

## **Research Article**

### **Prediction Method on Wellbore Stability Based on Analytic Hierarchy Process and Fuzzy Evaluation**

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**Abstract:** Wellbore stability are influenced by many complex factors, and there is no commonly used prediction method on wellbore stability until now. Use of an efficient decision making technique which can take into account several criteria in such conditions is necessary. Referring to traditional wellbore stability analyzing methods, the fuzzy comprehensive evaluation method of predicting wellbore stability principle is developed, which is based on physical and chemical performances of clay mineral. In application, it is modified by using analytic hierarchy process to confirm weight that minimizes the effect by human factors, applying the maximum subsection principle to obtain evaluation results. In applications of Hailar area, using this method to predict wellbore stability is of high reliability and strong practicability.

**Keywords:** Wellbore sloughing; Prediction; Fuzzy comprehensive evaluation; Analytic hierarchy process; AHP; Maximum subsection principle

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#### **INTRODUCTION**

Borehole stability technology is one of the key technologies in drilling process. Wellbore stability are influenced by many complex factors, and there is no commonly used prediction method on wellbore stability until now. Wellbore instability is a major problem confronted in drilling engineering, which can cause great difficulty for drilling and result in accidents, such as shrinkage, collapse, borehole enlargement, low cementing quality, etc. These accidents not only prolong drilling cycle, but also increase drilling costs[1-4]. Studies by many domestic and foreign scholars have been committed to the wellbore stability technology, but so far there is no method can accurately assess the wellbore stability condition. AHP is a process of modeling and quantizing complex system on the decision-making. In this paper, we focus on science and rationality of the prediction method using analytic hierarchy process(AHP) and fuzzy evaluation from the aspects of theory and practice. This method will provide a new way to scientific evaluation of wellbore stability for field drilling engineers.

#### **MATERIAL AND METHODS**

The analytic hierarchy process (AHP) is a structured technique for organizing and analyzing complex decisions, based on mathematics and psychology. It was developed by Thomas L. Saaty in the 1970s and has been extensively studied and refined

since then. It is a combination of qualitative and quantitative decision analysis method. Using this method, we divide complex problem into several levels or several factors, and make simple comparisons and calculations at each level or among various factors. AHP method is widely used in economic management, city planning, energy system analysis, etc. We can draw the weights of different factors to provide the basis for decision-making, control or prediction[5-6].

Weight plays a vital role in the fuzzy comprehensive evaluation, the conventional method based on experience of determining the weight of each factor sometimes cannot be accurately assigned weights, especially when many factors and the relationship between these factors are not clear. Therefore, the application of AHP to determine the weight of each factor can reduce man-made interference on the weights, so the weight distribution can be more reasonable[7-8].

An important feature of the analytic hierarchy process is the use of ratio of importance degree between each other to express two scheme of the relative importance degree. We make comparisons between  $n$  schemes in the criteria, and rate them according to their degree of importance denoted as importance ratio of  $i$  and  $j$  factor. Table 1 lists 9 levels of importance and assignment which are put forward by Saaty. Result

matrix according to the comparisons is called the judgment matrix.

In order to extract useful information from the judgment matrix and find the regularity to provide scientific basis for decision-making, we need to calculate weight vector of judgment matrix. For the judgment matrix satisfies the consistency condition, we can calculate the maximum eigenvector, then normalize and use it as the weight[9]. Therefore, constructing

judgment matrix meet consistency the requirements of the following:

If judgment matrix  $P$ , such as  $p_{ij} > 0$ ;  $p_{ii} = 1$ ;  $p_{ij} = 1/p_{ji}$ ;  $p_{ij} = p_{ik}/p_{jk}$  ( $i, j, k = 1, 2, \dots, n$ ) is established, then  $P$  is said to satisfy the complete consistency, and  $P$  is called consistent matrix. But in fact it is impossible for the pairwise comparison matrices to satisfy such above equations. So we step back and define if pairwise comparison matrices have certain consistency, we can call it acceptable consistency.

