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Research Article

The method of calculating the average decline rate by using the iterative method

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Abstract: In this paper, we used a simple iterative method to get the iterative formula of average decline rate,

$$\begin{cases} y_{n+1} = \dfrac{y_n^m + A - 1}{A} (y = 0, A = \dfrac{M}{a_1}), \\ x_n = 1 - y_n \end{cases}$$

and this formula is used to solve practical problems of solving the average decline rate.
Keywords: Iterative method, Average lapse rate, Differential mean value theorem

Questions raising

For the practical problems, such as calculating the departmental budget of national economy or estimating the oil well output, we often need to calculate the average increase (decrease) rate. For example:

For an oil well, its January output is a_1 ton,. If we want to guarantee annual oil production is M ton, what is the average monthly decline rate we need to control? $(12a_1 \geq M)$.

If we can find a simple and accurate algorithm to solve these kind of problems, it will be very useful and valuable. To solve these questions, let's see some knowledge of simple iterative method first.

Simple iterative method

Simple iterative method is a way to calculate approximate root of $\boldsymbol{x = g(x)}$ using the follow theorem.

Theorem: If $g(x)$ in $[a,b]$ has first order continuous derivative, and satisfy below conditions, $x = g(x)$ in $[a,b]$ has a unique root x^*.

When $0 < L < 1$, to any $x \in [a,b]$, $0 < g'(x) \leq L < 1$ is tenable.

Then for iterative equation $x_{k+1} = g(x_k)$, any initial value $\boldsymbol{x_0} \in [a,b]$ all converge to real root x^* of $x = g(x)$, and can deduce the following inequality.

$$\left| x^* - x_k \right| \leq \frac{1}{1-L} \left| x_{k+1} - x_k \right| \qquad (1)$$

$$\left| x^* - x_k \right| \leq \frac{L^k}{1-L} \left| x_1 - x_0 \right| \qquad (2)$$

Proof: As x^* is a real number root of $x = g(x)$ in $[a,b]$, according to mean value theorem

$$\left| x^* - x_k \right| = \left| g(x^*) - g(x_{k-1}) \right| = \left| g'(\xi) \right| \left| x^* - x_{k-1} \right| \leq L \left| x^* - x_{k-1} \right|$$

(ξ is between x^* and x_{k-1}) $(k = 1, 2, 3, \cdots\cdots)$.

We can know

$$\left|x^* - x_k\right| \le L\left|x^* - x_{k-1}\right|,\ \left|x^* - x_{k-1}\right| \le L\left|x^* - x_{k-2}\right|, \cdots\cdots,\ \left|x^* - x_1\right| \le L\left|x^* - x_0\right|,$$

so

$$\left|x^* - x_{k+1}\right| \le L^k\left|x^* - x_0\right|.$$

When $k \to \infty$: $x_{k+1} \to x^*$.

because

$$\left|x_{k+1} - x_k\right| = \left|(x^* - x_k) - (x^* - x_{k-1})\right| \ge \left|g'(\xi)\right|\left|x^* - x_k\right| - \left|x^* - x_{k+1}\right| \ge \left|x^* - x_k\right| - L\left|x^* - x\right|$$

We can conclude $\left|x_{k+1} - x_k\right| \ge (1-L)\left|x^* - x_k\right|$.

So that $\left|x^* - x_k\right| \le \dfrac{1}{1-L}\left|x_{k+1} - x_k\right|$.

So in equation（1）is proved.

Because $\left|x_{k+1} - x_k\right| = \left|g(x_k) - g(x_{k-1})\right| \le L\left|x_k - x_{k-1}\right| \quad (k = 1, 2, 3, \cdots\cdots)$,

so that

$$\left|x^* - x_k\right| \le \frac{1}{1-L}\left|x_{k+1} - x_k\right| \le \frac{L}{1-L}\left|x_k - x_{k-1}\right| \le \frac{L^2}{1-L}\left|x_{k-1} - x_{k-2}\right| \le \cdots\cdots \le \frac{L^k}{1-L}\left|x_1 - x_0\right|.$$

So in equation（2）is proved.

And though theorem 1, we can conclude：

When $x^* < x_0 \le b$, the distribution for approximate root of $x_{k+1} = g(x_k)$ in $[a,b]$ is

$$b \ge x_0 > x_1 > x_2 > \cdots > x^* > a \qquad (3)$$

When $x^* > x_0 \ge a$, the distribution for approximate root of $x_{k+1} = g(x_k)$ in $[a,b]$ is

$$a \le x_0 < x_1 < x_2 < \cdots < x^* < b \qquad (4)$$

According to mean value theorem

$$x^* - x_{k+1} = g(x^*) - g(x) = g'(\xi_k)(x^* - x_k)$$ （ξ_k is between x^* and x_k）.

