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Pre-stack Reverse Time Migration Method Based on Staggered Grid High Order Finite Difference

GAO Xin-cheng, LI Chun-sheng
Modern Education Technique Center of Northesat Petroleum University, Hei Long Jiang, 163318, China

***Corresponding Author:**
GAO Xin-cheng
Email: gxc@nepu.edu.cn

Abstract: This paper used one order stress - velocity wave equations and staggered grid high order finite difference algorithm to solve the problem. Using Random boundary condition and PML boundary condition to proceed the forward continuation of seismic wave field, which can not only reduce the storage of wave field, but also ensure the accuracy of wave field data. At the same time, we use GPU parallel computing to improve computation efficiency. The experimental results show that, the pre-stack reverse time migration algorithm can form clear image for big stratigraphic dip and migration aperture, and accurate image for complex tectonic geologic body.
Keywords: Reverse time migration; High order finite difference; Random boundary conditions; PML.

INTRODUCTION

In 1983, Whitmore et al proposed the reverse time migration method at the SEG annual international meeting whose main idea is use the reverse time migration results to pick up the horizon[1], update the speed model, iteration speed. Subsequently, McMechan et al[2] and Loewenthal et al[3] also made some in-depth studies on the reverse time migration, and proposed some ideas and methods, but these studies are post stack migration aspects. Moreover, the reverse time migration algorithm can produce huge amount of computation and storage, and the early hardware facilities can’t finish, which seriously hinders the application of reverse time migration method in actual production. In recent years, with the rapid development of computer hardware technology, high performance computing technology such as parallel computing, cluster computing and GPU acceleration are applied to seismic exploration field which greatly improves the computation efficiency in seismic data processing, and greatly promotes the development of reverse time migration technology[4]. At present, the Pre-stack reverse migration is the most accurate imaging method in seismic data migration, and has become the hotspot in the field of earth exploration.

In this paper, the staggered grid high order finite difference method is used to solve the first-order velocity stress wave equation. The seismic wave field forward process is accomplished by random boundary conditions and PML boundary conditions which can not only reduce the amount of the storage, but also ensure the accuracy of the wave field data. The low frequency noise is suppressed by the High pass filtering method and Laplacian operator filtering method. The computational efficiency is improved by using GPU accelerating technology, and the imaging results are analyzed by complex model experiments.

STAGGERED GRID HIGH ORDER FINITE DIFFERENCE FORWORD SIMULATION

The idea of staggered grid difference scheme is that the derivative of several groups of variables is dispersed in different locations of the grid , and solving the derivative of variable to time bases on transforming it to space, and using the relationship between them to carry out iterative calculations. Compared with the rule grid, it makes the local accuracy improve and the convergence speed is faster. In addition, because the first-order velocity stress wave equation contains only one order derivative, it can improve the accuracy of numerical simulation and the order of finite difference approximation. So this paper uses the first-order velocity stress wave equation, and uses the staggered grid high order finite difference method to solve the problem.

First-Order Stress Velocity Equation

The form of the first-order stress velocity equation of two-dimensional acoustic wave equation in isotropic medium can be expressed as this:

$$\left.\begin{aligned}\frac{\partial p}{\partial t} &= \rho v^2 (\frac{\partial v_x}{\partial x} + \frac{\partial v_z}{\partial z}) \\ \frac{\partial v_x}{\partial t} &= \frac{1}{\rho}\frac{\partial p}{\partial x} \\ \frac{\partial v_z}{\partial t} &= \frac{1}{\rho}\frac{\partial p}{\partial z}\end{aligned}\right\} \quad (1)$$

Among them, v_x, v_z are velocity of particles, u is normal stress, ρ is density, v_p is longitudinal wave velocity.

The speed and stress components are shown in the staggered position of the grid points as shown in Figure 1, when use staggered grid high order finite difference to solve the first-order velocity stress wave equation.

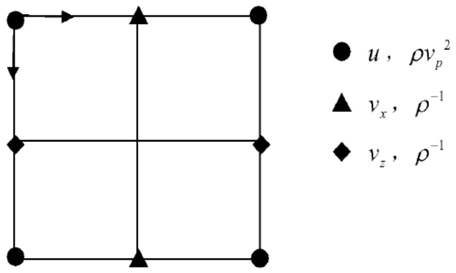

Fig-1: A sketch map of the velocity and stress staggered distribution on the grid points

