Scholars Journal of Economics, Business and Management
e-ISSN 2348-5302
p-ISSN 2348-8875
Sch J Econ Bus Manag, 2017; 4(11):799-809

Using Shadow Prices in a Chance Constrained Programming Model: A Case Study for Production System

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Article History
Received: 01.11.2017
Accepted: 13.11.2017
Published: 30.11.2017

DOI:
10.36347/sjebm.2017.v04i11.007

Abstract: Businesses should make production plans to continue their commercial life and to improve their profitability. Linear programming is one of the basic approaches used in solving production planning problems. However, deterministic structure of linear programming cannot express the randomness of variable coefficients that exists in many real production problems. In production planning, coefficients of variables and constraints such as prices or demand may not be in deterministic structure. In such cases, linear programming model gives inadequate results for the problem. Under uncertainty, stochastic programming approach comes to the fore as a method for solving production planning problems. It is possible to add uncertainty into to the model in decision-making process by using stochastic programming models. In solving phase of stochastic programming the basic approach is, converting the probabilistic structure of the problem into deterministic form and solving by mathematical methods. Chance constrained programming approach which is one of the stochastic programming techniques, offers the opportunity to examine random constraints in deterministic structure, according to specific statistical distributions. The study was conducted in the production system of an office products operating in Izmir. Firstly, the deterministic model of the production system is obtained. Then stochastic variables were defined in the model and the chance constrained programming was used for production planning of an office products manufacturing system which is still in operation. Deterministic and stochastic models were compared. It was found out, using proposed chance constrained model in production planning is valuable in terms of profit in long term.
Keywords: Stochastic Programming, Chance Constrained Programming, Manufacturing System.

INTRODUCTION

Production planning problem can be defined as linear programming problem that is aimed to maximize profit or minimize cost under deterministic constraints. A problem can be modeled as linear programming model only if some certain assumptions are valid. One of the notable assumptions of linear programming (LP) is certainty. The certainty assumption requires that the model parameter such as objective function coefficient (c_j), technological coefficient (a_{ij}) and right hand side (RHS) of a constraint (b_i) must be known with certainty. However, real life problems are not only consist of parameters which are deterministic. If there is an uncertainty in any model parameter of the problem, the solution of LP model cannot be valid. If any parameter of the LP problem is random, it would be called stochastic programming (SP) problem [1]. If decision variables are determined on different stage of solution process in SP problems, it can be modeled with multi stage recourse models. SP problems that have decision variables to be determined at once, can be modeled with chance constrained programming (CCP) approach. CCP can be defined as expressing random parameters with certain probability levels and presenting constraint and objective functions as probability functions [2].

Charnes and Cooper [3] introduced CCP in their pioneering study that they defined randomness with known probability distributions. In their study, they used different probability to express chance constraints. Miller and Wagner [4] examined random independent variables with joint probability constraint. Sengupta [6] used different probability distributions in CCP. Prekopa [5] examined random dependent variables with joint probability constraint. Hillier [7] proposed chance constrained integer programming (CCIP), with assigning only zero-one values to decision variables. Hamlen [8] used CCMIP in designing internal control system in management. Jagannathan [9] defined chance constraints by using information from sample unit. In the literature, a number of applications with various types of methods and problems can be found. Charnes *et al.* [10] used CCP to calculate critical path of projects. Agnew *et al.* [11] used CCP in portfolio selection problem. Cooper *et al*. [12] defined stochastic inputs and outputs with chance constraints in data envelopment analysis. Liu [13] used fuzzy logic in CCP. Hanasusanto *et al*. [14] reviewed wide classes of CCP which are robust. Jiang and Guan [15] studied distributional robust CCP in data driven setting. Pievatolo and Ruggeri [16] studied reliability of systems with missing data. Sakalli [17] combined CCP with simulated annealing algorithm. Ye and Xie [18] briefly analyzed stochastic modeling on reliable products.

