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Research Article

On The System of Double Equations $4x^2-y^2=z^2$ and $x^2+2y^2=w^2$

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Abstract: The system of double equations given by $4x^2 - y^2 = z^2$ and $x^2 + 2y^2 = w^2$ has only a finite number of integer solutions.
Keywords: System of double equations, Integer solutions.

INTRODUCTION

The solvability in integers of systems of simultaneous pell equations of the form $x^2 - ay^2 = 1, y^2 - bz^2 = 1$, where a and b are distinct non-square positive integers, have been discussed in [1-3].Using the theorems due to Siegel [4] and Baker[5], it is possible to show that the number of solutions to the above system of equations is always finite and it is possible to give a complete list of them. Indeed, M.A. Bennett [6] has proved that the above system of equations possesses at most three solutions in positive integers x, y, z .Further there are infinite families (a, b) for which the above system has at least two positive solutions .Mihai cipu [7] has proved that, for positive integers m and b, the number of simultaneous solutions in positive integers to $x^2 - (4m^2 - 1)y^2 = 1, y^2 - bz^2 = 1$ is at most one. In [8] the authors have showed that the system of pell equations $y^2 - 5x^2 = 4$ and $z^2 - 442x^2 = 441$ has no positive integer solutions. In this context one may refer [9,10]. The above results motivated us to search for the integral solutions of the simultaneous equations $4x^2 - y^2 = z^2$ and $x^2 + 2y^2 = w^2$.It is observed that there exists only eighteen integer solutions.

METHOD OF ANALYSIS

The system of double equations under consideration is

$$4x^2 - y^2 = z^2 \qquad (1)$$

$$x^2 + 2y^2 = w^2 \qquad (2)$$

At the outset it is noted that the quadruple $(1, 2, 0, 3)$ satisfies the system (1) and (2)
Due to symmetry the following quadruples

$$(-1,2,0,3),\quad (-1,2,0,-3),\quad (1,-2,0,3),\quad (1,-2,0,-3),$$

$$(-1,2,0,-3),\ (-1,-2,0,3), (-1,-2,0,-3), (1,2,0,-3)$$

also saisfy (1) and (2)

Now taking $z = 2w$ (3)

In (1) and subtracting (2) from (1), we have

$$x^2 = y^2 + w^2,$$

which is the well known Pythagorean equation satisfied by

$$x = r^2 + s^2, \quad y = r^2 - s^2 \ , \ \text{w=2rs}$$

and thus from (3),

$z = 4rs$

It is seen that the above values of x, y, z and w satisfy the system (1), (2) provided $r = s$. Thus the solution of the system (1) and (2) is ($(1,0,2,1)$

Because of symmetry of x, y and z, the following quadruples also satisfy the system (1) and (2):

$(-1,0,2,1),\quad (1,0,-2,1),\quad (1,0,2,-1),\quad (1,0,-2,-1),$

$(1,0,2,-1),\quad (-1,0,2,-1),\quad (-1,0,-2,1),\quad (-1,0,-2,-1)$

Alternatively, (2) is satisfied by $x = 2r^2 - s^2$, $y = 2rs$, w=$2r^2 + s^2$.Substituting the values of x and y in (1), it is observed that it will be satisfied provided $4r^4 + s^4 - 5r^2s^2$ is a perfect square, which is not possible as it can be written only as the difference of two squares.

Thus it seems that the system (1) and (2) has only a finite number of integer solutions

CONCLUSION

To conclude, one may search for the existence of other choices of solutions to the system under consideration

MSC 2000 Mathematics subject classification: 11D99

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