

On Ramsey's Rejection of Keynes's Mathematical Lattice Structure and Interval Valued Probabilities in the *A Treatise on Probability*

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Adjunct Lecturer, California State University, Dominguez Hills**Abstract****Review Article**

Henry E. Kyburg was able to demonstrate that Keynes had a qualitative, graphical understanding of interval probability based on his careful analysis of Keynes's diagram on page 39 of the *A Treatise on Probability*. Kyburg showed in four different papers, published in 1995, 1999, 2003 and 2010, that Keynes's diagram on page 39 is a mathematical lattice structure encompassing interval valued probability. However, Kyburg rejected any conclusion that Keynes had provided a mathematical, quantitative theory of interval valued probability. Kyburg did recognize that Keynes had an intuitive understanding about the nature of interval valued probability, but that the best that Keynes had been able to accomplish in this regard in his *A Treatise on Probability* was to offer some hints, ideas, or suggestions. This is the same conclusion put forth by all members of ISIPTA since 1999. Of course, Keynes had provided a complete mathematical, quantitative theory, in Parts II and III of the *A Treatise on Probability*, of interval valued probability in chapters XV, XVI, XVII, XX, and XXII. Keynes's interval valued theory was based on Boole's original theory of interval valued probability that Kyburg and all members of ISIPTA have overlooked. Kyburg makes a very interesting point about the deficiencies of Ramsey, as regards Keynes's graphical presentation (see Keynes, 1921, p.161, ft.2) that were based on Boole's approach. Boole's mathematical, lattice structure of upper and lower probabilities, that Boole demonstrated had least and greatest limits, narrowest limit, maximum limit, highest, inferior numerical limit, highest minor limit, greatest minor numerical limit, etc. (Boole, 1854, pp.288, 293, 305-313, 317-324) all involve a mathematical lattice structure. All of these terms mean that Boole is solving for a greatest lower bound and/or a least upper bound with his solutions methods, which involved using second order, quadratic equations and third order, cubic equations. Such greatest lower bounds and least upper bounds automatically specify a mathematical lattice structure to Boole's partial orderings of probabilities: "What is curious is that the mathematician -philosopher, Frank Ramsey, paid no attention to this structure in his review of the *Treatise* (Ramsey, 1922) ..." (Kyburg, 2010, p.26). Actually, Kyburg must have known that Ramsey had never made any comment on Keynes's graphical demonstration on page 39 of Keynes's *A Treatise on Probability* in any publication in his life. The reason is very simple. Ramsey knew that the mathematical lattice demonstration on p.39 showed that his theory of exact and precise, additive probability, illustrated by Keynes with the linear line OAI, is a very, very special case of Keynes's general theory. Therefore, Ramsey had to reject the foundation for the construction of any type of mathematical lattice structure, which was Keynes's Boolean relational, propositional logic. Kyburg recognized that Ramsey's criticism was directed at this foundation: "That there is an out-and-out conflict between Keynes and Ramsey ... becomes clear in the fourth part of Ramsey's essay ... Now it is all very well for Ramsey modestly to admit that he sees no logical relation of probability such as the one that Keynes seeks to draw our attention to [author's note-Kyburg has completely overlooked that such an objective, logical probability relation was first specified by Boole, not Keynes], but wants to go further than that. It is clear that Ramsey wants to claim that there is no such relation." (Kyburg, 2010, p.29; italics is Kyburg's). Ramsey, in fact, did claim this. I have noted many, many times in the papers that I have written on this topic over the last 15 years that are available at SSRN, Researchgate and Academia, that Keynes, if he had actually ever thought himself threatened by Ramsey's claims, could have, either with or without Bertrand Russell's assistance, completely and totally crushed Ramsey intellectually by simply making the following hypothetical one sentence statement at any Apostles meeting from 1922 to 1929: "My objective, logical probability relations are identical to Boole's logical, probability relations as discussed in chapter I and XVI of his *The Laws of Thought* (1854)." That, of course, would have been the beginning of the end of Ramsey's academic career.

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SECTION 1: INTRODUCTION

The paper will be organized in the following manner. Section Two will examine Ramsey's erroneous claims about "I think it is a pity that Mr Keynes did not see this clearly, because the exactitude of this correspondence would have provided quite as worthy material skepticism as did the numerical measurement of probability relations. Indeed, some of his arguments against their numerical measurement appear to apply quite equally well against their exact correspondence with degrees of belief..." (Ramsey, 1936, "Truth and Probability". In Kyburg and Smokler, (eds.), (1980; 2nd ed), p.27), using Kyburg's understanding of Keynes's mathematical lattice structure, that was used by Keynes to illustrate graphically Boole's partial ordered set representations that Boole carried out on pp.293-324 of *The Laws of Thought* (1854).

Section Three will examine Ramsey's claims that there are no relational, propositional logics, no mathematical lattices with specified lub's and glb's, and no such things as interval valued probabilities. Section Four will conclude the paper.