In our study, we used both chance constrained linear programming (CCLP) and chance constrained mixed integer programming (CCMIP) to solve the production planning problem of a manufacturer that products consumables goods for workplaces and offices. The objective function of the problem is to determine the optimum production plan for maximizing profit. Consumable office goods are defined as decision variables of the problem. Hence, unit sales profit of the goods are used as coefficient of objective function and known with certainty. Also technological coefficients have constant values. Operation time capacity for each operation and affordable amount of the raw materials are known exactly. Total manpower and maximum demand for each goods are chance constraints that are normally distributed. We determined confidence level of chance constraints with two different approaches. One of them is allocating confidence levels to RHS of demand constraints of each decision variables by their profits proportionally. In this way it is aimed to cover high profitable goods more than low profitable goods in solution. Second approach which is our main contribution to the CCP depends on using shadow prices of demand constraints from LP solution and allocating confidence levels proportionally to the decision variables. Average confidence level of demand constraints is considered similar in both approach in order to compare models.

In the literature there are numerous works which is dealt with duality and CCP, some of the remarkable are as follows. Charnes and Cooper [19] used duality relations in CCP for evaluating risk and performance levels. Charnes *et al.* [20] obtained dual CCP model from dual deterministic equivalents. Guo and Huang [21] used duality with both fuzzy and chance constraints in a two stage programming method. Moghaddam and Michelot [22] used shadow prices in determining joint cost allocation with LP. Bot *et al*. [23] investigated duality theorem in CCP. Ahmed *et al*. [24] proposed two new techniques to obtain dual values in CCP.

CHANCE CONSTRAINED PROGRAMMING

Basic assumption of CCP is to obtain solution under α (0,1) probability constraint in α confidence level. Chance term in CCP expresses the satisfaction probability of constraint. In solution process, stochastic parameter of constraint and objective functions are converted to corresponding deterministic parameter by using predetermined probability distribution and confidence levels, then problem can be solved by LP [25]. CCP provides flexibility by controlling the probability that a chance constraint may be violated [26]. CCP can be defined as allowing random constraint to restrict model by a confidence level of predetermined probability distribution. We can turn CCP into LP by adding expected value to confidence interval and then deterministic model can be easily solved with simplex method [27].
Chance constrained linear programming model is given as follows,

$$max(min)\, z = \sum_{j=1}^{n} c_j x_j$$

$$P\left[\sum_{j=1}^{n} a_{ij} x_j \le b_i\right] \ge 1-\alpha_i\,, \quad i=1,\dots,m \qquad (1.1)$$

$$x_j \ge 0 \qquad , \; j=1,\dots,n$$

$$\alpha_i \in (0,1), \; i=1,\dots,m$$

At least one of the model parameters, such as ,c_j , a_{ij} and b_i have to be random variables. Predetermined probabilities can be expressed as α_i. Decision variables x_j are deterministic [28].

Stochastic parameters in Equation (1.1) must have known probability distribution with α_i confidence levels. Stochastic problem have to turn into deterministic one by using information of distribution. In CCP problems we generally assume that parameters are normally distributed with known mean and variance. We can examine chance constraints with normally distributed parameters in different cases as follows

- Only a_{ij} coefficients are random variables,
- Only b_i constants are random variables,
- Only c_j coefficients are random variables,
- a_{ij} and b_i are random variables,
- a_{ij} and c_j are random variables,

- c_j and b_i are random variables,
- a_{ij}, c_j and b_i are random variables,

We can obtain last three of them from the first four cases. Hence, we examine the first four cases here [29].

Case 1: Only a_{ij} coefficients are random variables,
For all i and j let a_{ij} normally distributed random variables with mean E (a_{ij}) and variance Var (a_{ij}) and also assume that a_{ij} and a_{kl} with covariance cov $\{a_{ij}, a_{kl}\}$. Define a random variable r_i as follows (Taha, 2000: 801),

$$r_i = \sum_{j=1}^{n} a_{ij} x_j \quad , \text{ i=1,…,m}$$

Coefficients $a_{i1},\ldots,a_{in}$ are normally distributed random variables and x_j decision variables, then expected value and variance would be also normally distributed as follows,

$$E(r_i) = \sum_{j=i}^{n} E(a_{ij} x_j), \text{ i=1,…,m}$$

$$Var(r_i) = X^T V_i X, \text{ i=1,…,m}$$

Then V_i is the ith covariance matrix as follows,

$$V_i = \begin{bmatrix} \text{var}(a_{i1}) & \text{cov}(a_{i1}, a_{i2}) & \ldots & \text{cov}(a_{i1}, a_{in}) \\ \text{cov}(a_{i2}, a_{i1}) & \text{var}(a_{i2}) & \ldots & \text{cov}(a_{i2}, a_{in}) \\ . & . & . & . \\ \text{cov}(a_{in}, a_{i1}) & \text{cov}(a_{in}, a_{i2}) & \ldots & \text{var}(a_{in}) \end{bmatrix}$$

