Scholars Journal of Engineering and Technology (SJET) **ISSN 2321-435X (Online)**
Sch. J. Eng. Tech., 2014; 2(4B):561-565 **ISSN 2347-9523 (Print)**
©Scholars Academic and Scientific Publisher
(An International Publisher for Academic and Scientific Resources)
www.saspublisher.com

Research Article

New Matrix Representation of Kinematic Chains and Determination of Isomorphism

Syed Shane Haider Rizvi[1*], Ali Hasan[2], R.A. Khan[3]
[1,2] Mechanical Engineering Department, Jamia Millia Islamia(Central University), New Delhi-110025, India
[3] Mechanical Engineering Department, Galgotias University, Gr.Noida-201308, India

***Corresponding author**
Syed Shane Haider Rizvi
Email: amsalrizvi@gmail.com

Abstract: This paper presents a new method for representing a kinematic chain by a matrix known as chain identification matrix(CIM),which uniquily represents nonisomorphic chains as the eigen vectors of the eigen values of the chain identification matrix of the two isomorphic kinematic chains are same.
Keywords: Chain Identification Matrix, Eigen values, Eigen vectors, Isomorphism, Kinematic Chains

INTRODUCTION

Studies on the matrix representation and determination of isomorphism is avilable in the literature like the weighted adjacency matrix introduced by wilson[1]which assigns a number as a weight to different types of the joints in the kinematic chains , the adjacency matrix which shows the connectivety pattern of the links of the kinematic chains was used by Uicker[2] ,the degree matrix representation of a kinematic chain was presented by Mruthyunjaya[3] and the characteristics coefficients of this matrix is used for isomorphism identification, J.P. Cubillo [4] uses eigen values and eigen vectors of the adjacency matrices for isomorphism detection, Motion Property Adjacency Matrix was introduced by Zang [5] to relate the input and output of a mechanism. Twan. J. Li [6] introduces the method for representing a chain having gears and lower pairs with the help of a weighted graph known as combinatorial graph and the matrix representation of such graph is known as combinatorial Matrix which is used to detect isomorphism among geared kinematic chains. Rao [7] introduces hamming distance matrix to represent a kinematic chain, joint-joint matrix represention of the kinematic chains was inrtoduced by Ali Hasan [8] in which the connectivity pattern is shown interms of the degree of the link connected between two joints, canonocal adjecency matrix obtained from the relabelling of the peremeter topological graph canonocally was introduced by Hufeng and Ding[9], Nageshwara[10] gives the concept of the flow of motion from one link to the other link represented by a flow matrix, Buctcher [11] introduced a stratified adjacency matrix which is an adjecency matrix of simplified graph is obtained by eleminating the binary verteces from a contracted graph of a kinematic chain's monograph and a stratified code for the kinematic chain is obtained and used to eleminate the isomer, representation of a kinematic chain by physical connectivity matrix was introduced by Ali Hasan[12]in which the elements of this matrix are zero or a number which represent the type of the pair between the connection of two links and the sum of the coefficients of absolute characterstics polynomail of this matrix works as an indicator of isomorphism. Yang [13] gives a method of the comparison of two chains by their incident matrices.

PRELIMINARIES

- Definition1:Type of link depends upon the number of connections that a particular link forms if a link is connected with two links it is termed as a binary link, if it is connected with three links then it is termed as ternary link, if it is connected with four links then it is a quaternary link and so on.
- Definition2: Degree of a particular link depends upon its type the degree of a binary link is 2, the degree of a ternary link is 3, the degree of a quaternary link is 4 and so on.
- Definition3: A chain identification matrix(CIM) is a unique representation of the kinematic chain and can be formed by the following rule base.
  - ➢ CIM_{ij} = 0 if the i^{th} link is directly connected to j^{th} link.
  - ➢ CIM_{ij} = n where n is the number of the links shared by i^{th} and j^{th} link if they are not directly connected
  - ➢ CIM_{ij} = 0 if i^{th} and j^{th} link doesn't share any link.
  - ➢ CIM_{ij} = degree of the link if i=j

- Definition4: Isomorphism - Two kinematic chains are isomorphic if there is a one-to-one correspondence between the links of one chain to that of the other chain and if there is no such correspondence, the two chains are non isomorphic.

METHODS

Test for Isomorphism

Let CIM1 and CIM2 be the chain identification metrices of the two kinematic chains and Ψ1 and Ψ2 be the eigenvector matrices for the corresponding CIM1 and CIM2 matrices.

