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Research Article

Observations on the hyberbola $x^2 = 19y^2 - 3^t$

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Abstract: The binary quadratic equation $x^2 = 19y^2 - 3^t$ is considered and a few interesting properties among the solutions are presented. Employing the integral solutions of the equation under consideration, a few patterns of Pythagorean triangles and rectangles are observed
Keywords: Binary quadratic, integral solutions

INTRODUCTION

The binary quadratic equation of the form $y^2 = Dx^2 + 1$ where D is non-square positive integer has been studied by various mathematicians for its non-trivial integral solutions when D takes different integral values [1,2,3,4] .In [5] infinitely many Pythogorean triangles in each of which hypotenuse is four times the product of the generators added with unity are obtained by employing the non-integral solutions of binary quadratic equation $y^2 = 3x^2 + 1$.In [6],a special Pythogorean triangle is obtained by employing the integral solutions of $y^2 = 10x^2 + 1$. In [7], different patterns of infinitely many Pythogorean triangles are obtained by employing the non-integral solutions of $y^2 = 12x^2 + 1$.In this context one may also refer [8-14]. These results have motivated us to search for the integral solutions of yet another binary quadratic equation $x^2 = 19y^2 - 3^t$ representing a hyberbola. A few interesting properties among the solutions are presented.

METHOD OF ANALYSIS

The binary non-homogeneous quadratic Diophantine equation represents a hyberbola to be solved for its non-zero integral solutions is

$$x^2 = 19y^2 - 3^{2t+1} \tag{1}$$

whose initial solution is $x_0 = 7(3^{t-1}),\ y_0 = 2(3^{t-1}),\ t \ge 1$ (2)

To find the other solutions of (1),consider the pellian equation of (1) given by

$$x^2 = 19y^2 + 1 \tag{3}$$

Whose general solution $(\tilde{x}_n, \tilde{y}_n)$ is represented by

$$\left.\begin{aligned} \tilde{x}_n &= \frac{f_n}{2} \\ \tilde{y}_n &= \frac{g_n}{2\sqrt{19}} \end{aligned}\right\} \qquad (4)$$

where

$$\left.\begin{aligned} f_n &= \frac{1}{2}\left[(170+39\sqrt{19})^{n+1} + (170+39\sqrt{19})^{n+1}\right] \\ g_n &= \left[(170+39\sqrt{19})^{n+1} - (170+39\sqrt{19})^{n+1}\right] \end{aligned}\right\}$$

where $n = 0,1,2,$

Employing the lemma of Brahamagupta between the solution (x_0, y_0) and $(\tilde{x}_n, \tilde{y}_n)$.The general solution of (1) is found to be

$$\left.\begin{aligned} x_{n+1} &= (3^{t-1})\left[\frac{7}{2} f_n + \sqrt{19} g_n\right] \\ y_{n+1} &= (3^{t-1})\left[f_n + \frac{7}{2}\frac{g_n}{\sqrt{19}}\right], n \geq 0,1,2,3... \end{aligned}\right\} \quad \text{.................} (5)$$

A few numerical examples are presented in the table below:

n	x_n	y_n
0	$7(3^{t-1})$	$2(3^{t-1})$
1	$2672(3^{t-1})$	$613(3^{t-1})$
2	$908473(3^{t-1})$	$208418(3^{t-1})$
3	$308878148(3^{t-1})$	$70861507(3^{t-1})$

A few interesting properties are given below:

1. $x_{2n+1} \equiv 0 \pmod 2$

2. The recurrence relations satisfied by the values of x_{n+1} and y_{n+1} are respectively

$$x_{n+3} - 340x_{n+2} + x_{n+1} = 0, x_0 = 7(3^{t-1}), x_1 = 2672(3^{t-1})$$

$$y_{n+3} - 340y_{n+2} + y_{n+1} = 0, y_0 = 2(3^{t-1}), x_1 = 613(3^{t-1})$$

3. A few interesting relations among the solutions are exhibited below:

(a) $x_{n+2} = 741y_{n+1} + 170x_{n+1}$

(b) $x_{n+3} = 251940y_{n+1} + 57799x_{n+1}$

(c) $y_{n+2} = 170y_{n+1} + 39x_{n+1}$

(d) $y_{n+3} = 57799y_{n+1} + 13260x_{n+1}$

4. $6\left[\frac{4}{3^{t+2}}(19y_{2n+2} - \frac{7}{2}y_{2n+2}) + 2\right]$ is a nasty number

5. $\frac{4}{3^{t+2}}(19y_{3n+3} - \frac{7}{2}x_{3n+3} + 57y_{n+1} - \frac{21}{2}x_{n+1})$ is a cubical integer

6. Employing the solutions of (1),each of the following among the special Polygonal,Pyramidal,Star number,Centered Pyramidal number and Pronic numbers is congruent to zero under modulo 3^{2t+1}

(a) $\left(\frac{3P^3_{x-2}}{t_{3,x-2}}\right)^2 - 19\left(\frac{6P^4_{y-1}}{t_{3,2y-2}}\right)^2$

(b) $\left(\dfrac{P_x^5}{t_{3,x}}\right)^2 - 19\left(\dfrac{4P_y^5}{Ct_{4,y}-1}\right)^2$

(c) $\left(\dfrac{18P_{x-2}^3}{Ct_{6,x-2}-1}\right)^2 - 19\left(\dfrac{6P_{y-1}^5}{Ct_{6,y}-1}\right)^2$

(d) $\left(\dfrac{6P_x^3}{\Pr_x}\right)^2 - 19\left(\dfrac{6P_y^5}{S_{y+1}-1}\right)^2$

(7) The solutions of (1) interms of special integers namely,Generalized Lucas GL_n and Fibonacci GF_n numbers are exhibited below:

$$\tilde{x}_n = \frac{GL_{n+1}}{2}(340,-1)$$

$$\tilde{y}_n = 39GF_{n+1}(340,-1)$$

Remark:

It is worth to note that (1) may also be satisfied by $x_0 = 4(3^t), y_0 = 3^t, t \geq 0$

Following the analysis Presented above ,the sequence of integer solutions of (1) are obtained as

$$x_{n+1} = (3^t)\left[2f_n + \frac{\sqrt{19}}{2}g_n\right]$$

$$y_{n+1} = (3^t)\left[\frac{1}{2}f_n + \frac{2}{\sqrt{19}}g_n\right], n \geq 0,1,2,3..$$

CONCLUSION

In this paper ,we have presented non-zero distinct integer solutions of the pell equation $x^2 = 19y^2 - 3^t$ when t is odd. It is to be noted that the above pell equation has no integer solutions when t is even since the negative pell equation $x^2 = 19y^2 - 1$ has no integer solutions.

To conclude, one may search for other choices of negative pell equations for finding their integer solutions.

Mathematics Subject Classification:11D09

Notations

$t_{m,n}$: Polygonal number of rank n with size m

P_n^m : Pyramidal number of rank n with size m

$\Pr_n$: Pronic number of rank n

S_n : Star number of rank n

$Ct_{m,n}$: Centered Pyramidal number of rank n with size m

$GF_n(k,s)$: Generalized Fibonacci Sequences of rank n

$GL_n(k,s)$: Generalized Lucas Sequences of rank n

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