Then, constraint in equation (1.1) ,

$$P[r_i \le b_i] = P\left[\frac{r_i - E(r_i)}{\sqrt{\text{var}(r_i)}} \le \frac{b_i - E(r_i)}{\sqrt{\text{var}(r_i)}}\right] \ge 1 - \alpha_i$$

can be expressed as,

$$P[r_i \le b_i] = \Phi\left[\frac{b_i - E(r_i)}{\sqrt{\text{var}(r_i)}}\right]$$

here, Φ defines the distribution function of standard normal distribution. Let $K_{\alpha i}$, as the value of standard normal variable,

$$\Phi\left[\frac{b_i - E(r_i)}{\sqrt{\text{var}(r_i)}}\right] \ge \Phi(K_{\alpha i})$$

and equation can only be valid in cases as follows,

$$\left[\frac{b_i - E(r_i)}{\sqrt{\text{var}(r_i)}}\right] \ge K_{\alpha i}$$

In different notation we can express chance constraint as follows,

$$E(r_i) + K_{\alpha i}\sqrt{Var(r_i)} \le b_i$$

Similarly, r_i can be replaced with a_{ij}, and we obtain deterministic nonlinear constraint corresponding to probabilistic linear constraints as follows,

$$max(min) z = \sum_{j=1}^{n} c_j x_j$$

$$\sum_{j=1}^{n} E(a_{ij}) x_j + K_{\alpha i}\sqrt{X^T V_i X} \le b_i \quad \text{i=1,…,m} \quad x_j \ge 0 \quad \text{j=1,…,n}$$

if all a_{ij} are independent variables, then we obtain as follows,

$$\sum_{j=1}^{n} E(a_{ij})x_j + K_{\alpha i}\sqrt{\sum_{j=1}^{n} Var(a_{ij})x_j^2} \le b_i \quad \text{i=1,…,m}$$

Case 2: Only b_i constants are random variables,

Let b_i, normally distributed random variables with mean E (b_i) and variance Var (b_i). It can be expressed as follows,

$$P\left[\frac{b_i - E(b_i)}{\sqrt{\text{var}(b_i)}} \ge \frac{\sum_{j=1}^{n} a_{ij}x_j - E(b_i)}{\sqrt{\text{var}(b_i)}}\right] \ge 1-\alpha_i$$

and $1-\alpha_i = p_i$ then, standard normal variable can be expressed as follows,

$$\left[\frac{b_i - E(b_i)}{\sqrt{\text{var}(b_i)}}\right]$$

we obtain inequality as follows,

$$P\left[\frac{b_i - E(b_i)}{\sqrt{\text{var}(b_i)}} \le \frac{\sum_{j=1}^{n} a_{ij}x_j - E(b_i)}{\sqrt{\text{var}(b_i)}}\right] \le 1-p_i$$

If we replace standard normal variable as follows,

$$\Phi\left(\frac{\sum_{j=1}^{n} a_{ij}x_j - E(b_i)}{\sqrt{\text{var}(b_i)}}\right) \le \Phi(K_{pi})$$

this inequality only valid when following condition holds true as follows,

$$\left(\frac{\sum_{j=1}^{n} a_{ij}x_j - E(b_i)}{\sqrt{\text{var}(b_i)}}\right) \le K_{pi}\ , \quad \text{i=1,…,m}$$

or, can be defined as follows,

$$\sum_{j=1}^{n} a_{ij}x_j \le E(b_i) + K_{pi}\sqrt{\text{var}(b_i)}\ , \quad \text{i=1,…,m}$$

Probabilistic model corresponding to the deterministic model is as follows,

$$max(min)z = \sum_{j=1}^{n} c_j . x_j$$

$$\sum_{j=1}^{n} a_{ij}x_j \le E(b_i) + K_{pi}\sqrt{\text{var}(b_i)}\ , \text{i=1,…,m}$$

and can be solved by simplex method [28].