If the matrices Ψ1and Ψ2 becomes identical by the permutations of interchange of rows or columns at the same time the two chains under consideration are isomorphic.

EXAMPLES OF THE KINEMATIC CHAINS

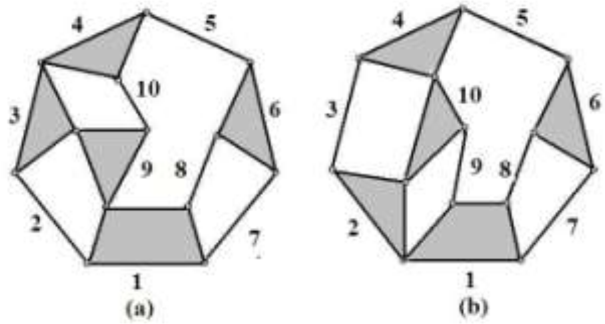

Fig. 1: Pair of 10- link 1-degree of freedom isomorphic chains

$$\text{CIM1(a)} = \begin{bmatrix} 4 & 0 & 2 & 0 & 0 & 2 & 0 & 0 & 0 & 1 \\ 0 & 2 & 0 & 1 & 0 & 0 & 1 & 1 & 2 & 0 \\ 2 & 0 & 3 & 0 & 1 & 0 & 0 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 & 0 & 1 & 0 & 0 & 2 & 0 \\ 0 & 0 & 1 & 0 & 2 & 0 & 1 & 1 & 0 & 1 \\ 2 & 0 & 0 & 1 & 0 & 3 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 2 & 2 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 2 & 2 & 1 & 0 \\ 0 & 2 & 0 & 2 & 0 & 0 & 1 & 1 & 3 & 0 \\ 1 & 0 & 2 & 0 & 1 & 0 & 0 & 0 & 0 & 2 \end{bmatrix} \qquad \text{CIM1(b)} = \begin{bmatrix} 4 & 0 & 1 & 0 & 0 & 2 & 0 & 0 & 0 & 2 \\ 0 & 3 & 0 & 2 & 0 & 0 & 1 & 1 & 2 & 0 \\ 1 & 0 & 2 & 0 & 1 & 0 & 0 & 0 & 0 & 2 \\ 0 & 2 & 0 & 3 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 2 & 0 & 1 & 1 & 0 & 1 \\ 2 & 0 & 0 & 1 & 0 & 3 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 2 & 2 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 2 & 2 & 1 & 0 \\ 0 & 2 & 0 & 1 & 0 & 0 & 1 & 1 & 2 & 0 \\ 2 & 0 & 2 & 0 & 1 & 0 & 0 & 0 & 0 & 3 \end{bmatrix}$$

CIM1(a) and CIM1(b) are the chain identification matrices of the ten link one degree of freedom chains shown in figure 1(a) and 1(b)

$$\Psi1(a) = \begin{pmatrix} -0.0000 & -0.3816 & -0.2544 & 0.0823 & 0.2780 & -0.3927 & 0.2492 & -0.2241 & 0.4878 & 0.4484 \\ -0.0000 & -0.2635 & -0.0528 & -0.5468 & -0.5612 & -0.3274 & -0.0654 & -0.0184 & -0.3354 & 0.2993 \\ 0.0000 & 0.4764 & 0.2847 & -0.3978 & 0.1790 & -0.0806 & -0.3683 & 0.2645 & 0.3936 & 0.3670 \\ -0.0000 & -0.3576 & 0.2786 & -0.0894 & 0.3500 & 0.4373 & -0.3571 & -0.4265 & -0.2626 & 0.3071 \\ 0.0000 & -0.0913 & -0.6368 & -0.2039 & 0.0462 & 0.5672 & 0.0505 & 0.4148 & 0.0305 & 0.2126 \\ 0.0000 & 0.3904 & 0.0862 & -0.0282 & -0.3876 & 0.3827 & 0.4491 & -0.4833 & 0.1825 & 0.2721 \\ 0.7071 & -0.0246 & 0.2937 & 0.0938 & 0.1347 & -0.0069 & 0.3835 & 0.3226 & -0.2550 & 0.2645 \\ -0.7071 & -0.0246 & 0.2937 & 0.0938 & 0.1347 & -0.0069 & 0.3835 & 0.3226 & -0.2550 & 0.2645 \\ 0.0000 & 0.4499 & -0.3889 & 0.4067 & 0.0654 & -0.2265 & -0.2480 & -0.1351 & -0.4366 & 0.3937 \\ -0.0000 & -0.2565 & 0.1925 & 0.5501 & -0.5102 & 0.1458 & -0.3324 & 0.2561 & 0.2660 & 0.2572 \end{pmatrix}$$

