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Research Article

Exact solutions to the regularized Burgers-BBM equation

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Abstract: The trial equation method is applied to the regularized Burgers-BBM equation, when the parameter p takes the different values, we have obtained all of its corresponding exact traveling wave solutions.
Keywords: traveling wave solution; the trial equation method; the regularized Burgers-BBM equation

INTRODUCTION

In this paper, we consider the regularized Burgers-BBM equation

$$u_t+\sigma(u+1)^p u_x-\delta u_{xx}-\mu u_{xxt}=0. \tag{1}$$

We use the trial equation method to Eq.(1) and give all of its possible exact traveling wave solutions. The trial equation method was proposed by Liu[1-5] in the past several years. The trial equation method is a powerful method to solve the nonlinear equations that can not change into the elementary integral form.

CLASSIFICATION

Taking the following transformation to the Eq.(1)

$u=u(\xi),\xi=x-ct$, Eq.(1) becomes

$$-cu'+\sigma(u+1)^p u'-\delta u''+\mu c u'''=0. \tag{2}$$

Denote $v=u+1$, Eq. (2) becomes

$$-cv'+\sigma v^p v'-\delta v''+\mu c v'''=0. \tag{3}$$

Integrating Eq.(3), we have

$$v''-\frac{\delta}{c\mu}v'=-\frac{\sigma}{(p+1)c\mu}v^{p+1}+\frac{1}{\mu}v-\frac{g_1}{c\mu}. \tag{4}$$

We will give the corresponding exact traveling wave solutions when the parameter p makes three different values.

Case1 $p=1$, Eq.(4) becomes

$$v''+Av'=Bv^2+Cv+D, \tag{5}$$

where $A=-\frac{\delta}{c\mu},B=-\frac{\delta}{2c\mu},C=\frac{1}{\mu},D=-\frac{g_1}{c\mu}$.

Denote

$$v=a_0+a_1\varphi+a_2\varphi^2. \tag{6}$$

Taking the Eq.(6) to the Eq.(5), then we obtain the following equation

$$2a_2(\varphi')^2+(2a_2\varphi+a_1)\varphi''+A(2a_2\varphi+a_1)\varphi=B(a_0+a_1\varphi+a_2\varphi^2)^2+C(a_0+a_1\varphi+a_2\varphi^2)+D \tag{7}$$

Denote $w=\varphi'$, Eq. (7) becomes

$$(2a_2\varphi+a_1)w\frac{dw}{d\varphi}+2a_2w^2+A(2a_2\varphi+a_1)w=B(a_0+a_1\varphi+a_2\varphi^2)^2+C(a_0+a_1\varphi+a_2\varphi^2)+D \tag{8}$$

Denote $$w = b_0 + b_1\varphi + b_2^2\varphi^2, \tag{9}$$

take the Eq.(9) into the Eq.(8), we have

$$\begin{aligned}&(2a_2\varphi + a_1)(b_0 + b_1\varphi + b_2^2\varphi)(2b_2\varphi + b_1) + 2a_2(b_0 + b_1\varphi + b_2^2\varphi)^2 \\ &+ A(2a_2\varphi + a_1)(b_0 + b_1\varphi + b_2^2\varphi) = B(a_0 + a_1\varphi + a_2\varphi^2)^2 + C(a_0 + a_1\varphi + a_2\varphi^2) + D\end{aligned} \tag{10}$$

According to the principle of balance, we can obtain the following equations by Eq.(10),

$$\begin{aligned}&6a_2b_2^2 = Ba_2^2; \\ &5a_2b_1b_2 + a_1b_2^2 + Aa_2b_2 = Ba_1a_2; \\ &4a_2b_1^2 + 8a_2b_0b_2 + 3a_1b_1b_2 + Aa_1b_2 + 2Aa_2b_1 = Ba_1^2 + 2Ba_0a_2 + Ca_2; \\ &6a_2b_0b_1 + 2a_1b_0b_2 + a_1b_1^2 + 2Aa_2b_0 + Aa_1b_1 = 2Ba_0a_1 + Ca_1; \\ &2a_2b_0^2 + a_1b_0b_1 + Aa_1b_0 = Ba_0^2 + Ca_0 + D.\end{aligned} \tag{11}$$

Solving this algebraic equations system (11), we obtain a family values of parameters:

$$a_2 = \frac{6}{B}b_2^2,\ a_1 = \frac{6A}{5B}b_2,\ a_0 = \frac{6}{B}b_0b_2 - \frac{C}{2B},\ b_1 = 0,\ A^2 = -100b_0b_2. \tag{12}$$

Becase of $b_0b_2 < 0$, take $b_0 = 1, b_2 = -1$,

$\varphi = \tanh(\xi - \xi_0)$, or $\varphi = \coth(\xi - \xi_0)$, we obtain the corresponding traveling wave solutions are:

$$u = \frac{12\mu + 1}{\sigma} - \frac{12\delta}{5\sigma}\tanh(x - ct - \xi_0)\frac{12c\mu}{5\sigma}\tanh^2(x - ct - \xi_0) - 1. \tag{13}$$

$$u = \frac{12\mu + 1}{\sigma} - \frac{12\delta}{5\sigma}\coth(x - ct - \xi_0)\frac{12c\mu}{5\sigma}\coth^2(x - ct - \xi_0) - 1\ . \tag{14}$$

Case2 $p = 2$, the Eq. (4) becomes

$$v'' + Av' = Bv^3 + Cv + D, \tag{15}$$

where $A = -\dfrac{\delta}{c\mu}, B = -\dfrac{\delta}{3c\mu}, C = \dfrac{1}{\mu}, D = -\dfrac{g_1}{c\mu}$.