Case 3: Only c_j coefficients are random variables,

Due to coefficient of decision variables in objective function c_j are normally distributed random variables, z objective function is also normally distributed with mean E(z). Hence, we can define objective function as follows,

$$max(min)\, E(z) = \sum_{j=1}^{n} E(c_j)x_j$$

and here, objective function with expected value would be a deterministic function [29].

Case 4: a_{ij} and b_i are normally distributed
Assume that random variable r_i is normally distributed and defined as follows,

$$r_i = \sum_{j=1}^{n} a_{ij}x_j - b_i$$

then chance constraint can be expressed as follows,

$$P[r_i \leq 0] \geq 1-\alpha_i \quad , \quad i=1,\ldots,m$$

Hence, it includes linear combination of normally distributed random variable,

$$P\left[\frac{r_i - E(r_i)}{\sqrt{\text{var}(r_i)}} \leq \frac{-E(r_i)}{\sqrt{\text{var}(r_i)}}\right] \geq 1-\alpha_i \quad i=1,\ldots,m$$

Standard normal random variable is defined as $\dfrac{r_i - E(r_i)}{\sqrt{\text{var}(r_i)}}$,

$\Phi(K_{\alpha i}) = 1-\alpha_i$ standard normal variable can be replaced with $K_{\alpha i}$, then we obtain inequality as follows,

$$\Phi\left[\frac{-E(r_i)}{\sqrt{\text{var}(r_i)}}\right] \geq \Phi(K_{\alpha i})$$

and it is only valid if following inequality holds,

$$\left[\frac{-E(r_i)}{\sqrt{\text{var}(r_i)}}\right] \geq K_{\alpha i}$$

in another notation we have,

$$E(r_i) + K_{\alpha i}\sqrt{\text{var}(r_i)} \leq 0 \quad , \quad i=1,\ldots,m$$

We obtain deterministic programming model corresponding to probabilistic model when r_i is replaced with random variable as follows,

$$max(min) z = \sum_{j=1}^{n} c_j x_j$$

$$E\left(\sum_{j=1}^{n} a_{ij}x_j - b_i\right) + K_{\alpha i}\sqrt{\text{var}\left(\sum_{j=1}^{n} a_{ij}x_j - b_i\right)} \leq 0 \quad , \ i=1,\ldots,m \quad x_j \geq 0 \quad j=1,\ldots,n$$

then it can be solved by simplex method (Taha, 2000: 803).

EXPERIMENTAL ANALYSIS

8 different types of products are manufactured in an office supplies and products factory in İzmir, Turkey. The firm have 32 number of raw materials as input and 29 number of operations in production process. 23 employees are assigned at the operations. Firm have limited capacity for each operation in a week. An employee can work for maximum 40 hours in a week. Firm does not want to manufacture above the demand levels for each product, due to shortage in depot place. Production under the demand levels cannot cause any problem for the firm, because they can easily supply them from outsource with no loss or profit. Firm wants to optimize their weekly production plan to maximize their profit under production constraints. In table 1 profit of each product per unit is given as follows,

Table-1: Profit of Products Per Unit

Products	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8
Unit Profit (TL)	14.0807	11.025	5.26138	23.0494	4.643	20.8212	0.9213	5.26138

In table 2 some of operations are listed. For example, operation 1 defines a cutting operation and required cutting times for each product.

Table-2: Operation and Required Time

i	Products	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8
1	Operation 1	1.723	115.2	0.716	0.716				0.0203
2	Operation 2	0.716	57.6	0.716	0.716				0.1477

In table 3, some of raw materials are listed. For example, Raw material 1 defines "Shring Pe Film" and amount of usage for each product.

Table-3: Raw Material and Required Amount

i	Products	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8
1	RawMaterial1	0.04	0.04			0.04	0.095		
2	RawMaterial2	0.016	0.005	0.016	0.016	0.005			0.016

Demand for products are stochastic variables with normal distribution and in table 4, mean and variance of demand for each product per week are listed.

