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Research Article

Pulsatile Flow of Casson Fluid in Mild Stenosed Artery with Periodic Body Acceleration and Slip Condition

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Abstract: The pulsatile flow of blood through a mild stenosed artery under periodic body acceleration is investigated in this theoretical analysis. The Casson fluid model of blood and the presence of slip (velocity discontinuity) at the flow boundaries (stenosed vessel walls) are given due consideration in the analysis. The geometry of the stenosis to be manifested in the arterial segment is assumed to be an axisymmetric surface generated by a smooth cosine curve. On using perturbation technique, the flow velocity u is expanded in terms of the Womersley frequency parameter α^2 (where $\alpha^2 << 1$) and analytic expressions for velocity profile, flow rate, wall shear stress and effective viscosity are derived. An extensive quantitative analysis is carried out by performing large scale numerical computations of the above flow variables having physiological significance. The diagrammatic representation of the flow variables with the change of parameters are given with scientific discussion. Finally comparisons are made with the other existing results to justify the applicability of the model.

Keywords: Casson fluid, pulsatile flow, body acceleration, Stenosis, slip velocity, effective viscosity

INTRODUCTION

The understanding of anatomy and physiology of an organic system depends much on the knowledge of blood flow through arteries. The cause and development of many arterial diseases are related to the flow characteristics of blood and the mechanical behavior of the blood vessel walls. The abnormal and unnatural growth in the arterial wall thickness at various locations of the cardio-vascular system is medically termed 'stenosis'. Its presence in one or more locations restricts the flow of blood through the lumen of the coronary arteries into the heart leading to cardiac ischemia.

Once the contraction develops, it brings about significant alterations in the blood flow, pressure distribution, wall shear stress and the impedance (flow resistance). The fact that hemodynamic factors play a commendable role in the genesis and growth of the disease, has attracted many researchers to explore modern approach and more and more sophisticated mathematical models for flow through stenotic arteries.

In some situations the human body is subject to body accelerations or variations, for example, when vibration therapy is applied to a patient with heart disease, during flying in a spacecraft, or sudden movement of the body during sports activities etc. In all such cases, a specific part of the whole body may be subjected to an external acceleration that may cause disturbance to the blood flow. Though human body has the natural capacity to adapt the changes, but prolonged exposure to such variations may lead to some serious health problems like abdominal pain, increase in pulse rate, and hemorrhage in the face, neck, lungs and brain. So the study of the effect of the magnitude, frequency and duration of the periodic acceleration may play a significant role in the diagnosis and treatment of health problems.

To illuminate the effects of body acceleration on physiologically important flow quantities, intensive theoretical investigations have been carried out worldwide for both, normal and stenotic arteries. In most of the investigations relevant to the domain under discussion, the Newtonian behavior of blood (single phase homogeneous viscous fluid) was considered. Due to physiological importance of body acceleration, many theoretical investigations have been carried out for the flow of blood under the influence of body acceleration with single or multiple stenosis in human artery. Usha and Perma [1], Elshehawery [2], El-Saheed [3] studied the effect of body acceleration on pulsatile flow of blood taking the Newtonian model for blood. Sud and Sekhon [4] have developed a mathematical model for blood flow through stenosed artery by taking laminar and unidirectional blood flow subject to periodic body acceleration. In most of the investigations mentioned above, the Newtonian behavior of blood (single phase homogeneous viscous fluid) and no slip velocity condition at the stenosed vessel wall were considered. This model of blood is accepted for high shear rate in case of a flow through narrow arteries of diameter $\leq 1000 \mu m$.

On the basis of experimental observations, Iqbal et al., [5] suggested that blood, being predominantly a suspension of erythrocytes in plasma exhibits remarkable Non-Newtonian behaviour when it flows through narrow arteries, at low shear rates, particularly in diseased state, when clotting effects in small arteries are present. H-B fluid model and Casson fluid models are used in the theoretical investigations of blood flow through narrow arteries. Experiments conducted on blood (Scott Blair [6], Cokelet et al. [7], Merril et al. [8]) with varying hematocrit, anticoagulants, temperature etc. suggest that the behavior of blood at low shear rates can be better described by Casson model. It is well known that the flow of blood through arteries is highly pulsatile due to the heart pulse pressure gradient. In the proposed investigation the pulsatile flow of blood under the influence of periodic body acceleration is studied treating the blood as a Casson fluid. Also, a number of studies (both theoretical and experimental) on blood flow have established the presence of slip (velocity discontinuity) at the flow boundaries. As a result of the presence of slip, the apparent (effective) viscosity will be lowered. Misra JC [9] have presented an analysis of blood acceleration with consideration of velocity slip at the stenosed vessel wall. Blood has been represented by a Newtonian fluid. Verma et. al. [10] has considered the Casson model for blood under no slip condition while investigating the influence of body acceleration on pulsatile flow of blood through a stenosed artery. Some other studies conducted on stenosed artery are given in [11-21].