**Table 1 Ratio Scale**

Factors of $i$ than $j$	Quantitative value
Equally important	1
Slightly more important	3
More important	5
Much more important	7
Extremely more important	9
Values between two adjacent judgment	2, 4, 6, 8

Calculating the maximum eigenvalue and the corresponding eigenvector for each judgment matrix to check consistency by consistency index, random consistency index and the ratio of consistency. If the test is passed, the feature vector (normalized) is the weight vector; if not, it is needed to reconstruct pairwise judgment matrix. The steps for pairwise judgment matrix consistency test are as follows:

- 1) Calculating the measure of inconsistent degree index  $CI$  for  $P$  judgment matrix :

$$CI = \frac{\lambda_{\max}(P) - n}{n - 1} \quad (1)$$

In this equation,  $\lambda_{\max}$  is the largest eigenvalue of matrix  $P$ . The larger values consistency index( $CI$ ) is, the more judgment matrix deviate from complete consistency; On the contrary, the smaller the  $CI$  value

is, the judgment matrix is closer to the complete consistency. Generally, the bigger order number( $n$ ) of judgment matrix is, the larger man-made deviation value from perfect consistency index( $CI$ ) will be; the smaller  $n$  is, the smaller man-made deviation value from perfect  $CI$  will be.

- 2) For the multi-order judgment matrix  $P$ , taking into account the deviation consistency may due to random reasons, so when the consistency of judgment matrix is tested, the mean random consistency standard  $RI$ (Random Index) is introduced.  $RI$  is only related to the order of matrix  $n$ . In general, the bigger order of matrix is, the greater possibility of uniform random deviation is. Table 2 gives the average consistency index of positive reciprocal matrix whose order is from 1 to 15 by 1000 times calculations.

**Table 2 The average random consistency index  $RI$  value**

Order of Matrix	RI						
1	0	5	1.12	9	1.46	13	1.56
2	0	6	1.25	10	1.49	14	1.58
3	0.52	7	1.35	11	1.52	15	1.59
4	0.90	8	1.42	12	1.54		

- 3) Random consistency ratio  $CR$ . The complete consistency of judgment matrix always exists when  $n < 3$ . Ratio of consistency index of judgment matrix  $CI$  and mean random with the same order consistency index  $RI$  is called random consistency ratio  $CR$ .

$$CR = \frac{CI}{RI} \quad (2)$$

When  $CR < 0.10$ , then we can say that judgment matrix meet required consistency, or the level of inconsistency is acceptable. When  $CR > 0.10$ , adjustment

is needed to modify the judgment matrix, which can meet  $CR < 0.10$  to satisfy consistency.

For the matrix that already meets the consistency, we can use sum-product method to get feature vector, so it can be normalized as weights. The specific steps are as follows.

1) Get each column of the judgment matrix be normalized, the general term for the element is:

$$\overline{p_{ij}} = \frac{p_{ij}}{\sum_1^n p_{ij}} \quad (i, j=1, 2, 3, \dots, n) \quad (3)$$

2) Plus up each column of the normalized judgment matrix by the line as follows:

$$\overline{W_{ij}} = \sum_1^n \overline{p_{ij}} \quad (i, j=1, 2, 3, \dots, n) \quad (4)$$

3) Normalized vector  $W = (\overline{W}_1, \overline{W}_2, \dots, \overline{W}_n)^T$ :

$$W_i = \frac{\overline{W}_i}{\sum_1^n \overline{W}_i} \quad (i, j=1, 2, 3, \dots, n) \quad (5)$$

we can get:

$$W = (W_1, W_2, \dots, W_n)^T \quad (6)$$

Despite the usefulness of AHP handling multi-criteria decision making problems, there are fuzziness and vagueness in many decision-making problems and the method may lead to the inexact judgments of decision makers. The comprehensive evaluation is to make a reasonable comprehensive evaluation of attributes and factors for multi-attributes things, or things which overall quality is affected by many factors. Fuzzy comprehensive evaluation method uses the theory and method of fuzzy mathematics to achieve exact solution that is not easy to define the real world or a mathematical method of imprecise incomplete information by comprehensive evaluation. This method can naturally handle human thinking initiative and fuzziness. So, for these factors involved fuzzy factors, make reasonable evaluation, in most cases, using the fuzzy mathematics method evaluation is a feasible and effective way [10-13]. General steps for fuzzy comprehensive evaluation method are as follows:

**Establishing factor set**

Assuming that the research issues are  $n$  factors, the factors of evaluation objects as elements of an ordinary set called the factor set, namely  $U = \{u_1, u_2, \dots, u_n\}$ .