Table-4: Mean and Variance Of Demand

	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8
E(Ti)	750	1126	438	419	397	1010	995	158
Var(Ti)	6400	10000	1600	1600	1600	10000	10000	400

Total manpower capacity per week is normally distributed random variable. Its' expected value and variance was calculated, E(P)= 662400 second, Var(P)=57600 second, respectively. Problem was modeled with variables and notations as follows,

i *products* *i=1:8,*
j *operations* *j=1:29,*
h *raw material* *h=1:32,*
P *total manpower,*
a_{ij} *required time for product i in operation j,*
b_{hi} *required amount of raw material k in production of product i,*
c_i *profit of product i,*
T_i *demand of product i,*
J_j *total capacity for operation j,*
H_h *total amount of raw material h capacity,*

General form of the problem was modeled as closed form and the model is given as follows;

Objective Function;

$$max\ z = \sum_{i=1}^{n} c_i x_i$$

Constraints;

$$\sum_{i=1}^{n} a_{ji} x_i \leq J_j \qquad j=1,\dots,m \qquad (2.1)$$

$$\sum_{i=1}^{n} x_i \sum_{j=1}^{m} a_{ji} \leq P \qquad (2.2)$$

$$\sum_{i=1}^{n} b_{hi} x_i \leq H_h \qquad h=1,\dots,p \qquad (2.3)$$

$$x_i \leq T_i \qquad i=1,\dots,n \qquad (2.4)$$

$$x_i \geq 0 \qquad i=1,\dots,n \qquad (2.5)$$

The goal of the model is to maximize profit. Equation (2.1) limits each operation time. Equation (2.2) restricts production with total manpower capacity. Equation (2.3) enforces production under each raw material capacity. Equation

(2.4) limits the manufacturing of products under demand level. Equation (2.5) ensures decision variables cannot have negative values.

Model 1 and Model 2

Firstly, we assumed all variables were deterministic in model 1. Then we assumed all variables were integers in model 2. Objective function and constraints of both model 1 and model 2 are same and as follows,

$Z_{max}=14.0807X_1+11.025X_2+5.26138X_3+23.0494X_4+4.643X_5+20.8212X_6+0.9213X_7+5.26138X_8$

Required time for each product in operations: Constraint Group 1

$1.723X_1+115.2X_2+0.716X_3+0.716X_4+0.0203X_8 \leq 57600$
$0.716X_1+57.6X_2+0.716X_3+0.716X_4+0.1477X_8 \leq 28800$
$1.111X_1+0.555X_2+1.11X_3+1.11X_4+0.0074X_8 \leq 28800$
$1.111X_1+0.555X_2+1.11X_3+1.11X_4+0.3702X_8 \leq 28800$
$0.117X_1+0.117X_2+0.117X_3+0.117X_4+0.0278X_8 \leq 115200$
$6X_1+0.173X_2+1.2X_3+1.2X_4+0.175X_8 \leq 28800$
$24X_1+24X_2+5.136X_3+5.136X_4+0.0444X_8 \leq 57600$
$8.154X_1+13X_2+8.154X_3+8.154X_4+0.13X_6+0.2778X_8 \leq 57600$
$197.8X_1+41.98X_2+197.8X_3+197.8X_4+1.678X_5+31.072X_6+5.966X_7+0.0222X_8 \leq 57600$
$1.124X_2 \leq 28800$
$0.172X_2 \leq 115200$
$0.198X_2 \leq 28800$
$0.404X_2 \leq 57600$
$0.929X_2 \leq 28800$
$0.922X_2+0.32X_6 \leq 230400$
$0.73X_2 \leq 115200$
$0.498X_2+0.533X_6 \leq 28800$
$0.793X_5+0.656X_6 \leq 57600$
$0.925X_5+1.557X_6 \leq 28800$
$0.458X_5 \leq 28800$
$0.175X_5 \leq 115200$
$1.286X_5+0.642X_6 \leq 28800$
$0.493X_7 \leq 28800$
$0.116X_7+0.1667X_8 \leq 28800$
$0.163X_7 \leq 28800$
$0.326X_7 \leq 57600$
$0.49X_7 \leq 57600$
$0.0041X_8 \leq 28800$
$0.2477X_8 \leq 28800$

