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Research Article

Study on Mathematical Model Parameters of City-Flood Numerical Inversion

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Abstract: This article elaborately introduced the theory and actuality of city flood. It clarifies the process of rectified model: diffusion coefficient function instead of using ground elevation function, rainfall integrated functions formation permeability function and drainage function function terms. Then it chooses finite-different method to solve convection-diffusion equations. It applied to this model. Classical optimal inversion method-Best Motion method is used, constructed arithmetic for the related parameter of equation. Apparently, this model could exactly describe city flood. Inverse identification is feasible in the process of simulation.
Keywords: city-flood; Numerical Inversion; Mathematical model

INTRODUCTION

City-flood (storm) is particularly serious problems currently facing China. Short period of time, heavy rainfall caused waterlogging disasters has become increasingly prominent. It not only threatens the city's flood control and drainage tasks, but also for the people living in the city are also a great impact. Such as Beijing, 2009-2011, many sections of city formation of waterlogging, big congestion, traffic jam. Especially, July 2012, heavy rainfall resulting in mudslides, traffic disruption, and municipal water projects with multiple injuries. The loss of economic statistics preliminary closes to 10 billion. Concern is casualty's people unexpectedly to 37 due to this City-flood at 17:00 on the 22nd.

Currently, most studies on improperly research in inverse problem from perspective of mathematics, inverse problem solving method can't be used to treating engineering problems. In this field of study, C. G. Collierl, S. Cavdar, Wiggins RA and so many scholar research work hardly [1-9]. However, there is no effective method to complete mathematical model parameters of City-Flood numerical inversion.

City-Flood Mathematical Model

In this paper, City Flood refers to change of rainfall accumulation under normal operation of drainage system in city. Grades of rainfall intensity are light rain, moderate rain, heavy rain, and rainstorm. The rainstorm refers to the rainfall intensity are great. Following two kinds of circumstances: rainfall exceeds 16mm/hour or 50mm/day.

Assumptions and Descriptions

Since the actual physical characteristics of urban flooding, we can modify the model as follows:

$$k(x,y)\left(\frac{\partial^2 u(x,y,t)}{\partial x^2}+\frac{\partial^2 u(x,y,t)}{\partial y^2}\right)+\frac{\partial k(x,y)}{\partial x}\cdot\frac{\partial u(x,y,t)}{\partial x}+\frac{\partial k(x,y)}{\partial y}\cdot\frac{\partial u(x,y,t)}{\partial y}+\cdots$$
$$+F(x,y,t)+P(x,y,t)+S(x,y,t)=\frac{\partial u(x,y,t)}{\partial t}$$

Where $F(x,y,t)$ is Rainfall function, $P(x,y,t)$ is Confluence function of drainage system, $S(x,y,t)$ is Confluence function of water permeation, $k(x,y)$ is diffusion coefficient, represented by topographic function. $\frac{\partial k(x,y)}{\partial x}$ and $\frac{\partial k(x,y)}{\partial y}$ are convection coefficient, $u(x,y,t)$ is water level. Boundary conditions: $u|_{\partial\Omega}=0, t=0$ ..Initial conditions: $u|_{\Omega}=0, t=0$.

Relevant parameters and Functions

Space step: $\Delta x = \Delta y = 1m$, time step: $\Delta t = 1\min$, region grid dissection: 40*40, four region: $[0\sim 20]\times[0\sim 20]$, $[0\sim 20]\times[20\sim 40]$, $[20\sim 40]\times[0\sim 20]$, $[20\sim 40]\times[20\sim 40]$, number of grid direction X is m, number of grid direction Y is n.

Consider actual geographical terrain, diffusion coefficient is a relationship with the terrain elevation and geology. Geological changes of the same city are little change. Diffusion coefficient as following terrain function is given by

$$Z(x,y) = 100(x_2 - x_1^2)^2 + (1-x_1)^2.$$

where $x_1 = [-2,2]$, $x_2 = [-1,3]$.

Suppose rainfall function $F(x,y,t)$ as follows:

$$F(x,y,t) = F_0 \cdot e^{-a\sqrt{b(x-x_0)^2+(y-y_0)^2}}$$

where F_0 is rainfall of precipitation Center, a and m are coefficient, $F_0 = v\cdot t$, v is intensity of rainfall, assumed $v = 0.01m/\min$. t is time, assumed $a = 0.02$, $b = 1.2$.

Suppose drainage system function as follows, Drainage converges system had relationship with structure and timing of drainage network, simulation of displacement. This paper mainly described drainage capacity, as negative value

$$P = \frac{1}{n_c}\cdot A_f \cdot R_f^{\frac{2}{3}} \cdot S_f^{\frac{1}{2}} \cdot t$$

where $n_c = 0.02$ is roughness coefficient, A_f is cross-sectional area of pipe. $R_f = 0.25m$ is hydraulic radius of pipeline. S_f is friction gradient, $S_f = \frac{n^2 \cdot u\sqrt{u^2+v^2}}{d^{4/3}}$. n is ground roughness coefficient. d is depth from the ground. u,v are surface water flow rate.

Supposed $n = 0.04$, $d = 1.5m$, $u = v = 0.1m/s$, pipe diameter is 1m.It is average penetration in area except drain unit. It has two drains. Lowest terrain as (31, 21) and (10, 10). With drainage depth reflects displacement, assumed saturation at 40 minutes. Following shows contour maps of displacement at 10~40min.

