

Research Article

Fuzzy Distance Measure

B. Pardha saradhi¹, N.Ravi Shankar²¹Dr. L.B .college, Visakhapatnam, India²GITAM University, Visakhapatnam, India

*Corresponding author

B. Pardha saradhi

Email: pardhu07@gmail.com

Abstract: This presents a new distance method to find the distance between trapezoidal fuzzy numbers using centroid of centroids. In addition, we prove that our proposed distance measure satisfies metric space properties. So far in literature, This type of measure is first time introduced and applied by centroid of centroids using trapezoidal fuzzy numbers.

Keywords: trapezoidal fuzzy, centroid

INTRODUCTION

The theory of fuzzy sets has been associated to many fields that need to administer uncertain and inconclusive data. These areas are approximate reasoning, decision making, optimization and control, where fuzzy number ranking is a very vital constituent in the process of decision [2], Fuzzy set theory and its application discussed in [3]. In [4] approximation and aggregation of trapezoidal fuzzy number and metric and orders in space discussed in [5]. Distance between intuitionist fuzzy sets and interval-valued fuzzy sets based on Hausdorff metric explained [6]. The relation between fuzzy number and its centroid evinced [7]. They had explicitly solved the related linear system in order to characterize the ingredients of each of the trapezoidal fuzzy numbers. Albeit, the uniqueness of the problem is that this system was not investigated by them. In the paper [9], the correct centroid formulae for fuzzy numbers and justify them from the viewpoint of analytical geometry is discussed. The ranking of fuzzy numbers by sign distance was explained [10]. The distance between the two arbitrary trapezoidal fuzzy numbers is the distance between its centroid explained in [20]. The compilation of the paper is as follows. Section - 2 consists of some basic concepts and the construction of centroid of centroid trapezoidal fuzzy numbers. Section - 3 is composed of a new distance measure putting into use the centroid of centroids formula as well as left-right spreads. In addition, the metric properties of the suggested distance are explored, finally example is furnished.

CENTROID OF CENTROIDS FORMULA FOR TRAPEZOIDAL FUZZY NUMBER

The membership function of a trapezoidal fuzzy number $\tilde{A} = (a, b, c, d; w)$ is given by:

$$f_{\tilde{A}}(x) = \begin{cases} \frac{w(x-a)}{b-a}, & a \leq x < b, \\ w, & b \leq x \leq c, \\ \frac{w(x-d)}{c-d}, & c < x \leq d, \\ 0, & \text{otherwise.} \end{cases} \quad (1)$$

If $w=1$, then $\tilde{A} = (a, b, c, d; 1)$ is a normalized trapezoidal fuzzy number, otherwise \tilde{A} is a generalized or non normal trapezoidal fuzzy number if $0 < w < 1$.

The Fig. 1 represents a trapezoidal fuzzy number \tilde{A} :

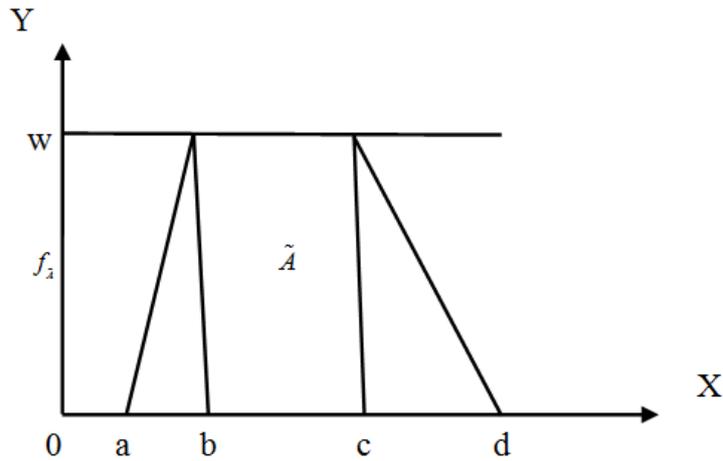


Fig-1: Trapezoidal fuzzy number

Let us consider \tilde{A} to be a given trapezoidal fuzzy number such that $\tilde{A} = (a_1, a_2, a_3, a_4)$ then the centroid of centroids point of \tilde{A} is obtained from the paper [19]

$$G_{\tilde{A}} = \left(\frac{a_1 + 2a_2 + 5a_3 + a_4}{9}, \frac{4w}{9} \right) = (\varepsilon_{\tilde{A}}, \varepsilon'_{\tilde{A}}) \tag{2}$$

where $\varepsilon_{\tilde{A}} = \frac{a_1 + 2a_2 + 5a_3 + a_4}{9}$ and $\varepsilon'_{\tilde{A}} = \frac{4w}{9}$