It gives an opportunity to consider the problem of blood flow through a stenosed segment of an artery under the influence of body acceleration when the rheology of blood is described by Casson fluid model. Also the effects of velocity slip on the flow variables will be given due consideration in the analysis. Perturbation technique has been adopted to get the analytical solution of the problem. The geometry of the stenosis to be manifested in the arterial segment is given due consideration in the present investigation. An extensive quantitative analysis is carried out by performing large scale numerical computations of the measurable flow variables having more physiological significance by developing computer codes. Their graphical representations have been presented at the end of the paper with appropriate scientific discussions. Finally comparisons are made with the other existing results to justify the applicability of the present model.

MATHEMATICAL CALCULATION

Let us consider a one-dimensional pulsatile flow of blood in an artery in the presence of externally imposed periodic body acceleration with mild stenosis. We consider the flow to be axially symmetric, laminar, fully developed by considering blood as a Casson fluid. The geometry of the flow is shown in *fig.1* and is given by:

$$\bar{R}(\bar{z}) = \begin{cases} \bar{R}_0 - \frac{\bar{\delta}}{2} \left(1 + \cos \frac{\pi \bar{z}}{\bar{z}_0} \right); & for \, |\bar{z}| \le \bar{z}_0 \\ \bar{R}_0; & for |\bar{z}| > \bar{z}_0 \end{cases}$$
(1)





where R (z) is the radius of the obstructed artery, R_0 is the constant radius of the normal artery, L_0 is the length of the stenosis, L is the length of the artery, d is the location of the stenosis and δ is the maximum height of the stenosis. The periodic body acceleration $F(\bar{t})$ in the axial direction is given by:

 $F(\bar{t}) = a_0 \cos(\omega_b \bar{t} + \emptyset)$ (2) Where a_0 is the amplitude, $\omega_b = 2\pi . f_b$, f_b is the frequency in Hz, \emptyset is the lead angle of $F(\bar{t})$. The frequency of body acceleration f_b is assumed to be small, so that wave effects can be neglected.

The pressure gradient at any \overline{z} is given by

$$-\frac{\partial p}{\partial \bar{z}} = A_0 + A_1 \cos(\omega_p \bar{t}) \tag{3}$$

Where A_0 is the steady component of pressure gradient, A_1 is the amplitude of the fluctuating component $\omega_p=2\pi$. f_p , f_p is the pulse frequency. The momentum equation governing the flow in cylindrical co-ordinate system is given by:

$$\overline{\rho} \cdot \frac{\partial \overline{u}}{\partial \overline{t}} = -\frac{\partial \overline{\rho}}{\partial \overline{z}} + \frac{1}{\overline{r}} \cdot \frac{\partial (\overline{r} \cdot \overline{\tau}_{\overline{r},\overline{z}})}{\partial \overline{r}} + F(\overline{t})$$
(4)
$$\frac{\partial \overline{\rho}}{\partial \overline{r}} = 0$$
(5)

Where \bar{r}, \bar{z} denotes the radial and axial co-ordinates respectively, $\bar{\rho}$ denotes density, \bar{u} is the axial velocity of blood, \bar{t} is time, \bar{p} is pressure and $\bar{\tau}$ the shear stress.

For Casson fluid the relation between shear stress and shear rate is given by

$$\sqrt{\overline{\tau}} = \sqrt{\overline{\tau}_{y}} + \sqrt{\mu} \cdot \left(-\frac{\partial \overline{u}}{\partial \overline{r}}\right) ; \quad if \, \overline{\tau} \ge \overline{\tau}_{y} \\
\frac{\partial \overline{u}}{\partial \overline{r}} = 0 \quad (6)$$

Where $\overline{\tau}$ denotes yield stress and $\overline{\mu}$ denotes the viscosity of blood. The boundary conditions are:

$$\overline{u} = \overline{u}_s \text{ at } \overline{r} = \overline{R}(\overline{z})$$
 (7)
And $\overline{\tau}$ is finite at $\overline{r} = 0$ (8)