$$
\Psi1(b) = \begin{pmatrix}
0.0000 & 0.3816 & 0.2544 & -0.0823 & -0.2780 & 0.3927 & 0.2492 & -0.2241 & 0.4878 & 0.4484 \\
-0.0000 & -0.4499 & 0.3889 & -0.4067 & -0.0654 & 0.2265 & -0.2480 & -0.1351 & -0.4366 & 0.3937 \\
0.0000 & 0.2565 & -0.1925 & -0.5501 & 0.5102 & -0.1458 & -0.3324 & 0.2561 & 0.2660 & 0.2572 \\
0.0000 & 0.3576 & -0.2786 & 0.0894 & -0.3500 & -0.4373 & -0.3571 & -0.4265 & -0.2626 & 0.3071 \\
0.0000 & 0.0913 & 0.6368 & 0.2039 & -0.0462 & -0.5672 & 0.0505 & 0.4148 & 0.0305 & 0.2126 \\
-0.0000 & -0.3904 & -0.0862 & 0.0282 & 0.3876 & -0.3827 & 0.4491 & -0.4833 & 0.1825 & 0.2721 \\
0.7071 & 0.0246 & -0.2937 & -0.0938 & -0.1347 & 0.0069 & 0.3835 & 0.3226 & -0.2550 & 0.2645 \\
-0.7071 & 0.0246 & -0.2937 & -0.0938 & -0.1347 & 0.0069 & 0.3835 & 0.3226 & -0.2550 & 0.2645 \\
0.0000 & 0.2635 & 0.0528 & 0.5468 & 0.5612 & 0.3274 & -0.0654 & -0.0184 & -0.3354 & 0.2993 \\
-0.0000 & -0.4764 & -0.2847 & 0.3978 & -0.1790 & 0.0806 & -0.3683 & 0.2645 & 0.3936 & 0.3670
\end{pmatrix}
$$

Ψ1(a) and Ψ1(b) are the eigenvector matrices obtained by the use of MATLAB for the ten link one degree of freedom chains shown in Fig. 1.

It can be observed easily that by the following permutations of the change of the rows in the matrix Ψ1(b) at the same time it becomes identical to the matrix Ψ1(a).

R3→R10, R2→R9, R10→R3, R9→R2

Which shows that both the chains are having one to one correspondence between their links hence the two chains are isomorphic and both the chains has already being shown isomrphic by F.G. Kong[14] using artificial neural network method and Ashoak[15] using link adjacency values.

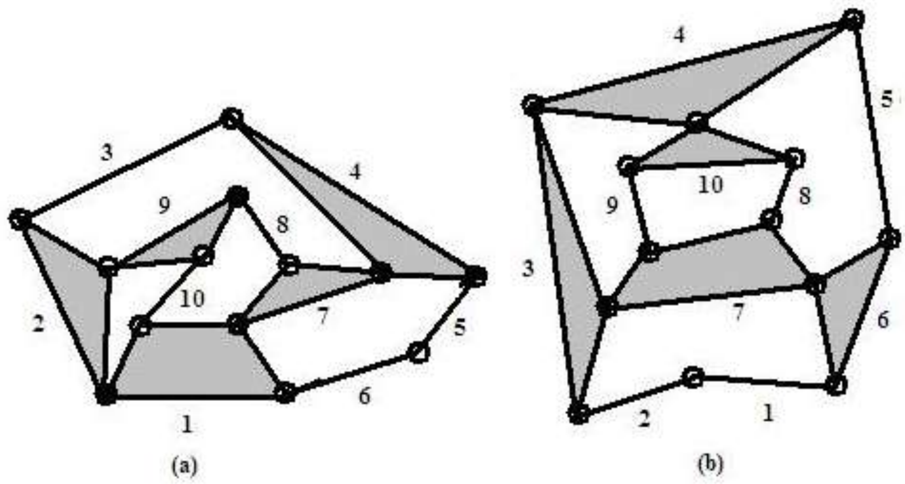

Fig. 2: Pair of 10-link 1-degree of freedom nonisomorphic chains

$$CIM2(a) = \begin{bmatrix} 4 & 0 & 1 & 1 & 1 & 0 & 0 & 1 & 2 & 0 \\ 0 & 3 & 0 & 1 & 0 & 1 & 1 & 1 & 0 & 2 \\ 1 & 0 & 2 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 3 & 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 2 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 2 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 1 & 3 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 & 0 & 2 & 0 & 1 \\ 2 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 3 & 0 \\ 0 & 2 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 2 \end{bmatrix}$$

