sum_{j=1}^{r} D(v_i^j)ec(v_i^j) + \sum_{i=1}^{n} D(v_{i,i+1})ec(v_{i,i+1}) +$$
$$\sum_{i=1}^{n}\sum_{j=1}^{r} D(v_{i,i+1}^j)ec(v_{i,i+1}^j)$$
$$=[r+n+2rn+n(2+3r)]3+n[(3r+1)+(2+3r)(n-1)+2(1+2r)+(n-2)(3+4r)]4+$$
$$[1+2(r-1)+2n+3nr+n(4r+3)]4r+5nr[1+2(r-1)+(2+3r)+(3+4r)(n-1)$$
$$+2(2+3r)+(n-2)(4+5r)]]+[r+2(1+2r)+(2+3r)+(n-1)(3+4r)]5+$$
$$6nr[1+2(r-1)+2(2+3r)+2(3+4r)+(4+5r)(n-2)].$$

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