Denote

$$\begin{aligned}&v' = F(v) = a_m v^m + \Lambda + a_1 v + a_0 \\ &v'' = F'(v)F(v).\end{aligned} \tag{16}$$

According to the principle of balance, we get $m = 2$,then we have

$$v' = a_2v^2 + a_1v + a_0 \tag{17}$$

$$v'' = 2a_2^2.v^3 + 3a_1a_2v^2 + (2a_0a_2 + a_1^2)v + a_0a_1. \tag{18}$$

We obtain the following equations by the Eq.(17) and Eq.(18),

$$\begin{aligned}&2a_2 = B; \\ &3a_1a_2 + Aa_2 = 0; \\ &2a_0a_2 + a_1^2 + Aa_1 = C; \\ &a_0a_1 + Aa_0 = D.\end{aligned} \tag{19}$$

Solving them, we have

$$a_2 = \pm\sqrt{-\frac{\sigma}{2c\mu}},\ a_1 = \frac{\delta}{3c\mu},\ a_0 = \frac{3g_1}{2\delta}. \tag{20}$$

Integrating Eq.(17) , we have

$$a_2(\xi-\xi_0)=\int \frac{dv}{v^2+\frac{a_1}{a_2}v+\frac{a_0}{a_2}}. \quad (21)$$

Denote
$$F(v)=v^2+\frac{a_1}{a_2}v+\frac{a_0}{a_2}\ , \quad (22)$$

the discrimination of (22) is

$$\Delta=\left(\frac{a_1}{a_2}\right)^2-\frac{4a_0}{a_2}. \quad (23)$$

According to the discrimination , we give the corresponding traveling wave solutions to Eq.(21).

Case 2.1 $\Delta=0$,

$$F(v)=\left(v+\frac{a_1}{2a_2}\right)^2, \quad (24)$$

the corresponding traveling wave solutions are:

$$u=\mu\sqrt{\frac{-2c\mu}{\sigma}}\cdot\frac{1}{x-ct-\xi_0}-\frac{\delta}{6}\sqrt{\frac{-2}{c\mu\sigma}}-1\ . \quad (25)$$

Case 2.2 $\Delta>0$,

$$F(v)=\left(v+\frac{a_1}{2a_2}\right)^2-\frac{a_1^2}{2a_2}+a_0\ , \quad (26)$$

the corresponding traveling wave solutions are:

$$x-ct-\xi_0=-\frac{2}{\sqrt{a_1^2-4a_0a_2}}\operatorname{arctan}\frac{2(u+1)+a_1}{\sqrt{a_1^2-4a_0a_2}}; \quad (27)$$

$$x-ct-\xi_0=\frac{1}{\sqrt{a_1^2-4a_0a_2}}\ln\left|\frac{2(u+1)+a_1-\sqrt{a_1^2-4a_0a_2}}{2(u+1)+a_1+\sqrt{a_1^2-4a_0a_2}}\right|. \quad (28)$$

Case 2.3 $\Delta<0$, the corresponding traveling wave solutions are:

$$x-ct-\xi_0=\frac{2}{\sqrt{4a_0a_2-a_1^2}}\operatorname{arctan}\frac{2(u+1)+a_1}{\sqrt{4a_0a_2-a_1^2}}. \quad (29)$$

Case 3 $p=4$, the Eq. (4) becomes

$$v''+Av'=Bv^5+Cv+D\ , \quad (30)$$

where $A=-\frac{\delta}{c\mu},B=-\frac{\delta}{5c\mu},C=\frac{1}{\mu},D=-\frac{g_1}{c\mu}$.

Denote
$$v'=a_3v^3+a_2v^2+a_1v+a_0 \quad (31)$$

$$v''=3a_3v^5+5a_2a_3v^4+\left(4a_1a_3+2a_2^2\right)v^3+\left(3a_0a_3+3a_1a_2\right)v^2+\left(2a_0a_2+a_1^2\right)v+a_0a_1. \quad (32)$$

We obtain the following equations by the Eq.(31) and Eq.(32),

$$
\begin{aligned}
&3a_{32}^2 = B;\\
&5a_3a_2 = 0;\\
&4a_1a_3 + 2a_2^2 + Aa_3 = 0;\\
&3a_0a_1 + 3a_1a_2 + Aa_2 = 0;\\
&2a_0a_2 + a_1^2 + Aa_1 = C;\\
&a_0a_1 + Aa_0 = D.
\end{aligned} \tag{33}
$$

Solving them, we have

$$a_3 = \pm\sqrt{-\frac{\sigma}{2c\mu}},\ a_2 = 0, a_1 = \frac{\delta}{4c\mu},\ a_0 = \frac{3g_1}{2\delta}. \tag{34}$$

Take the Eq.(34) to the Eq. (31) and integrate it，we have

$$\xi - \xi_0 = \int \frac{dv}{a_3v^3 + a_1v}. \tag{35}$$

We obtain the corresponding traveling wave solutions of the Eq.(35):

$$x - ct - \xi_0 = \frac{2c\mu}{\delta}\ln\left|\frac{(u+1)^2}{\frac{\delta}{4c\mu} \pm \sqrt{\frac{-\sigma}{2c\mu}}(u+1)^2}\right| \tag{36}$$

CONCLUSION

In this paper, all possible traveling wave solutions for the regularized Burgers-BBM equation have been given. The results show that trial equation method is powerful method to solve nonlinear-problems arising in mathematical physics.

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