**Establishing evaluation set**

The evaluation set  $V = \{v_1, v_2, \dots, v_m\}$  is set consisting of  $m$  kinds of evaluation factors. The elements of the number and the name was determined according to the actual problems.

**Determination of membership function**

The thought of membership degree is the basic idea of fuzzy mathematics. To determine the degree of membership, fuzzy statistical method and expertise method are commonly used. After establishing the membership function, according to the level of delineation, we should count the number values of the factors in line with a particular level, and calculate its share in the ratio of the factors. The ratio is fuzzy evaluation matrix  $R$  in  $r_{ij}$ .

$$R = \begin{pmatrix} r_{11} & r_{12} & \dots & r_{1m} \\ r_{21} & r_{22} & \dots & r_{2m} \\ \dots & \dots & \dots & \dots \\ r_{n1} & r_{n2} & \dots & r_{nm} \end{pmatrix} \quad (7)$$

**Establishing weight set**

Weight is the influence of single factor in all evaluation factors. Practice in a large amount of drilling has proved that for different wells, the importance of the above factors is generally not the same. For the selected shale stability evaluation factors, their impact on the final evaluation purpose and results are different. Therefore, in order to reflect the importance of each factor, factor should be given the appropriate weights. Each set which is made up of a collection of weights called factor weight set  $A = (a_1, a_2, \dots, a_n)$  [14]. Meanwhile, the number of each weight should satisfy the normalization condition, namely  $a_1 + a_2 + \dots + a_n = 1$ .

**Establishing fuzzy comprehensive evaluation model**

Fuzzy comprehensive evaluation method is based on a variety of factors into account to evaluate the stability of the shale so that it can be a valuable reference. By the knowledge of fuzzy mathematics, the fuzzy comprehensive evaluation formula is:

$$B = AR \quad (7)$$

In the equation,  $A$  is the weight set,  $A = (a_1, a_2, \dots, a_n)$ ;  $R$  is fuzzy comprehensive evaluation matrix,  $R = (r_{ij})_{n \times m}$ ;  $B$  is evaluation result,

$$B = (b_1, b_2, \dots, b_m), \quad b_j = \sum_{i=1}^n a_i \cdot r_{ij}$$

**EXPERIMENTAL SECTION**

The physicochemical properties of clay mineral is the basis for studying the mechanism of wellbore instability and technical strategies. The cation exchange capacity(CEC), potential measurement, expansion rate experiment, rolling recovery experiment, determination of formation water activity and other indoor experiment are used to acquire the experimental data so we can evaluate the expansion and dispersion of clay minerals. 15 groups of shale samples experiment data from Yimin group of Wuerxun depression in Hailar area is showed in Table 3 [15-16].

**Table 3 Hailar area shale sample data wellbore stability analysis**

$\rho / \text{g/cm}^3$	expansibility %	Rate of recovery/%	CEC/ mmol/100g	$\zeta$	Water activity
2.70	12.55	89.12	8.71	-16.34	0.79
2.61	11.69	97.36	7.65	-17.61	0.76
2.72	12.12	97.23	7.91	-13.82	0.83
2.67	11.97	82.83	8.03	-11.88	0.80
2.63	8.31	96.19	7.45	-14.16	0.75
2.75	11.42	89.46	7.44	-8.13	0.74
2.74	11.14	97.22	5.91	-11.16	0.81
2.72	11.46	87.62	6.13	-12.75	0.85
2.66	9.78	94.21	7.31	-8.61	0.84
2.64	6.68	99.41	2.26	-8.83	0.83
2.73	4.94	92.85	7.48	-10.28	0.81
2.69	5.62	96.83	7.64	-10.47	0.85
2.63	7.59	88.13	4.24	-9.16	0.84
2.73	11.03	84.84	2.73	-7.14	0.83
2.59	6.65	98.31	5.16	-7.51	0.82

**Determining the factor set**

From the table above, to evaluate mud shale stability, we identified 6 factors, which consists of a collection of  $U = \{u_1, u_2, u_3, u_4, u_5, u_6\}$ . Among them,  $u_1$  is the density;  $u_2$  is the expansion;  $u_3$  is recovery rate;  $u_4$  is the cation exchange capacity;  $u_5$  is for Zeta potential;  $u_6$  is for water activity.