Raw Material Capacity: Constraint Group 2

$0.04X_1+0.04X_2+0.04X_5+0.095X_6 \leq 30000$
$0.016X_1+0.005X_2+0.016X_3+0.016X_4+0.005X_5+0.016X_8 \leq 30000$
$0.008X_1+0.0075X_2+0.008X_3+0.008X_4+0.008X_8 \leq 15000$
$0.0056X_1+0.0022X_3+0.056X_4+0.0022X_8 \leq 25000$
$5.85X_1 \leq 100000$
$13X_1 \leq 100000$
$0.4368X_1+0.4368X_3+0.4368X_8 \leq 150000$
$0.7X_2 \leq 100000$
$27.5X_2+275X_6 \leq 500000$
$55X_2+550X_6 \leq 1000000$
$27.5X_2+275X_6 \leq 500000$
$1.18125X_2 \leq 25000$
$0.87975X_2 \leq 10000$
$0.04X_3+0.04X_4+0.65X_7+0.04X_8 \leq 80000$
$0.4672X_3+0.4672X_4+0.4672X_8 \leq 100000$
$5.568X_3+5.568X_8 \leq 100000$
$60.72X_4 \leq 200000$
$60.72X_4 \leq 200000$
$0.30792X_4 \leq 10000$

$55X_5 \leq 100000$
$55X_5 \leq 100000$
$3.257X_5 \leq 50000$
$250X_6 \leq 500000$
$8.34X_6 \leq 100000$
$0.0025X_6 \leq 10000$
$0.05X_7 \leq 10000$
$5X_7 \leq 100000$
$5.1X_7 \leq 1000000$
$5X_7 \leq 100000$
$22X_7 \leq 300000$
$2.5X_7 \leq 50000$
$0.00675X_7 \leq 10000$

Total manpower constraint and demand constraints: Constraint Group 3
$241X_1+258.157X_2+216.059X_3+216.059X_4+5.315X_5+34.91X_6+7.554X_7+1.5113X_8 \leq 662400$
$X_1 \leq 750$
$X_2 \leq 1126$
$X_3 \leq 438$
$X_4 \leq 419$
$X_5 \leq 397$
$X_6 \leq 1010$
$X_7 \leq 995$
$X_8 \leq 158$

In model 1 and model 2 we assumed total manpower constraint and demand constraints are deterministic and problem was solved by linear programming method.

Model 3 and Model 4

Objective function and constraint group 1 and 2 are same for models 1, 2, 3 and 4. We assumed b_i were normally distributed random variables in constraint group 3 and expressed them as chance constraints. Hence, models 3 and 4 were created as CCLP and CCIP, respectively, as follows,
$Z_{max}=14.0807X_1+11.025X_2+5.26138X_3+23.0494X_4+4.643X_5+20.8212X_6+0.9213X_7+5.26138X_8$

Constraint Group 1: $\sum_{i=1}^{n} a_{ji}x_i \leq J_j$

Constraint Group 2: $\sum_{i=1}^{n} b_{hi}x_i \leq H_h$

Constraint Group 3:
$P(241X_1+258.157X_2+216.059X_3+216.059X_4+5.315X_5+34.91X_6+7.554X_7+1.5113X_8 \leq 662400) \geq 0.05$
$P(X_1 \leq 750) \geq 0.10$
$P(X_2 \leq 1126) \geq 0.10$
$P(X_3 \leq 438) \geq 0.15$
$P(X_4 \leq 419) \geq 0.05$
$P(X_5 \leq 397) \geq 0.15$
$P(X_6 \leq 1010) \geq 0.05$
$P(X_7 \leq 995) \geq 0.20$
$P(X_8 \leq 158) \geq 0.15$

Here, in constraint group 3, first constraint corresponds to total manpower constraint and rest used for demand constraints. Confidence level of demand constraints was proportionally determined for products with their profits. We aimed to allow production of more profitable products by stretching their demand constraints. This strategy also traditionally used by firm in their production process. We used mean, variance and confidence level for each demand and obtained deterministic equivalents of chance constrained as follows,

Constraint Group 3:
$241X_1+258.157X_2+216.059X_3+216.059X_4+5.315X_5+34.91X_6+7.554X_7+1.5113X_8 \leq 662794.8$
$X_1 \leq 852.8$

$X_2 \leq 1254.5$
$X_3 \leq 479.44$
$X_4 \leq 484.8$
$X_5 \leq 438.44$
$X_6 \leq 1174.5$
$X_7 \leq 1079.5$
$X_8 \leq 178.72$