Staggered Grid High Order Finite Difference

According to the approximate formula derivation of higher order difference of finite difference method in time and space, we find that the high order difference of time can greatly increases the amount of calculation and the time frequency dispersion in the time sampling of seismic exploration is not obvious, and increasing the time step to reduce the computation will lead to the instability of the algorithm. So, based on the comprehensive consideration of the accuracy and the operational efficiency, this paper uses the form of time two order and space higher order difference. The time two order and space 2N order finite difference format is given below. Supposing $P_{i,j}^{k+1/2}$, $V_{i+1/2,j}^{k}$, $W_{i,j+1/2}^{k}$ are discrete value of stress p, velocity v_x, v_z, the difference form of the equation is[5]：

$$P_{i,j}^{k+1/2} = P_{i,j}^{k-1/2} - \frac{\Delta t \rho v_p^2}{\Delta x}\left\{\sum_{n=1}^{N} C_n^{(N)}\left[V_{i+(2n-1)/2,j}^{k} - V_{i-(2n-1)/2,j}^{k}\right]\right\} - \frac{\Delta t \rho v_p^2}{\Delta z}\left\{\sum_{n=1}^{N} C_n^{(N)}\left[W_{i,j+(2n-1)/2}^{k} - W_{i,j-(2n-1)/2}^{k}\right]\right\} \quad (2)$$

$$V_{i+1/2,j}^{k} = V_{i+1/2,j}^{k-1} - \frac{\Delta t}{\Delta x \rho}\left\{\sum_{n=1}^{N} C_n^{(N)}\left[P_{i+1/2+(2n-1)/2,j}^{k-1/2} - P_{i+1/2-(2n-1)/2,j}^{k-1/2}\right]\right\} \quad (3)$$

$$W_{i,j+1/2}^{k} = W_{i,j+1/2}^{k-1} - \frac{\Delta t}{\Delta x \rho}\left\{\sum_{n=1}^{N} C_n^{(N)}\left[P_{i,j+1/2+(2n-1)/2}^{k-1/2} - P_{i,j+1/2-(2n-1)/2}^{k-1/2}\right]\right\} \quad (4)$$

Among them, $P_{i,j}^{k+1/2}$ represents the value of $p(x,z,t+\frac{\Delta t}{2})$, $V_{i+1/2,j}^{k}$ represents the value of $v_x(x+\frac{\Delta x}{2},z,t)$, $W_{i,j+1/2}^{k}$ represents the value of $v_z(x,z+\frac{\Delta z}{2},t)$, Δx and Δz respectively represent grid spacing of x and z direction, Δt represents time step.

Differential Stability Condition

The stability condition of high order difference of the staggered grid for first-order elastic wave equation in the two-dimensional case[6]：

$$0 \le \sum_{m=1}^{M} \frac{(-1)^{m-1}}{(2m-1)!} L_x^{2m} d^{2m} \le 1 \quad 0 \le \sum_{m=1}^{M} \frac{(-1)^{m-1}}{(2m-1)!} L_z^{2m} d^{2m} \le 1 \tag{5}$$

Among them： $L_x = \Delta t \sqrt{\frac{C_{11}}{\rho \Delta x^2} + \frac{C_{44}}{\rho \Delta z^2}}$, $L_z = \Delta t \sqrt{\frac{C_{44}}{\rho \Delta x^2} + \frac{C_{33}}{\rho \Delta z^2}}$, $d = \sum_{n=1}^{N} C_n^N (-1)^{n-1}$

Boundary Conditions

PML boundary conditions

The idea of PML is that adding absorption and attenuation layer of a certain thickness in the boundary of computational region, which lets wave of the absorbing layer of the boundary is gradually attenuated by the propagation distance ,and do not produce reflection at the edge of the boundary. So, the effect of eliminating the boundary reflex can be achieved[7] . PML boundary condition principle as shown in Figure 2, the attenuation factor is equal to zero in effective computation area O, the attenuation factor is greater than zero in absorption layer A、B、C, the cosine attenuation factor of the directions of x and z can be expressed as:

When x direction, d_z =0.

$$d_x = R(1-\cos[\frac{\pi(pml - x)}{2pml}]) \qquad x = 1,2,3 \ldots pml \tag{6}$$

When z direction, d_x =0.

$$d_z = R(1-\cos[\frac{\pi(pml - z)}{2pml}]) \qquad z = 1,2,3 \ldots pml \tag{7}$$

Among formula, d_x and d_z are the attenuation factor in the direction of x and z. R is amplitude factor of attenuation.