**Determining the evaluation set**

According to the wellbore stability evaluation, we determined for 5 kinds of results. It is divided to Grade

1, Grade 2, Grade 3, Grade 4, Grade 5 based on the stability descendingly.  $V = \{v_1, v_2, v_3, v_4, v_5\}$ , wherein  $v_1$  represents a very stable state;  $v_2$  represents a relatively stable state;  $v_3$  is for a stability state;  $v_4$  indicates a instability state;  $v_5$  represents a very instable state.

**Determining the weight set**

According to the relationship between various factors in this case, we can determine the judgment matrix, as shown in table 4.

**Table-4: The judgment matrix,**

P	P <sub>1</sub>	P <sub>2</sub>	P <sub>3</sub>	P <sub>4</sub>	P <sub>5</sub>	P <sub>6</sub>
P <sub>1</sub>	1	1/5	1/5	1/4	1/3	1/2
P <sub>2</sub>	5	1	1	2	3	4
P <sub>3</sub>	5	1	1	2	3	4
P <sub>4</sub>	4	1/2	1/2	1	2	2
P <sub>5</sub>	3	1/3	1/3	1/2	1	2
P <sub>6</sub>	2	1/4	1/4	1/2	1/2	1

Note: The matrix A is a judgment based on the information data, observations and repeated research experience determined carefully by experts;  $P_1$  is the density,  $P_2$  is expandability,  $P_3$  is recovery,  $P_4$  is cation exchange capacity,  $P_5$  is  $\zeta$  potential,  $P_6$  is water activity.

According to the above proposed method of testing the consistency of judgment matrix, consistency index and random consistency index were calculated :  $CI = 0.026$  8,  $RI = 1.25$ , resulting in a value of consistency ratio:  $CR = 0.033$  92.

In this case, the judgment matrix  $P$  is not completely consistent, but due to the random consistency ratio  $CR = 0.03392 < 0.10$ , so that the consistency of judgment matrix  $P$  has satisfactory. In

accordance with the product and method, we can calculate the weight vector:

$$A=W=(0.05, 0.3, 0.3, 0.17, 0.11, 0.07) \quad (8)$$

**Determining fuzzy comprehensive evaluation matrix**

1. Establish the membership function of various factors. Due to various factors are in different dimensions, we cannot directly calculate the matrix. We should first establish the membership function of various factors, which each factor is normalized. In this case, membership functions is a

gradient distribution function :

$$\mu_1(x) = \frac{x-b}{a-b} \quad (a \leq x \leq b) \quad (9)$$

Wherein:  $a$  is a minimum value of the evaluation factors;  $b$  an evaluation factor for the maximum. The maximum benefit of this model is in determining the membership function on the data were normalized.

- Determine the evaluation matrix. We can use that function to calculate the degree of membership  $\mu$ .

$$R = \begin{pmatrix} 2/15 & 1/5 & 2/15 & 2/15 & 2/5 \\ 2/15 & 1/5 & 1/15 & 1/15 & 8/15 \\ 2/15 & 4/15 & 0 & 2/15 & 7/15 \\ 2/15 & 1/15 & 2/15 & 2/15 & 8/15 \\ 2/15 & 2/15 & 2/15 & 1/15 & 2/15 \\ 2/15 & 2/15 & 1/15 & 1/15 & 7/15 \end{pmatrix} \quad (10)$$

If  $0 \leq \mu < 0.2$ , it is of  $v_1$  level of the concentration evaluation;  $0.2 \leq \mu < 0.4$ , it is of  $v_2$  level of the concentration of the evaluation;  $0.4 \leq \mu < 0.6$ ,  $v_3$  level;  $0.6 \leq \mu < 0.8$ ,  $v_4$  stage;  $0.8 \leq \mu < 1.0$ , is the part of  $v_5$  level of the concentration evaluation.