Model 5 and Model 6:

We proposed two new models with using shadow prices of demand constraints that we obtained by solving model 1. In model 5 and 6 we proportionally determined confidence level of demand constraints with their shadow prices in model 1 and rest of the other constrained were remained unchanged as follows,

$Z_{max}=14.0807X_1+11.025X_2+5.26138X_3+23.0494X_4+4.643X_5+20.8212X_6+0.9213X_7+5.26138X_8$

Constraint Group 1: $\sum_{i=1}^{n} a_{ji} x_i \leq J_j$

Constraint Group 2: $\sum_{i=1}^{n} b_{hi} x_i \leq H_h$

Constraint Group 3

$P(241X_1+258.157X_2+216.059X_3+216.059X_4+5.315X_5+34.91X_6+7.554X_7+1.5113X_8 \leq 662400) \geq 0.05$
$P(X_1 \leq 750) \geq 0.20$
$P(X_2 \leq 1126) \geq 0.20$
$P(X_3 \leq 438) \geq 0.20$
$P(X_4 \leq 419) \geq 0.20$
$P(X_5 \leq 397) \geq 0.05$
$P(X_6 \leq 1010) \geq 0.05$
$P(X_7 \leq 995) \geq 0.20$
$P(X_8 \leq 158) \geq 0.05$
and their deterministic equivalents as follows,

Constraint Group 3

$241X_1+258.157X_2+216.059X_3+216.059X_4+5.315X_5+34.91X_6+7.554X_7+1.5113X_8 \leq 662794.8$
$X_1 \leq 817.6$
$X_2 \leq 1210.5$
$X_3 \leq 471.8$
$X_4 \leq 452.8$
$X_5 \leq 462.8$
$X_6 \leq 1174.5$
$X_7 \leq 1079.5$
$X_8 \leq 190.9$

Models were coded and solved in MATLAB program. All models were solved and obtained results can be seen in table 5. Deterministic LP model 1 and deterministic IP model 2, were achieved objective function values as 29918 and 29915, respectively. They have relatively lower objective values, as expected.

We compared model 3 and 4 with model 5 and 6, respectively. In accordance to comparisons it can be seen as model 5 and 6 have higher objective function values than model 3 and model 4. In models 3 and 4 average probability of violating a chance constraint is 0.889, while in models 5 and 6 this average probability is 0.867. Obtained objective values under chance constraints with average probabilities can be seen as a proof that the determination of confidence level by using shadow prices like in models 5 and 6 are more effective than using objective coefficients for the same purpose as in models 3 and 4. Hence, firm would have better results in the long term with models 5 and 6.

Table-5: **Model Results**

Model No	LP Model 1	IP Model 2	CCLP Model 3	CCIP Model 4	CCLP Model 5	CCIP Model 6
X_1	0	0	0	0	0	0
X_2	499.59	499	485.14	485	484.162	484
X_3	0	0	0	0	0	0
X_4	0	0	0	0	0	0
X_5	397	397	438.44	438	462.8	462
X_6	1010	1010	1174.5	1174	1174.50	1174
X_7	766.78	770	0	3	0	3
X_8	158	158	178.72	178	190.9	190
Max Z	29918	29915	32779	32764	32945.56	32928

DISCUSSION

In real life problems assuming random variables as deterministic would cause problems such as obtaining unfeasible solutions with losing opportunities in long term. We can gain advantage of using information of probability distribution in problem and model would become more flexible and under control with confidence level. Determining confidence level is an important issue and related with problem and variable type in CCP. Model will be more reliable when confidence level of chance constraint is suitable.

In models 3 and 4, we assigned confidence levels to chance constraints according to their objective coefficients. However, in models 5 and 6 confidence levels are determined by dual values that we obtained in model 1. It can be seen from model results that models which is based on dual values have better performance in determination of confidence levels. Using dual values for calculation of b_i random variable can be seen as an better alternative. Corresponding confidence levels should be updated when stock levels are changed.

As a result, in problems that have random parameters, using suitable probability distribution and confidence levels have better results than using only their expected values in short, medium and long terms. CCP provides seizing every possible opportunity and gaining advantage from encountered randomness in application stage of model.

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