Hence, for any triangular fuzzy number with a piecewise linear membership function, its centroid of centroids can be derived by

$$G'_{\tilde{A}} = \left(\frac{a_1 + 7a_2 + a_4}{9}, \frac{4w}{9} \right) = (\varepsilon_{\tilde{A}}, \varepsilon'_{\tilde{A}}) \tag{3}$$

where $\varepsilon_{\tilde{A}} = \frac{a_1 + 7a_2 + a_4}{9}$ and $\varepsilon'_{\tilde{A}} = \frac{4w}{9}$

A briefly review about the relation between trapezoidal fuzzy number and its centroid formula is given. Let us consider the left and right spreads, $(l_{\tilde{A}}, r_{\tilde{A}})$ where $l_{\tilde{A}} = a_2 - a_1$ and $r_{\tilde{A}} = a_4 - a_3$ and centroid of centroids point $(cc_{\tilde{A}}, c'c'_{\tilde{A}})$, where

$$cc_{\tilde{A}} = \frac{a_1 + 2a_2 + 5a_3 + a_4}{9}, c'c'_{\tilde{A}} = \frac{4w}{9} \tag{4}$$

The Fig. 3 represents centroid of centroid trapezoidal fuzzy number

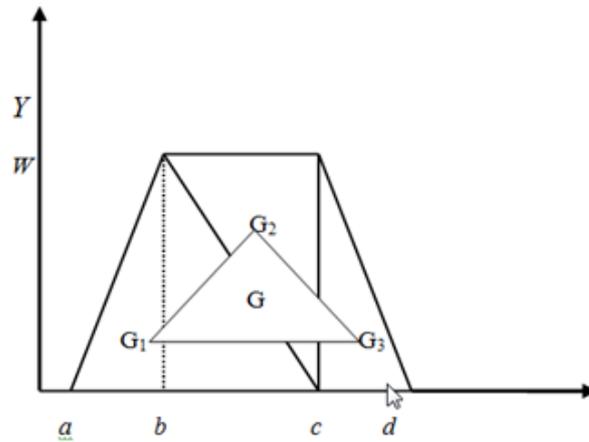


Fig-2: Trapezoidal fuzzy number

PROPOSED DISTANCE MEASURE FOR TRAPEZOIDAL FUZZY NUMBERS

In this section, we propose a new distance measure for trapezoidal fuzzy numbers using their centroid of centroids point and left - right spread.

Let us consider trapezoidal fuzzy numbers $\tilde{A} = (a_1, a_2, a_3, a_4)$ and $\tilde{B} = (b_1, b_2, b_3, b_4)$ with centroid of centroids points $(cc_{\tilde{A}}, c'c'_{\tilde{A}})$ and $(cc_{\tilde{B}}, c'c'_{\tilde{B}})$, left and right spreads $(l_{\tilde{A}}, r_{\tilde{A}})$ and $(l_{\tilde{B}}, r_{\tilde{B}})$, respectively. The distance measure of Trapezoidal fuzzy Numbers

$$f_d(\tilde{A}, \tilde{B}) = \max \{ |cc_{\tilde{A}} - cc_{\tilde{B}}|, |l_{\tilde{A}} - l_{\tilde{B}}|, |r_{\tilde{A}} - r_{\tilde{B}}| \} \tag{5}$$

And this f_d satisfies the metric distance property. The following theorem proved the metric distance.

Theorem:

The distance measure f_d under the consideration of the centroid of centroids points $(cc_{\tilde{A}}, c'c'_{\tilde{A}})$ and $(cc_{\tilde{B}}, c'c'_{\tilde{B}})$ of positive fuzzy Numbers and also it's, left and right spreads $(l_{\tilde{A}}, r_{\tilde{A}})$ and $(l_{\tilde{B}}, r_{\tilde{B}})$ respectively. Then the following property are yield:

$$f_d(\tilde{A}, \tilde{B}) \geq 0, \tag{6}$$

$$f_d(\tilde{A}, \tilde{B}) = f_d(\tilde{B}, \tilde{A}), \tag{7}$$

$$f_d(\tilde{A}, \tilde{B}) = 0 \Rightarrow \tilde{A} = \tilde{B}, \tag{8}$$

$$f_d(\tilde{A}, \tilde{C}) \leq f_d(\tilde{A}, \tilde{B}) + f_d(\tilde{B}, \tilde{C}), \tag{9}$$

for all positive fuzzy numbers $\tilde{A}, \tilde{B}, \tilde{C}$.

Proof:

Consider $\tilde{A} = (a_1, a_2, a_3, a_4)$ and $\tilde{B} = (b_1, b_2, b_3, b_4)$ are positive fuzzy numbers.