$$CIM2(b) = \begin{bmatrix} 2 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 3 & 0 & 1 & 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 3 & 0 & 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 2 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 & 3 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 1 & 0 & 4 & 0 & 0 & 2 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 & 2 & 2 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 & 2 & 2 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 2 & 0 & 0 & 3 \end{bmatrix}$$

CIM2(a) and CIM2(b) are the chain identification matrices of the ten link one degree of freedom chains shown in Fig-2.

Ψ2(a) and Ψ2(b) are the eigenvector matrices obtained by the use of MATLAB for the ten link one degree of freedom chains shown in figure-2

It can be concluded upon examining the matrix Ψ2(a) and Ψ2(b) that the eigenvectors of the chains shown in Fig-2 are not same which revels the fact that the two chains are nonisomorphic but in past, the apparoch of comparision of characterstics polynomail of the adjacency matrix given by Uicker [2] fails to determine the fact that the two chains are nonisomorphic which was pointed out by Mruthyunjaya[16].

$$\Psi 2(a) = \begin{pmatrix} 0.0000 & -0.4442 & 0.0607 & 0.0000 & -0.4275 & 0.1427 & -0.0810 & -0.3355 & -0.5109 & 0.4646 \\ -0.3536 & -0.0993 & 0.4162 & -0.5000 & 0.1095 & 0.1130 & -0.2082 & -0.0354 & 0.4776 & 0.3801 \\ 0.0000 & -0.0985 & -0.5841 & -0.5000 & 0.3839 & 0.1460 & 0.1917 & 0.2685 & -0.2486 & 0.2445 \\ 0.3536 & 0.0547 & 0.1201 & 0.0000 & 0.3263 & -0.3360 & 0.5156 & -0.5203 & 0.1262 & 0.292 \\ -0.0000 & 0.5276 & 0.3158 & 0.0000 & -0.1971 & 0.4917 & 0.4745 & 0.2258 & -0.1668 & 0.1932 \\ -0.3536 & 0.1842 & -0.4101 & -0.0000 & -0.5500 & -0.4152 & 0.2454 & 0.0530 & 0.2790 & 0.2390 \\ 0.0000 & -0.4043 & 0.1580 & 0.5000 & 0.1837 & -0.1243 & 0.1606 & 0.5823 & 0.1090 & 0.3683 \\ -0.3536 & 0.2064 & -0.3260 & 0.5000 & 0.3097 & 0.3832 & -0.1771 & -0.3492 & 0.1201 & 0.2563 \\ -0.0000 & 0.4978 & 0.1412 & 0.0000 & 0.1680 & -0.4613 & -0.4445 & 0.1402 & -0.4006 & 0.3370 \\ 0.7071 & 0.1183 & -0.2200 & -0.0000 & -0.2285 & 0.2085 & -0.3277 & 0.0944 & 0.3752 & 0.2912 \end{pmatrix}$$

$$\Psi 2(b) = \begin{pmatrix} -0.0000 & -0.5276 & -0.3158 & -0.0000 & -0.1971 & -0.4917 & 0.4745 & 0.2258 & 0.1668 & 0.1932 \\ -0.0000 & -0.2723 & 0.5161 & -0.5000 & -0.0118 & 0.2405 & 0.3960 & -0.3905 & -0.0238 & 0.2040 \\ 0.0000 & 0.4043 & -0.1580 & -0.5000 & 0.1837 & 0.1243 & 0.1606 & 0.5823 & -0.1090 & 0.3683 \\ -0.0000 & 0.0112 & -0.3102 & 0.0000 & 0.6478 & -0.2877 & -0.0575 & -0.4789 & -0.2224 & 0.3451 \\ 0.0000 & 0.0985 & 0.5841 & 0.5000 & 0.3839 & -0.1460 & 0.1917 & 0.2685 & 0.2486 & 0.2445 \\ 0.0000 & 0.0334 & -0.2261 & 0.5000 & -0.2120 & 0.5107 & 0.3649 & -0.0767 & -0.3814 & 0.3278 \\ 0.0000 & 0.4442 & -0.0607 & -0.0000 & -0.4275 & -0.1427 & -0.0810 & -0.3355 & 0.5109 & 0.4646 \\ 0.7071 & -0.1183 & 0.2200 & -0.0000 & -0.2285 & -0.2085 & -0.3277 & 0.0944 & -0.3752 & 0.2912 \\ -0.7071 & -0.1183 & 0.2200 & -0.0000 & -0.2285 & -0.2085 & -0.3277 & 0.0944 & -0.3752 & 0.2912 \\ -0.0000 & -0.4978 & -0.1412 & 0.0000 & 0.1680 & 0.4613 & -0.4445 & 0.1402 & 0.4006 & 0.3370 \end{pmatrix}$$