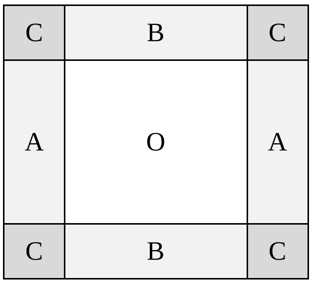

Fig- 2: Schematic diagram of PML

Random boundary conditions

The idea of random boundary conditions is to eliminate the coherence of the artificial boundary reflected wave, so that the boundary reflection can not be imaged[8]. The constructed random boundary functions are as follows:

$$v(x,z) = V(x,z) - r * d \tag{8}$$

Among them, $v(x,z)$ is stochastic velocity function of boundary points, $V(x,z)$ is primitive speed function of boundary points, r is random number, d is space distance between velocity points and inner boundary. The randomness of the random function varies with the distance, and it is proportional to the distance of the point and the normal range. That is, the distance from the normal region is farther, the randomness of the speed is larger.

IMPLEMENTATION AND ANALYSIS OF PRE-STACK REVERSE TIME MIGRATION

Basic Principle of Reverse Time Migration

The imaging principle of reverse time migration comes from the time consistency imaging principle of Claerbout, that is imaging in the point which the time of arrival of downlink wave within stratum and the time of arrival of the upper traveling wave are consistent. Pre-stack reverse time migration is mainly for the extension and imaging of the wave field, and it is concretely divided into three steps:

- Step 1: The wave field at the source propagates forward along time direction from zero moment to biggest moment, saving wave field information at every moment;

- Step 2: The wave field at detection point propagates reverse along time direction from biggest moment to zone moment, saving wave field information at every moment;
- Step 3: Reading two wave fields at the same time, and imaging operation is done by imaging conditions.

The most direct way to achieve time consistency imaging principle is zero delay cross-correlation between source and receiver, that is use forward wave field $S(x,z,t)$ and reverse push wave field $R(x,z,t)$ to taking cross-correlation, specific formula can be expressed as[9]:

$$I(x,z) = \sum_t S(x,z,t)R(x,z,t) \tag{9}$$

Among formula, x and z are expressed respectively in horizontal and depth direction coordinates; Function $S(x,z,t)R(x,z,t)$ indicates that the whole wave field in the plane is imaged at a time, the summation explains that the image in imaging space is the superposition of the images at each time.

Pre-Stack Reverse Time Migration Algorithm

During the whole process of pre-stack reverse time migration, in consideration of the factors such as the amount of computation, the amount of memory and the low frequency noise. In this paper, first, the random boundary conditions and PML completely absorbing boundary conditions are used in the forward wave field; second, GPU accelerating technology is adopted in the whole process of calculation[10]; finally, the High pass filtering and Laplacian operator filtering method are used to remove noise after imaging[11 12].

Specific algorithm steps are as follows:

- Step 1: Using the random boundary conditions, the wave field are extrapolated by using wave equation finite difference method forward, using GPU accelerating technology to calculate it until $T_{\max}$ moment, saving the last two time values of wave field.
- Step 2: Using PML boundary conditions, the wave field are extrapolated by using wave equation finite difference method forward, using GPU accelerating technology to calculate it until $T_{\max}$ moment, saving the detection point data for each time.
- Step 3: Reading the last two time values of wave field , the wave field are extrapolated by using wave equation finite difference method reversely, using GPU accelerating technology to calculate it until zone moment, calculating the wave field values for each moment.
- Step 4: Reading the detection point data, the wave field are extrapolated by using wave equation finite difference method reversely, using GPU accelerating technology to calculate it until zone moment, calculating the wave field values for each moment.
- Step 5: Reading two wave field data of Step 3 and Step 4 at the same time, using cross-correlation imaging condition to image from $T_{\max}$ moment to zone moment.
- Step 6: The cyclic calculation of the multi-shots data are added to N_{shot} , completing the superposition of imaging results of multi-shots.
- Step 7: The filtering method is used to suppress the low frequency noise of the imaging results.
- Step 8: Output imaging results.

EXPERIMENT AND ANALYSIS

In this paper, the finite difference numerical algorithm of time two order and space sixteen order is used to accomplish the seismic wave field calculation of following model, and according to the principle of time coherence, the imaging points are extracted from the cross-correlation imaging conditions. Selecting the Salt-2D speed model as shown in Figure 3, the model data were 380 shots, grid size dx=dz=25m, grid points nx=650, nz=150, the Ricker wavelet of 16Hz is adopted in the source.