From Eq. 5 ,all the values of $f_d(\tilde{A}, \tilde{B}) \geq 0$ are positive so $f_d(\tilde{A}, \tilde{B}) \geq 0$, and $f_d(\tilde{A}, \tilde{B}) = f_d(\tilde{B}, \tilde{A})$ for the reason that $|cc_{\tilde{A}} - cc_{\tilde{B}}| = |cc_{\tilde{B}} - cc_{\tilde{A}}|$, $|l_{\tilde{A}} - l_{\tilde{B}}| = |l_{\tilde{B}} - l_{\tilde{A}}|$, $|r_{\tilde{A}} - r_{\tilde{B}}| = |r_{\tilde{B}} - r_{\tilde{A}}|$. Now for $a_i \leq b_i, i = 1, 2, 3, 4$. and suppose that

$$\begin{aligned} f_d(\tilde{A}, \tilde{B}) = 0 &\Rightarrow \max \{ |cc_{\tilde{A}} - cc_{\tilde{B}}|, |l_{\tilde{A}} - l_{\tilde{B}}|, |r_{\tilde{A}} - r_{\tilde{B}}| \} = 0 \\ \Rightarrow |cc_{\tilde{A}} - cc_{\tilde{B}}| = 0, |l_{\tilde{A}} - l_{\tilde{B}}| = 0, |r_{\tilde{A}} - r_{\tilde{B}}| = 0 \\ &cc_{\tilde{A}} = cc_{\tilde{B}}, l_{\tilde{A}} = l_{\tilde{B}}, r_{\tilde{A}} = r_{\tilde{B}}. \\ l_{\tilde{A}} = l_{\tilde{B}} = a_2 - a_1 = b_2 - b_1 = k; r_{\tilde{A}} = r_{\tilde{B}} = a_4 - a_3 = b_4 - b_3 = t \end{aligned} \tag{10}$$

$$\begin{aligned} \text{From Eq.4, } cc_{\tilde{A}} &= \frac{a_1 + 2a_2 + 5a_3 + a_4}{9}, cc_{\tilde{B}} = \frac{b_1 + 2b_2 + 5b_3 + b_4}{9} \\ \omega_1 &= \frac{a_1 + 2a_2 + 5a_3 + a_4}{9}, \omega_2 = \frac{b_1 + 2b_2 + 5b_3 + b_4}{9} \end{aligned} \tag{11}$$

Where $cc_{\tilde{A}} = \omega_1 = \omega_2 = cc_{\tilde{B}}$

$$9\omega_1 = a_1 + 2a_2 + 5a_3 + a_4 \text{ and } 9\omega_2 = b_1 + 2b_2 + 5b_3 + b_4$$

$$a_1 = 9\omega_1 - 2a_2 - 5a_3 - a_4 \text{ and } b_1 = 9\omega_2 - 2b_2 - 5b_3 - b_4$$

From Eq.10, $a_1 = 9\omega_1 - 2(a_1 + k) - 5a_3 - (a_3 + t)$, $b_1 = 9\omega_2 - 2(b_1 + k) - 5b_3 - (b_3 + t)$

$$a_1 = 9\omega_1 - 2a_1 - 2k - 5a_3 - a_3 - t, b_1 = 9\omega_2 - 2(b_1 + k) - 5b_3 - (b_3 + t)$$

$$3a_1 = 9\omega_1 - 2k - 6a_3 - t, 3b_1 = 9\omega_2 - 2k - 6b_3 - t$$

$$3a_1 = 9\omega_1 - 2k - t - 6a_3, 3b_1 = 9\omega_2 - 2k - t - 6b_3 \tag{12}$$

From Eq.12, $(b_1 - a_1) + 2(b_3 - a_3) = 0$

We conclude that $a_1 = b_1, a_2 = b_2, a_3 = b_3, a_4 = b_4$ and $\tilde{A} = \tilde{B}$.

$$\begin{aligned} \text{Finally } f_d(\tilde{A}, \tilde{C}) &= \max \{ |cc_{\tilde{A}} - cc_{\tilde{C}}|, |l_{\tilde{A}} - l_{\tilde{C}}|, |r_{\tilde{A}} - r_{\tilde{C}}| \} \\ &= \max \{ |cc_{\tilde{A}} - cc_{\tilde{B}} + cc_{\tilde{B}} - cc_{\tilde{C}}|, |l_{\tilde{A}} - l_{\tilde{B}} + l_{\tilde{B}} - l_{\tilde{C}}|, |r_{\tilde{A}} - r_{\tilde{B}} + r_{\tilde{B}} - r_{\tilde{C}}| \} \\ &\leq \max \{ (|cc_{\tilde{A}} - cc_{\tilde{B}}| + |cc_{\tilde{B}} - cc_{\tilde{C}}|), (|l_{\tilde{A}} - l_{\tilde{B}}| + |l_{\tilde{B}} - l_{\tilde{C}}|), (|r_{\tilde{A}} - r_{\tilde{B}}| + |r_{\tilde{B}} - r_{\tilde{C}}|) \} \\ &\leq \max \{ (|cc_{\tilde{A}} - cc_{\tilde{B}}|), (|l_{\tilde{A}} - l_{\tilde{B}}|), (|r_{\tilde{A}} - r_{\tilde{B}}|) \} + \max \{ |cc_{\tilde{B}} - cc_{\tilde{C}}| |l_{\tilde{B}} - l_{\tilde{C}}| |r_{\tilde{B}} - r_{\tilde{C}}| \} \\ &\leq f_d(\tilde{A}, \tilde{B}) + f_d(\tilde{B}, \tilde{C}) \quad // \end{aligned}$$

