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Research Article

Fractional Form Jacobi Elliptic Function Solutions For the second order Benjamin Ono equation

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Abstract: Fractional form Jacobi elliptic function solutions for the second order Benjamin Ono equation were given. Some solutions are new.
Keywords: Jacobi elliptic function, Benjamin Ono equation

INTRODUCTION

To find the periodic solutions for nonlinear evolution equations, Porubov et.al introduced Weirestrass elliptic function expansion method [1-3]. Liu Shi-Kuo et.al introduced Jacobi elliptic function expansion method [4]. Jacobi elliptic function solutions given in these papers all are polynomial form, fractional form Jacobi elliptic function solutions have not been given. In the paper, we will give fractional form Jacobi elliptic function solutions the second order Benjamin Ono equation. Some solutions are new.

The solutions of the second order Benjamin Ono equation

We consider the second order Benjamin Ono equation

$$u_{tt}+2p_1u_x^2+2p_1uu_{xx}+q_1u_{xxxx}=0. \qquad (1)$$

Take traveling wave transformation

$$u=u(\xi_1),\quad \xi_1=x+ct \qquad (2)$$

Substituting (2) into Eq. (1) yields

$$c^2u_{\xi_1\xi_1}+2p_1u_{\xi_1}^2+2p_1uu_{\xi_1\xi_1}+q_1u_{\xi_1\xi_1\xi_1\xi_1}=0, \qquad (3)$$

Integrating (3) and letting the integral constant be zero yield

$$c^2u_{\xi_1}+2p_1uu_{\xi_1}+q_1u_{\xi_1\xi_1\xi_1}=0. \qquad (4)$$

Integrating Eq.(4) gives

$$c^2u+p_1u^2+q_1u_{\xi_1\xi_1}=c_1, \qquad (5)$$

where c_1 is integral constant. Integrating (5), we obtain

$$(u_{\xi_1})^2=-\frac{2p_1}{3q_1}u^3-\frac{c^2}{q_1}u^2+2c_1u+c_2 \qquad (6)$$

where c_1 and c_2 are integral constants. Letting

$$w=(-\frac{2p_1}{3q_1})^{\frac{1}{3}}u, \qquad (7)$$

$$\xi=(-\frac{2p_1}{3q_1})^{\frac{1}{3}}\xi_1, \qquad (8)$$

Equation (6) becomes

$$\left(\frac{dw}{d\xi}\right)^2 = w^3 + a_2 w^2 + a_1 w + a_0, \tag{9}$$

where

$$a_2 = -\frac{c^2}{q_1}(-\frac{2p_1}{3q_1})^{-\frac{2}{3}},\ a_1 = 2c_1(-\frac{2p_1}{3q_1})^{-\frac{1}{3}},\ a_0 = c_2. \tag{10}$$

Denotes $F(w)$ the right polynomial of (9), that is,

$$F(w) = w^3 + a_2 w^2 + a_1 w + a_0. \tag{11}$$

The discrimination of $F(w)$ is

$$\Delta = -27\left(\frac{2}{27}a_2{}^3 + a_0 - \frac{a_1 a_2}{3}\right)^2 - 4\left(a_1 - \frac{a_2{}^2}{3}\right)^3. \tag{12}$$

If $\Delta = 0$, then equation (6) can be solved with elementary functions, see [5] in detail. If $\Delta \neq 0$, then equation (6) can be solved with Jacobi elliptic functions. We obtain the following results

Case 2.1: $F(w) = 0$ has three distinct roots $\alpha < \beta < \gamma$, Respectively, as $\alpha < w < \beta$, the solutions of equation (1) are

$$u = (-\frac{2p_1}{3q_1})^{-\frac{1}{3}}\alpha + (-\frac{2p_1}{3q_1})^{-\frac{1}{3}}(\beta - \alpha)\mathrm{sn}^2\left(\frac{\sqrt{\gamma-\alpha}}{2}(-\frac{2p_1}{3q_1})^{\frac{1}{3}}(\xi_1 - \xi_0), m\right). \tag{13}$$

As $w > \gamma$, the solutions of equation (1) are

$$u = (-\frac{2p_1}{3q_1})^{-\frac{1}{3}} \frac{-\beta sn(\frac{\sqrt{\gamma-\alpha}}{2}(-\frac{2p_1}{3q_1})^{\frac{1}{3}}(\xi - \xi_0), m) + \gamma}{cn(\frac{\sqrt{\gamma-\alpha}}{2}(-\frac{2p_1}{3q_1})^{\frac{1}{3}}(\xi - \xi_0), m)}. \tag{14}$$

Case 2.2: $F(w) = 0$ has only one real root. Letting

$$F(w) = (w - \alpha)(w^2 + pw + q), \tag{15}$$

where $p^2 - 4q < 0$. As $w > \alpha$, respectively, he solutions of equation (1) are

$$u = (-\frac{2p_1}{3q_1})^{-\frac{1}{3}}(\alpha + \frac{2\sqrt{\alpha^2 + p\alpha + q}}{1 + \mathrm{cn}\left((\alpha^2 + p\alpha + q)^{\frac{1}{4}}(-\frac{2p_1}{3q_1})^{\frac{1}{3}}(\xi_1 - \xi_0), m\right)}). \tag{16}$$

They are new Jacobi elliptic function solutions.

Other nonlinear evolution equations like mKdv equation, nonlinear Klein-Gordon equation, Sine-Gordon equation, Boussinesq equation and symmetry long wave equation and so on, can be dealt with similarly.

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