When $k = 0$,

$$x^* - x_1 = g'(\xi_0)(x^* - x_0)$$ （ξ_0 is between x^* and x_k）.

Because $g'(\xi_0)$ satisfies $0 < g'(x) \le L < 1$,

so when $x^* < x_0 \le b$,

$$x_0 > x_1 > x^* > a \qquad (5)$$

By mathematical induction, we can conclude $x_0 > x_1 > x_2 > \cdots > x^* > a$, on the same logic, when $x^* > x_0 \ge a$, $a \le x_0 < x_1 < x_2 < \cdots < x^* < b$.By in equation (3) and (4), when $k \to \infty$, monotonic classification $\{x_k\}$ tend to x^*.

Application

For an oil well, its January output is a_1 tons. If we want to guarantee annual oil production is M ton, what is the average monthly decline rate we need to control? $(12a_1 \ge M)$.

Set n months of oil production is a_n tons, the average monthly decline rate is x, exactly complete the task of M tons. According to the following equation,

$$a_1 = a_1,$$

$$a_2 = a_1(1-x)\ ,$$
$$a_3 = a_2(1-x) = a_1(1-x)^2\ , \qquad (6)$$
....
$$a_{12} = a_{11}(1-x) = a_{10}(1-x)^2 = \cdots = a_1(1-x)^{11}\ .$$

By (6) and $a_1 + a_2 + \cdots + a_{12} = M$,

$$a_1\left[1+(1-x)+(1-x)^2+\cdots\cdots+(1-x)^{11}\right] = M \qquad (7)$$

Set $1-x=y$ ($0<y<1$), according to (7),

$$y^{11}+y^{10}+\cdots\cdots+y+1=\frac{M}{a_1}=A \qquad (8)$$

As $y \neq 1$, so

$$y^{11}+y^{10}+\cdots\cdots+y+1=\frac{1-y^{12}}{1-y}=A \qquad (9)$$

And the equivalent equation of equation (9) is

$$y=\frac{y^{12}+A-1}{A}=g(y) \qquad (10)$$

Based on facts, we can conclude root x^* of equation (7) is in the range of $(0,1)$, and $1-x=y$,so that equation (8)'s root y^* is in the range of $(0,1)$.

By equality (10),

$$g'(y^*)=\frac{12(y^*)^{11}}{A} \qquad (11)$$

Substituting y^* into equation (8)

$$y^{*11}+y^{*10}+\cdots\cdots+y^*+1=A \qquad (12)$$

From (11) and (12), we can get

$$0<g'(y^*)=\frac{12(y^*)^{11}}{y^{*11}+y^{*10}+\cdots+y^*+1}<1\ (0<y^*<1) \qquad (13)$$

It means the function value of $g'(y\ )$ at y^* of equation (8) satisfies $0<g'(y^*)<1$.

And $g'(y\ )=\frac{12y^{11}}{A}$ in $(0,1)$ is continuous and monotone increasing, and always stay positive.

So that there must be a section $(0,b)$ $(0<b<1)$ contains y^*, and constant q^* in $(0,b)$ satisfies $0<g'(y)<q^*<1$.

So $g(y)=\frac{y^{12}+A-1}{A}$ is accord with theorem conditions.

Set $y_0=0$, sequence $\{y_n\}$ of $y_{n+1}=g(y_n)=\frac{y_n^{\ 12}+A-1}{A}$ must converge to the root of equation (10) y^* , that is $\lim_{n\to\infty} y_n = y^*$.

So that through $x_n - 1 = y_n$, we can get the approximate root of equation (10) x_n, and calculate the average monthly oil decline rate x. Thus we can get a formula as follows:

$$\begin{cases} y_{n+1} = \dfrac{y_n^m + A - 1}{A} \qquad (y_0 = 0, A = \dfrac{M}{a_1}), \\ x_n = 1 - y_n \end{cases} \qquad (14)$$

Its error estimation can be calculated by Theorem (2).

For example an oil recovery well produced oil 1000 tons, and it has a mission to produce 5000 tons oil for one year. So what is the average oil production decline rate we need to keep?

By formula (14):

$$y_1 = \frac{0^{12} + 4}{5} = 0.8 \qquad (A = \frac{5000}{1000} = 5, \ A - 1 = 4)$$

$$y_2 = \frac{y_1^{12} + 4}{5} = 0.81374$$

$$y_3 = \frac{y_2^{12} + 4}{5} = 0.81686$$

$$y_7 = \frac{y_6^{12} + 4}{5} = 0.81793$$

$$x_7 = 1 - y_7 = 0.18207$$

So we need to keep the average oil production decline rate less than or equal to 0.18207 % if we want to complete the mission of oil-producing 5000 tons.

In this case, 12 months can be replaced by m months completing M tons. Using formula (14), we can easily calculate the approximation of the average monthly decline rate.

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