CONCLUSION

The method proposed in this paper is capable of checking the isomorphism among planer kinematic chains of any size and also in those chains, for which the some of the other apporoches seems difficult or fails. It is the easiest and quick among the methods proposed so far and the worth of this method is because of reason on that it not only identifies all known 16 kinematic chains of 8 links, 230 kinematic chains of 10 links having 1-d.o.f and 40 kinematic chains of 9 links having 2-d.o.f but also successful on all the counter examples reported earlier literature.

REFERENCES

1. Wilson R; Introduction to graph theory. 3rd edition, Longman, 1972.
2. Uicker JJ, Raicu A; A method for the identification and recognition of equivalence of kinematic chains. Journal of Mechanism and Machine Theory, 1975; 10(5): 375-383.
3. Mruthyunjaya TS, Raghavan MR; Structural analysis of kinematic chains and mechanisms based on matrix representation. Transaction of the ASME, Journal of Mechanical Design, 1979; 101: 488 – 518.
4. Cubillo JP, Wan J; Comments on mechanism kinematic chain isomorphism identification using adjacent matrices. Journal of Mechanism and Machine Theory, 2005; 40(2): 131–139.
5. Zhang WJ, Li Q; On a New approach to mechanism topology identification. Journal of Mechanical Design ASME, 1999; 121(1): 57-64.
6. Li TJ, Cao WQ, Chu JK; The topological representation and detection of isomorphism among geared linkage kinematic chains. Proceedings of ASME Design Engineering Technical Conference, 1988.
7. Rao AC; Kinematic chains, isomorphism, inversions, and type of freedom using the concept of hamming distances. Indian J of Tech., 1988; 26: 105-109.
8. Khan RA, Alihasan AM, Aas M; Structural synthesis of planar kinematic chains using matrix. Journal of Intelligent System Research, 2007; 1(1): 11-23.
9. Ding H, Huang Z; The establishment of the canonical perimeter topological graph of kinematic chains and isomorphism identification. Journal of Mechanical Design ASME, 2007; 129(9): 915–923.
10. Rao CN, Rao AC; Selection of best frame, input and output links for function generators modeled as probabilistic system. Journal of Mechanism and Machine Theory, 1996; 31(7): 973 –983.
11. Butcher EA, Hartman C; Efficient enumeration and hierarchical classification of planar simple -jointed kinematic chains: Application to12- and 14-bar single degree-of-freedom chains. Journal of Mechanism and Machine Theory, 2005; 40: 1030-1050.
12. Hasan A; Isomorphism and inversions of kinematic chains up to 10-links. 13th National Conference on Mechanisms and Machines IISc, Bangalore, India, December 12-13, 2007.
13. Yang F, Deng Z, Tao J, Li L; A new method for isomorphism identification in topological graphs using incident matrices. Journal of Mechanism and Machine Theory, 2012; 49: 298–307.
14. Kong FG, Li Q, Zhang WJ; An artificial neural network approach to mechanism kinematic chain isomorphism identification. Journal of Mechanism and Machine Theory, 1999; 34(2): 271-283.
15. Dargar A, Khan RA, Hasan A; Application of link adjacency values to detect isomorphism among kinematic chains. International Journal of Materials and Design, Springer, 2010; 6: 157-162.
16. Mruthyunjaya TS, Balasubramanian HR; In quest of a reliable and efficient computational test for the detection of isomorphism in kinematic chains. Journal of Mechanism and Machine Theory, 1987; 22(2): 131-139.