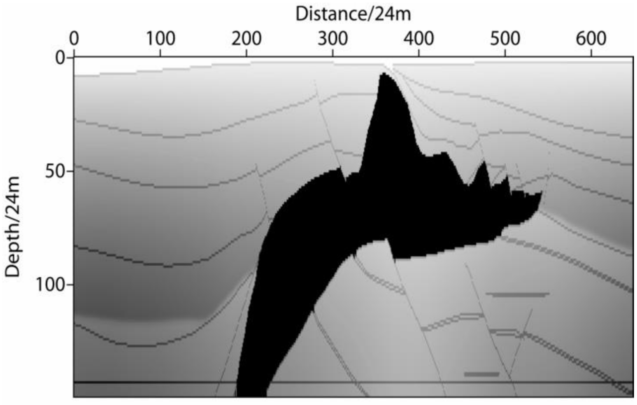

Fig-3: Salt-2D velocity model

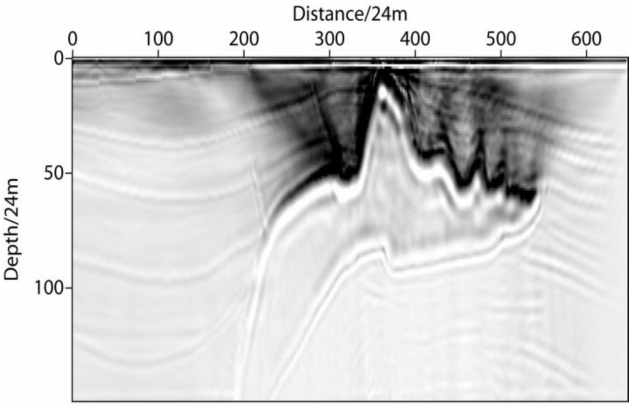

Fig- 4: RTM result for Salt-2D model

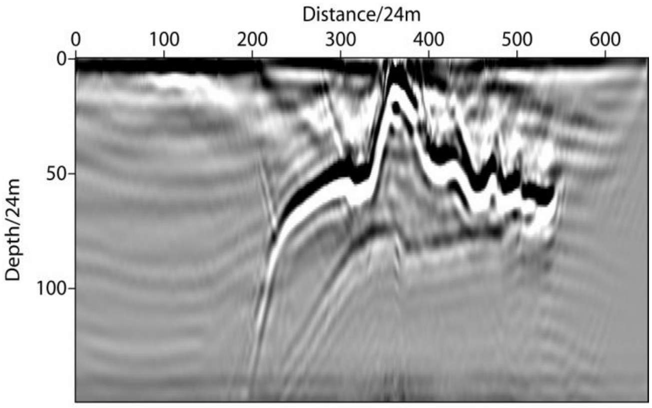

Fig-5: High pass filtering method to suppress noise

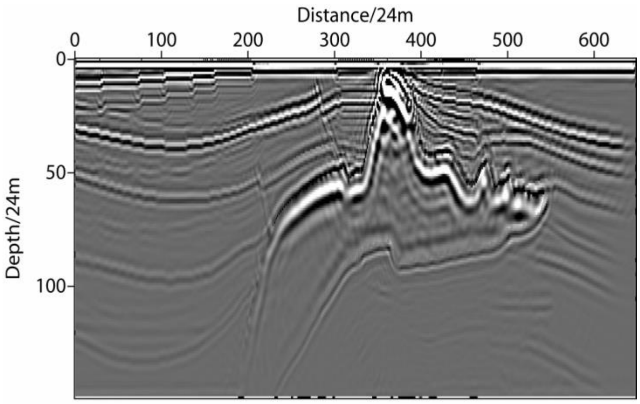

Fig-6: Laplacian method to suppress the noise

Figure 4 is the result of the reverse time migration, and we can see the large amount of low frequency noise, especially the shallow noise. The results of noise suppression by high pass filtering method are shown in Figure 5, and we can see that the horizontal stratification is not clear, the tomography is not obvious, and the lower interface of salt dome can't be imaged accurately. The imaging results after using Laplacian operator to suppress noise are shown in Figure 6. We can see that it can effectively suppress the low frequency noise, the upper and lower interface of salt dome can both be imaged accurately, stratification is clear, the effect of tomography is better.

CONCLUSION

According to the study of pre-stack reverse time migration method, this paper find that the reverse time migration method based on two-way wave equation is not affected by the stratigraphic dip and migration aperture, it can form accurate imaging for complex tectonic geologic body. This way is very simple and the imaging accuracy is high. Using random boundary conditions can greatly reduce the storage, and the results of reverse time imaging of PML boundary condition are more accurate and no noise jamming. GPU accelerating technology can largely improves computation efficiency. Using Laplacian operator to suppress low-frequency noise can obtain high precision imaging results. So, reverse time migration method use full wave field to image, there is no restriction on the direction of the wave propagation in the shot. It can image for variety of seismic wave, such as prism wave, rotary wave and reflected transmission wave, and it can also obtain good imaging effect for salt dome boundary and internal structure.

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