Section 2: Kyburg's Analysis of Keynes's Mathematical Lattice Structure

Kyburg, despite his other deficiencies with respect to Keynes's mathematical demonstrations concerning his non numerical(interval) probabilities, which are based on Boole, is the only academic who, based on his study of the diagram on p.39 of Keynes's *A Treatise on Probability*, correctly recognized that Keynes's diagram was an early illustration of a mathematical lattice structure. All other academicians, particularly Heterodox, Post Keynesian and Institutional economist, argue that Keynes is demonstrating his comparative theory of ordinal probability:

"This argument suggests, as does the illustration in Keynes (TP, p .39) ... that probabilities form a lattice structure... Upper and lower bounds for any probabilities exist, of course, --namely, 0 and 1---but the question is whether the meet and join of any two probabilities exist. A definitive answer is hard to come by for Keynes.... Nevertheless, the list of properties on page 41 of TP suggests that the answer is affirmative. Probabilities lie on paths, each of which runs from 0 to 1." (Kyburg, 2011, p.25).

[author's note-this is not the case for the interval valued probability, OVA, which is the glb and lies between 0 and .5. Kyburg was unable to realize that the draftsman, who provided the diagram to Keynes for publication in the TP, made three errors-First, OVA was supposed to have been drawn "intermediate" (Keynes, 1921, p.39) between 0 and 1 at .5; in fact, it is drawn 2/3 of the way between 0 and 1. Given that OVA is symmetric, the estimated value of V, which lies symmetrically in the interval OA, is .25 [0, .25, .50) and

not [0, .335, .67]. This makes it clear that Keynes did not think it possible "... that there should be an unbounded sequence of ever greater lower bounds... though it is of course mathematically possible." (Kyburg, 2011, p.25); Second, the ZWY path was incorrectly drawn (all of the paths represent second order quadratic equations; and Third, U, on the OUI path, should be drawn at the vertex of OUI, not off to the right of the vertex. See the ten papers by Brady in the References]

Keynes's extensive footnote 2 on p.161 makes it clear that Keynes is providing a diagram on p.39 of Boole's technical analysis, which involves solving for the roots of second and third order equations (inequations), which would require nonlinear paths that are non-additive.

The linear and additive path is OAI, which would be required in order to implement Ramsey's claims that all rational decision makers can specify complete orders, as well as precisely calculate the mathematical expectation. This represents Ramsey's view of decision theory. It is thus easy to conclude from this diagram that Ramsey's theory is a very special case of Keynes's far more general theory.

Kyburg also disposed of Ramsey's incoherent and inchoate claims about degrees of probability and degrees of belief. Ramsey made the following bewildering claim:

"But if, as Mr. Keynes holds, these things are not always expressible by numbers, then we cannot give his statement that the degree of the one is the same as the degree of the other such a simple interpretation but must suppose him to mean only that there is a one-one correspondence between probability relations and the degrees of belief which [p.161] they justify.

This correspondence must clearly preserve the relations of greater and less, and so make the manifold of probability relations and that of degrees of belief similar in Mr Russell's sense. I think it is a pity that Mr Keynes did not see this clearly, because the exactitude of this correspondence would have provided quite as worthy material skepticism as did the numerical measurement of probability relations. Indeed, some of his arguments against their numerical measurement appear to apply quite equally well against their exact correspondence with degrees of belief; for instance, he argues that if rates of insurance correspond to subjective, i.e. actual, degrees of belief, these are not rationally determined, and we cannot infer that probability relations can be similarly measured. It might be argued that the true conclusion in such a case was not that, as Mr Keynes thinks, to the non-numerical probability relation corresponds a non-numerical degree of rational belief, but that degrees of belief, which were always numerical, did not correspond one to one with the probability relations justifying them."

(Ramsey,1926, Truth and Probability. In Kyburg and Smokler, (eds.),1980, p.27).

Kyburg's Assessment of Ramsey Claims is Very Critical:

"This is an odd thing to say, since it is apparently Keynes's intuitions about rational belief that lead him to this view about probability, rather than vice versa .In any event, since the structure of this manifold of probabilities is very different from the structure of the reals between 0 and 1, to which Ramsey wished to reduce all degrees of belief and all probabilities, it is a pity that Ramsey did not provide more motivation for his drastic reduction of Keynes's rich manifold of probabilities[author's note -Kyburg is referring to Keynes's mathematical, lattice structure in the diagram on page 39, with its glb and lub of the TP, as opposed to Ramsey's "structure", which is the horizontal, linear line OAI]to the simple (alleged) structure of degrees of belief."(Kyburg,2010,p.27).

Kyburg's point is that Ramsey's structure can't be represented as a mathematical lattice structure unless one wants to view the single, horizontal, linear line, OAI, as a degenerate lattice structure.

It is clear, then, that only in the special Keynes case where $V(a/h) = w$ and $w=1$, where $0 \leq w \leq 1$, so that all probabilities become linear and additive when the lower probability bound =the upper probability bound, that Ramsey's theory becomes a special case of Keynes's general theory.