- According to the gradient distribution function to determine the membership degree of each single factor, through calculation, matrix can be obtained by the following:

### RESULTS AND DISCUSSION

After determining the evaluation matrix and weight set, it can be judged by these two according to equation  $B=AR$ .

$$B=WR=(0.05, 0.3, 0.3, 0.17, 0.11, 0.07) \times \begin{pmatrix} 2/15 & 1/5 & 2/15 & 2/15 & 2/5 \\ 2/15 & 1/5 & 1/15 & 1/15 & 8/15 \\ 2/15 & 4/15 & 0 & 2/15 & 7/15 \\ 2/15 & 1/15 & 2/15 & 2/15 & 8/15 \\ 2/15 & 2/15 & 2/15 & 1/15 & 2/15 \\ 2/15 & 2/15 & 1/15 & 1/15 & 7/15 \end{pmatrix} = (0.133, 0.185, 0.069, 0.125, 0.487) \quad (11)$$

So we can get evaluation results:  $B=(b_1, b_2, b_3, b_4, b_5) = (0.133, 0.185, 0.069, 0.125, 0.487)$

Before using the principle of maximum membership degree to process evaluation results, the results need to be discussed in order to test the effectiveness of the principle of maximum degree of

membership[17-18]. Evaluation index is calculated in accordance with the principle of maximum membership to determine the effectiveness of the method.

$$\beta = \max_{1 \leq j \leq n} b_j / \sum_{j=1}^n b_j = 0.487 / (0.133 + 0.185 + 0.069 + 0.125 + 0.487) = 0.487 \quad (12)$$

$$\beta' = \frac{\beta - (1/n)}{1 - (1/n)} = \frac{n\beta - 1}{n - 1} = \frac{5 \times 0.487 - 1}{5 - 1} = 0.35875 \quad (13)$$

$$\gamma = \sec b_j = 0.185 / (0.133 + 0.185 + 0.069 + 0.125 + 0.487) = 0.185 \quad (14)$$

$$\gamma' = 2\gamma = 2 \times 0.185 = 0.37 \quad (15)$$

$$\alpha = \beta' / \gamma' = 0.35875 / 0.37 = 0.97 \quad (16)$$

Evaluation  $\alpha = 0.97$ , which is very close to 1, indicating that the results of the implementation of the principle of maximum membership is very effective and of relatively high credibility. So using the principle of maximum membership degree to process evaluation results  $b_j$  ( $j=1, 2, 3, 4, 5$ ), that is, taking the maximum of the evaluation results as the final results of the

evaluation. From the results we can see the Level 5 value is the greatest, and therefore the results of its evaluation of the wellbore is very unstable. This result is in line with the actual drilling of the well segment, indicating that the evaluation results are correct. The method can accurately evaluate the situation shale wellbore stability, and it can be greatly useful for on-site drilling decision-makers.

## CONCLUSION

1. Cation exchange capacity (CEC), Zeta potential, expansion rate, roll recovery, formation water activity are measured in laboratory experiments. Physical and chemical properties of soil mineral, dispersion and dilatibility of clay mineral are evaluated. The research lays the foundation of studying borehole instability mechanism and anti-sloughing measures.
2. On the basis of various factors affecting the comprehensive analysis of wellbore stability, combined with the specific circumstances of oil and gas wells engineering practice and relevant experimental data and field data, the use of improved fuzzy comprehensive evaluation method wellbore shale stability for Yimin group of Wuerxun depression upper strata in Hailar region is evaluated and found that the shale in this area is very unstable. This result and the actual situation of the well drilling segment found is consistent.
3. By analyzing the introduction of AHP and the maximum membership principle, the use of fuzzy comprehensive evaluation method for predicting wellbore stability is simple and accurate. This provides a scientific evaluation of new methods for decision-makers for field drilling wellbore stability.

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