Example:

Consider two Trapezoidal fuzzy Numbers $\tilde{A}_1 = (0.1, 0.2, 0.4, 0.5; 1)$,

$$\tilde{A}_2 = (0.2, 0.5, 0.8, 0.9; 1)$$

From the Eq. 2 the centroid of centroids point of \tilde{A}_1 and \tilde{A}_2 are

$$G_{\tilde{A}_1}(\bar{x}_0, \bar{y}_0) = \left(\frac{0.1 + 2(0.2) + 5(0.4) + 0.5}{9}, \frac{4(1)}{9} \right) = (0.3, 0.444)$$

$$G_{\tilde{A}_2}(\bar{x}_0, \bar{y}_0) = \left(\frac{0.2 + 2(0.5) + 5(0.8) + 0.9}{9}, \frac{4(1)}{9} \right) = (0.68, 0.444)$$

From section 2, the centroid of centroids is

$$(cc_{\tilde{A}_1}, c'c'_{\tilde{A}_1}) = (0.3, 0.444), (cc_{\tilde{A}_2}, c'c'_{\tilde{A}_2}) = (0.68, 0.444)$$

left and right spreads are $(l_{\tilde{A}_1}, r_{\tilde{A}_1}) = (0.1, 0.1)$ and $(l_{\tilde{A}_2}, r_{\tilde{A}_2}) = (0.3, 0.1)$.

From the Eq.4, the distance measure of Trapezoidal fuzzy Numbers is

$$\begin{aligned} f_d(\tilde{A}_1, \tilde{A}_2) &= \max \{ |cc_{\tilde{A}_1} - cc_{\tilde{A}_2}|, |l_{\tilde{A}_1} - l_{\tilde{A}_2}|, |r_{\tilde{A}_1} - r_{\tilde{A}_2}| \} \\ &= \max \{ |0.3 - 0.68|, |0.1 - 0.3|, |0.1 - 0.1| \} \\ &= 0.38 \end{aligned}$$

The Fig. 3 represents a trapezoidal fuzzy numbers \tilde{A}_1, \tilde{A}_2 :

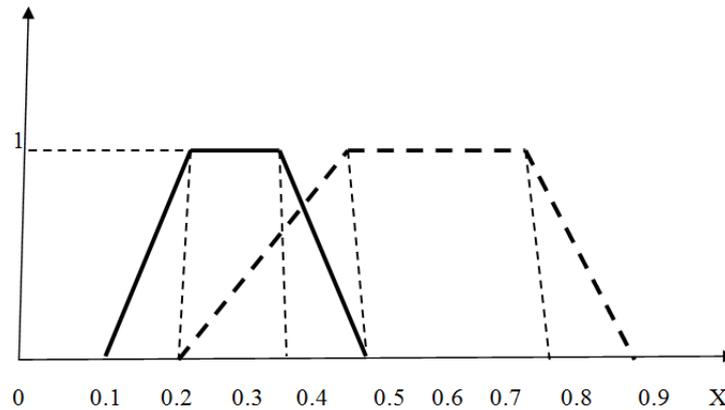


Fig-3: Trapezoidal fuzzy number \tilde{A}_1, \tilde{A}_2

ADVANTAGES

The centroid of centroids of the two positive trapezoidal fuzzy numbers contains equal second coordinates of the form $\frac{4w}{9}$ so the distance formula [20] can be reduced. Moreover it is very comfortable to evaluate the distance measures of Trapezoidal fuzzy Numbers. Further the distance measure is very useful to evaluate the fuzzy critical path method using Fuzzy Topsis.

CONCLUSION

A novel distance measure is found for the space of all Trapezoidal fuzzy numbers putting into use the centroid of the centroids point alongside left-right spread of trapezoidal fuzzy numbers. In addition to this, an investigation regarding the metric properties of the suggested distance measure has been conducted. As a matter of fact, it is shown that if the distance between centroid of centroids points of two arbitrary trapezoidal fuzzy numbers and the distance between the left and the right extend to zero, then the two given fuzzy numbers are equal.

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