Where \overline{u}_s is the slip velocity at the stenotic wall. Introducing the non-dimensional variables:

$$u = \frac{\bar{u}}{A_0 R_0^2 / 4\mu}; \quad z = \frac{\bar{z}}{R_0}; \quad z_0 = \frac{\bar{z}_0}{R_0}; \quad t = \omega_p \bar{t};$$

$$\delta = \frac{\bar{\delta}}{R_0}; \quad \tau = \frac{\bar{\tau}}{A_0 R_0 / 2}; \quad \theta = \frac{\bar{\tau}_y}{A_0 R_0 / 2};$$

$$R(z) = \frac{\bar{R}(\bar{z})}{R_0}; \quad r = \frac{\bar{r}}{R_0}; \quad e = \frac{A_1}{A_0};$$

$$B = \frac{a_0}{A_0}; \quad \omega = \frac{\omega_b}{\omega_p}; \quad u_s = \frac{\bar{u}_s}{A_0 R_0^2 / 4\mu}$$
(9)

The non-dimensional equation (4) becomes:

 $\alpha^{2} \cdot \frac{\partial u}{\partial t} = 4(1 + e\cos(t)) + 4B\cos(\omega t + \emptyset) + \frac{2}{r}\frac{\partial}{\partial r}(r$ (10)

Where $\alpha^2 = \frac{\omega_p R_0^2}{\mu/\rho}$, α is called Womersley frequency parameter.

Equation (6) changes to:

$$\sqrt{\tau} = \sqrt{\theta} + \frac{1}{\sqrt{2}} \sqrt{-\frac{\partial u}{\partial r}}; \quad if\tau \ge \theta, \qquad and$$
$$\frac{\partial u}{\partial r} = 0; \quad if\tau \le \theta \tag{11}$$

The boundary conditions (equations (7), (8)) reduce to

$$u = u_s \text{ at } r = R(z)$$
(12)
And τ is finite at $r = 0$ (13)

The geometry of the stenosis in the non-dimensional form is given by:

$$R(z) = \begin{cases} 1 - \frac{\delta}{2} \left(1 + \cos \frac{\pi z}{z_0} \right); & for |z| \le z_0 \\ 1; & for |z| > z_0 \end{cases}$$
(14)

METHOD OF SOLUTION

On using perturbation method, the velocity *u*, and shear stress, τ are expressed in terms of Womersley parameter, α^2 (where $\alpha \ll 1$)

$$\begin{aligned} u(z,r,t) &= u_0(z,r,t) + \alpha^2 u_1(z,r,t) + \dots \dots \\ (15) \\ \tau(z,r,t) &= \tau_0(z,r,t) + \\ \alpha^2 \tau_1(z,r,t) + \dots \dots (16) \end{aligned}$$

Substituting (15) and (16) in equation (10) and equating the constant terms and α^2 , we get:

$$\frac{\partial}{\partial r}(r\tau_0) = -2r[(1+e\cos t) + B\cos(\omega t + \emptyset)]$$
(17)

$$\frac{\partial u_0}{\partial t} = \frac{2}{r} \cdot \frac{\partial}{\partial r} (r\tau_1)$$
(18)

Integrating equation (17) and using boundary condition (13), we get: $\tau_0 = -f(t) \cdot r$ (19)

where,
$$f(t) = [(1 + e\cos t) + B\cos(\omega t + \emptyset)]$$
(20)

Substituting u from equation (15) into condition (12), we get

$$u_0 = u_s, u_1 = 0 \ atr = R(z)$$
 (21)

Substituting (15) and (16) in (11), we get $\frac{\partial \mu_0}{\partial \mu}$

$$-\frac{\partial u_0}{\partial r} = 2[\theta + |\tau| - 2.\sqrt{\theta \tau_0}]$$
(22)
$$-\frac{\partial u_1}{\partial r} = 2|\tau_1| \cdot \left[1 - \sqrt{\theta/\tau_0}\right]$$
(23)

Integrating equation (22), and using the relation (19) and the boundary condition (21), we obtain

$$u_{0} = f(t)R^{2} \left[1 - \left(\frac{r}{R}\right)^{2} - \frac{8}{3} \frac{k}{\sqrt{R}} \left\{ 1 - \left(\frac{r}{R}\right)^{3/2} \right\} \frac{2 \cdot k^{2}}{R} \left\{ 1 - \left(\frac{r}{R}\right)^{3} \right\} \right] + u_{s} \quad (24)$$

Where $k^{2} = \frac{\theta}{f(t)}$.

