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Research Article

A Method of Finding Solutions to the Cubic equation $x^3+px^2+qx+r=0$

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Abstract: A method of finding real and complex solutions to the cubic equation $x^3+px^2+qx+r=0$ is illustrated through solving quartic equation.
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INTRODUCTION:

In mathematics, a cubic function is a function of the form $f(x)=ax^3+bx^2+cx+d$,where 'a' is non-zero or in other words, a function defined by a polynomial of degree 3.Setting $f(x)=0$ produces a cubic equation of the form $ax^3+bx^2+cx+d=0$. Usually, the co-efficients a, b, c, d are real numbers. To solve a cubic equation is to find the zero of a cubic function, that is to say, to find a particular value of x for which $f(x)=0$. Cubic equations are rich in variety. There are various methods to solve a cubic equation [1-5] namely, Cardano's method, Vieta's substitution, Lagrange's method, Trigonometric(or Hyperbolic)method, factorization and Omarkhayyam's solutions.

It is worth mentioning here that solving the quartic equation requires solving its resolvent cubic equation and thus, solving cubic equation is a necessary part of solving the quartic equation. Thus, we have motivated to search for an elegant method of finding solutions to the cubic equation $ax^3+bx^2+cx+d=0$ through solving quartic equation.

METHOD OF ANALYSIS:

The cubic equations to be solved is

$$x^3+p^2+qx+r=0 \qquad (1)$$

Where p, q, r are non-zero numbers.

Multiplying both sides of (1) by $(x+p), p\neq 0$ we have

$$x^4+2px^3+(p^2+q)x^2+(pq+r)x+pr=0 \qquad (2)$$

Assuming the LHS of equation (2) can be expressed as the product of two quadratic factors in x. (2) is written as

$$(x^2+(p+\lambda)x+c)(x^2+(p-\lambda)x+d)=0 \qquad (3)$$

Where λ is any non zero number.

Comparing the coefficients of corresponding terms in (2) & (3) we have

$$c+d=\lambda^2+q \qquad (4)$$

$$(p+\lambda)d+(p-\lambda)d=pq+r \qquad (5)$$

$$cd=pr \qquad (6)$$

Now, $c-d=\pm\sqrt{(c+d)^2-4cd}$

$$c-d=\pm\sqrt{(\lambda^2+q)^2-4pr} \qquad (7)$$

From (4) & (7), the values of c & d are given by

$$c=\frac{1}{2}\left[\lambda^2+q\pm\sqrt{(\lambda^2+q)^2-4pr}\right] \qquad (8)$$

$$d=\frac{1}{2}\left[\lambda^2+q\mp\sqrt{(\lambda^2+q)^2-4pr}\right] \qquad (9)$$

Choose λ such that the square root of the RHS of (8) & (9) is removed. There are two sets of values for c and d . Select those values of c, d that satisfy the equation (5).

Now, substituting the suitable values of λ, c, d in (3) and solving each of the two quadratic equations, four values for x are obtained. Neglecting the value $x=-p$, the other values of x represent the solution of (1).

A few examples are presented in the following table.

Table: Solutions of $x^3+px^2+qx+r=0.$

p	q	r	λ	c	d	solutions
1	-10	8	1	-8	-1	1, 2, -4
6	11	6	1	6	6	-1, -2, -3
2	-13	10	1	-10	-2	1, 2, -5
-7	14	-8	1	8	7	1, 2, 4
-5	-2	24	2	-10	12	-2, 3, 4
1	1	-3	1	3	-1	1,-1$\pm i\sqrt{2}$
-12	39	-28	1	28	12	1, 4, 7
-6	13	-10	2	5	12	2, 2$\pm i$
7/2	7/2	-1	-1	7/2	1	1/2, 1, 2

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