Section 3: On Ramsey's 1926 Hallucination that There was No Such Thing as a Relational, Propositional Logic

Consider Ramsey's hallucination below:

"But let us now return to a more fundamental criticism of Mr Keynes' views, which is the obvious one that there really do not seem to be any such things as the probability relations he describes. He supposes that, at any rate in certain cases, they can be perceived; but speaking for myself I feel confident that this is not true. I do not perceive them, and if I am to be persuaded that they exist it must be by argument; moreover, I shrewdly suspect that others do not perceive them either, because they are able to come to so very little agreement as to which of them relates any two given propositions." (Ramsey,1926," Truth and Probability. In Kyburg and Smokler (eds.), (2nd ed.), p.27).

Note that this is a continuation of Ramsey's false claims, as analyzed above in Section 2, that were demolished by Kyburg based on Kyburg's understanding of Keynes's elementary, mathematical lattice structure.

This conclusion follows directly from Ramsey's nonsense claims in 1922 about some Axiom I that is supposed to have been in Keynes's *A Treatise on Probability* that stated that "... Mr Keynes accounts for

this by supposing that between any two propositions, taken as premiss and conclusion, there holds one and only one relation of a certain sort called probability relations; and that if, in any given case, the relation is that of degree α , from full belief in the premiss, we should, if we were rational, proceed to a belief of degree α in the conclusion." (Ramsey,1926, "Truth and Probability". In Kyburg and Smokler, (eds.), (2nd ed.)1980, (pp.26-27).

The 1922 version is "...First, he thinks that between any two non-self-contradictory propositions there holds a probability relation (Axiom I), for example between 'My carpet is blue' and 'Napoleon was a great general...'" (Ramsey,1922, p.3).

It is obvious that there is no logical relation in the bizarre examples chosen by Ramsey. All of Ramsey's examples are of two propositions that are unrelated. All of the propositions in a Boolean relational logic must be related, so Ramsey's

"But let us now return to a more fundamental criticism of Mr Keynes' views, which is the obvious one that there really do not seem to be any such things as the probability relations he describes..." is simply intellectual gobbledygook.

Ramsey was simply totally ignorant of the fundamental, Boolean, relational, propositional logic developed by Boole in 1854.This ignorance first showed up in Ramsey's November, 1921 Apostles paper criticizing Bertrand Russell's use of propositions. This November paper is strikingly similar to Ramsey's critique of Keynes's *A Treatise on Probability* that appeared in 1922 in the January issue of Cambridge Magazine without any referee examining the paper before it was published. Russell refuted Ramsey's critique quite simply with a devastating counter example to Ramsey's bizarre "My carpet is green; Napoleon was a great general", example, constantly used by C. Misak (2016,2020) as supposedly demonstrating how poorly constructed was Keynes's logical foundation in the TP.

Russell's counter example to Ramsey's use of two unrelated propositions was "2+2 =4; Napoleon disliked poodles." (Russell,1922, p.120, *ft).

It is very surprising that no economist, philosopher or academician has ever cited Russell, in either the 20th or 21st century, since Russell's simple example immediately and completely demolishes Ramsey's 1922(and 1926) review of Keynes's TP.

Ramsey's many, many errors are repeated in, for example, the works of Bateman, Brathwaite, Bures, Carabelli, Clarke, Davis, Dequech, Gerrard, Gillies, Good, Hacking, Lang, Levi, Marsay, Mellor, Misak, Methven, Monk, O'Donnell, Rowbottom, Runde, Skidelsky, Wheeler, Winslow and Zabell. See the references.

SECTION 4: CONCLUSIONS

Ramsey rejected any type of mathematical lattice structure, as well as any type of imprecise probability construct or interval probability representation, because of (a), his commitment to the complete ordering of all probabilities, (b) his commitment to the belief that all probabilities must be real numbers and (c), that all decision makers, who are rational, can calculate exact, mathematical expectations for all future events. This is the only way that Ramsey could continue to maintain his claim that the use of mathematical expectations was the hallmark of rationality. Keynes's total and complete rejection of the applicability of mathematical expectations, in general, demonstrates that Ramsey's approach is inherently contradictory to Keynes's approach, unless Keynes's $V(a/h) = w$ has a value of $w=1$, given $0 \leq w \leq 1$. In this case, the lower probability bound = the upper probability bound, so that the probabilities are linear, numerical, precise and additive.

Kyburg showed on four different occasions that Ramsey's 1922 and 1926 reviews of Keynes's *A Treatise on Probability* are badly flawed. These flaws revolve around Ramsey's rejection of Keynes's (a) mathematical lattice structure of imprecise probability, (b) interval valued probability and the Boolean relational, propositional logic. All of Keynes's results depend on Keynes's application throughout the TP of Boole's relational, propositional logic. This explains Ramsey's bizarre claims about (a) not being able to perceive any such relation and (b) no one else could perceive such a relation, either, including Keynes. This follows from Ramsey's very severe confusion of Moorean, metaphysical, Platonic relations for Boolean, probability relation between premises and conclusions, which is a formal, mathematical, symbolic logic.

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