Similarly the solutions for τ_1 and u_1 can be obtained using equations (18), (23), and (24) as:

$$\tau_{1} = \frac{f'(t).R^{3}}{8} \left[22 \left(\frac{r}{R}\right) - \left(\frac{r}{R}\right)^{3} - \frac{8}{21} \frac{k}{\sqrt{R}} \left\{ 7 \left(\frac{r}{R}\right) - 4 \left(\frac{r}{R}\right)^{5/2} \right\} \right]$$
(25)
$$u_{1} = \frac{f'(t)R^{4}}{16} \left[\left(\frac{r}{R}\right)^{4} + 4 \left(\frac{r}{R}\right)^{2} + 3 + \frac{k}{\sqrt{R}} \left\{ \frac{16}{3} \left(\frac{r}{R}\right)^{2} - \frac{424}{147} \left(\frac{r}{R}\right)^{7/2} + \frac{16}{3} \left(\frac{r}{R}\right)^{3/2} - \frac{1144}{147} \right\} + \frac{k^{2}}{R} \left\{ \frac{128}{63} \left(\frac{r}{R}\right)^{3} - \frac{64}{9} \left(\frac{r}{R}\right)^{3/2} + \frac{320}{63} \right\} \right]$$
(26)

Using equations (15) and (16), the total velocity distribution and wall shear stress can be written as

$$u = f(t) \cdot R^{2} \left[1 - \left(\frac{r}{R}\right)^{2} - \frac{8}{3} \frac{k}{\sqrt{R}} \left\{ 1 - \left(\frac{r}{R}\right)^{3/2} \right\} + \frac{2k^{2}}{R} \left\{ 1 - \frac{r}{R} \right\} + \frac{\alpha^{2}R^{2}C}{16} \left\{ \left(\frac{r}{R}\right)^{4} - 4\left(\frac{r}{R}\right)^{2} + 3 + \frac{k}{\sqrt{R}} \left\{ \frac{16}{3}\left(\frac{r}{R}\right)^{2} - \frac{424}{147} \left(\frac{r}{R}\right)^{7/2} + \frac{16}{3}\left(\frac{r}{R}\right)^{3/2} - \frac{1144}{147} \right\} + \frac{k^{2}}{R} \left\{ \frac{128}{63}\left(\frac{r}{R}\right)^{3} - \frac{64}{9}\left(\frac{r}{R}\right)^{3/2} + \frac{320}{63} \right\} \right] + u_{s}$$
(27)
$$\tau_{w} = f(t)R \left\{ 1 + \frac{\alpha^{2}R^{2}C}{8} \left(1 - \frac{8k}{7\sqrt{R}}\right) \right\}$$
(28)

Where
$$C = \frac{f'(t)}{f(t)}$$

The volumetric flow rate flow rate *Q* is given by:

$$Q(z,t) = 4 \int_{0}^{R(z)} r.u(z,r,t)dr$$

Where $Q(z,t) = \frac{\bar{Q}(\bar{z},\bar{t})}{\pi\bar{R}_{0}A_{0}/8\bar{\mu}}$
$$= f(t)R^{4} \left[\frac{1}{4} + \frac{4}{7}\frac{k}{\sqrt{R}} + \frac{1}{3}\left(\frac{k}{\sqrt{R}}\right)^{2} + \frac{\alpha^{2}R^{2}C}{16}\left\{\frac{2}{3} + \frac{120}{177}\frac{k}{\sqrt{R}} + \frac{32}{35}\left(\frac{k}{\sqrt{R}}\right)^{2}\right\} + 2u_{s}R^{2}$$
(29)

The effective viscosity $\bar{\mu}_e$ is defined as:

$$\bar{\mu}_e = \frac{\pi \cdot \left(-\frac{\partial \bar{p}}{\partial \bar{z}}\right) \left(\bar{k}(\bar{z})\right)^4}{\bar{q}(\bar{z},\bar{t})} \quad [22] \tag{30}$$

The non-dimensional form of effective viscosity is

$$\mu_e = \frac{(R(z))^2}{Q(z,t)} (1 + e \cos t)$$
$$= R^2 (1 + e \cos t) \left[f(t) R^2 \left\{ \frac{1}{4} + \frac{4}{7} \frac{k}{\sqrt{R}} + \frac{1}{3} \frac{k^2}{R} + \frac{a^2}{R} + \frac$$

RESULTS & DISCUSSION

In order to have an estimate of the quantitative effects of various parameters involved in this analysis, the relevant computational work has been performed for some specific cases using available experimental data. The purpose of this numerical computation is to bring out the effects of periodic body acceleration, slip velocity, stenotic height on the pulsatile flow of blood through stenosed artery taking the non-Newtonian (Casson fluid model) for blood. On using perturbation method, the velocity u is expanded in terms of the Womersley frequency parameter α^2 (where $\alpha^2 \ll 1$). The assumption of the small value of α is valid for physiological situations in small blood vessels. In most of the earlier investigations, the boundary condition is no-slip condition (velocity continuity). In this investigation two values of slip velocity $u_s = 0$ (no slip) and $u_s = 0.05$ are taken.

As velocity profiles play an important role in the flow field so the results for the axial velocity profiles of the streaming blood are studied under slip ($u_s = 0.05$) and no slip condition ($u_s = 0$) and body acceleration parameter B = 0, 1, 2.

Fig.2 and Fig.3 illustrates that the axial velocity increase with radial distance and attains its maximum value at the axis (r = 0) and the minimum at the boundary (r = R (z)). In the presence of body acceleration, velocity increases rapidly. As the body acceleration increases, the plug region shrinks so more flow takes place. Fig.4 illustrates that the axial velocity increase with radial distance and

attains its maximum value at the axis (r = 0) and the minimum at the boundary (r = R (z)). It also shows the variations of the axial velocity with B, δ and θ .Fig.5, Fig.6and Fig.7 illustrates that the axial velocity increase with radial distance and attains its maximum value at the axis (r = 0) and the minimum at the boundary (r = R(z)). The axial velocity increases with the decrease in δ , the maximum height of the stenosis.Fig.8 and Fig.9 illustrates that for B=0, the volumetric flow flux (Q) directly proportional to e, with the increasing value of δ and u_s , where e is the ratio of amplitude of the fluctuating component to steady component of pressure gradient. And also the value of O increases with the increase in θ . Fig.10 and Fig.11 illustrates that for B=0, the volumetric flow flux (Q) directly proportional to e, with the increasing value of u_s . And also the value of Q increases with the increase in \Box . When the \Box increases, the rate of change of Q also increases proportionally. Fig.12 illustrates the volumetric flow flux (Q) directly proportional to \Box , with the increasing value of B. But Q decreases with the increase in □.Fig.13 and Fig.14 illustrates that for B=0, the volumetric flow flux (Q) decreases with the increase in t till a certain limit, then increases with t. And also the value of Q increases with the increase in □.Fig.15 and Fig.16 illustrates that for B=1, the volumetric flow flux (Q) decreases with the increase in t till a certain limit, then increases with t. And also the value of O increases with the increase in \Box and the rate of change in Q increases with \Box_{\Box} .Fig.17 and Fig.18 illustrates that, the wall shear stress (\Box) decreases with the increase in t till a certain limit, then increases with t. And also the value of \Box is more or less independent of \Box or e. Fig.19 also illustrates that the wall shear stress (\Box) decreases with the increase in t till a certain limit, then increases with t. It shows the variation of \Box with B and \Box .Fig.20 and Fig.21 illustrates that the effective viscosity (\Box_{\Box}) decreases with the increase in axial distance (z) till a certain limit, then increases with z. The rate of change of \Box_{\Box} increases with the increase in value of stenosis height (\Box) .









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CONCLUSION

The pulsatile flow of blood through narrow stenosed arterial segments with periodic body acceleration has been studied in this analysis, treating blood as a Casson fluid. The axial velocity slip at the constricted wall is given due consideration in the present investigation. It is observed that the body acceleration parameter, radius of stenosis, the slip velocity and the Casson fluid parameters are the strong parameters influencing the flow qualitatively and quantitatively. It is interesting to note that the body acceleration parameter enhances the flow rate. As expected, axial velocity and flow rate both show an increasing tendency with the wall slip but effective viscosity decreases due to slip. Also it is observed that the stenosis height plays a dominant role in the estimation of flow rate and effective viscosity. It is of importance to mention that the effect of yield stress and stenosis reduces the flow rate while the flow resistance is seen to be increased substantially due to the presence of stenosis and yield stress. The presence of body acceleration is to increase the flow rate but reduce the flow resistance. It is interesting to note that the model developed in the paper will throw light on the influence of various parameters on flow characteristics and to ascertain which of the parameters has the most dominating role. Also it is hoped that the analytical study will help the physicians in estimating the severity of stenosis and its consequence. So this may help in taking decisions regarding the treatment of the patients, whether medicine may be applied or the patient will undergo surgery. This study can be further extended by the introduction of more rheological and physical parameters in the analysis.

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