Infiltrate function on special regional:

$$S = \begin{cases} -0.005\cdot t\cdot x/m, x\in[0,20]\times[0,20] \\ 0 \qquad , x \in other\ area \end{cases}$$

It assumed saturation after 20 minutes. Following shows contour maps of infiltration at 10~20min.

City-Flood Mathematical Model Numerical Inversion

Definite solution problems of two-dimensional diffusion equation as City-Flood mathematical model:

$$\frac{\partial u}{\partial t} - (k(x,y)\frac{\partial^2 u}{\partial x^2} + k(x,y)\frac{\partial^2 u}{\partial y^2} + \frac{\partial k}{\partial x}\frac{\partial u}{\partial x} + \frac{\partial k}{\partial y}\frac{\partial u}{\partial y} + f) = 0 \qquad (1)$$

Where $f = F + P + S,\ (x,y)\in\Omega[M\times N],\ (x,y)\in\Omega,\ t\in[0,T)$.

Boundary condition

$$u|_{\partial\Omega} = 0, (x,y)\in\partial\Omega, t\in[0,T) \qquad (2)$$

Initial condition

$$u|_{\Omega} = 0, (x,y)\in\Omega, t = 0 \qquad (3)$$

An inverse problem of the diffusion equation as following: formula (1~3) additional boundary conditions:

$$u(x,y,t) = u_\eta(x,y,t), (x,y)\in\Gamma_\eta \subset \Gamma \qquad (4)$$

To determine the unknown function $k(x,y,t)$ .So, this problem is from known function f and $u_\eta(x,y,t)$ to solve $k(x,y,t)$. $u_\eta(x,y,t)$ is ground water level when it rains. The water level can be monitored by some observation points. It can used to additional boundary conditions.For example, T as sampling period, $t = iT(i = 0,1,\cdots,I)$, this problem is use known function f and $u_\eta(x,y,iT)(i=0,1,\cdots,I)$ to solve $k(x,y,t)$.

Assume $\{\varphi_i(x,y)\}$ and $\{\phi_j(t)\}$ respectively region Ω and $[0,\infty)$ basis function or family of orthogonal basis functions.

$$k(x,y,t) = \sum_{i=1}^{\infty}\sum_{j=1}^{\infty} k_{i,j}\varphi_i(x,y)\phi_j(t) \tag{5}$$

$$k_{nm}(x,y,t) = \sum_{i=1}^{n}\sum_{j=1}^{m} k_{i,j}\varphi_i(x,y)\phi_j(t) \tag{6}$$

$$\Phi_{nm}(x,y,t) = (\varphi_1(x,y)\phi_1(t), \varphi_1(x,y)\phi_2(t), \cdots, \varphi_n(x,y)\phi_m(t))^T$$

$$k_{nm} = (k_{11}, k_{12}, \cdots, k_{1m}, \cdots, k_{n1}, \cdots, k_{nm})^T \in R^{nm}$$

Where R^{nm} is $n\times m$ real space, T as transpose。

$$k(x,y,t) = \lim_{n,m\to\infty} k_{nm}(x,y,t) = \lim_{n,m\to\infty} \Phi_{nm}^T(x,y,t)k_{nm} \tag{7}$$

For appropriate n m , sum of finite base $k_{nm}(x,y,t)$ approximation $k(x,y,t)$ .So, this inverse problem Converted to known definite solution problems (1), (2),(3)and $u_\eta(x,y,iT)(i=0,1,\cdots,I)$ to solve $k_{nm}(x,y,t)$, notation k_{nm} .

Assumption $k_{nm}(x,y,t)$ concerning solution of definite solution problem (1),(2) and (3), notation $u(k_{nm};x,y,t)$ .So,the inverse problem conversion to optimization problem as following:

$$\min_{k_{nm}\in R^{nm}} \| u_\eta(x,y,t) - u(k_{nm};x,y,t) \|^2_{(x,y,t)\in\Gamma_{\eta t}} \tag{8}$$

Where $u(k_{nm};x,y,t)$ satisfy equation (1), $\Gamma_{\eta t} = \{(x,y,t)\in\Gamma_\eta, t\in[0,IT]\}$.

Numerical iterative process as following:

$$k_{nm}^{l+1} = k_{nm}^l + \delta k_{nm}^l, l = 0,1,2,\cdots \tag{9}$$

Where k_{nm}^0 is initial guess value, small undisturbed $\delta k_{nm}^l(\|\delta k_{nm}^l\| < \|k_{nm}^l\|)$ of k_{nm}^l can solved use minimum value of objective function:

$$F[\delta k_{nm}^l] = \| u_\eta(x,y,t) - u(k_{nm}^l + \delta k_{nm}^l; x,y,t)\|^2_{(x,y,t)\in\Gamma_{\eta t}} + \alpha S[\delta k_{nm}^l]$$

Where α is regularizing parameter, $S[\delta k_{nm}^l]$ as stability functional.

If discrete data $(x_i, y_i)(j=1,2,\cdots,J)$ of $u_\eta(x,y,t)$ in Γ_η and sampling point $(iT)(i=1,2,\cdots,I)$ of time t is known, in addition

$$u(k_{nm}^l + \delta k_{nm}^l; x,y,t) = u(k_{nm}^l;x,y,t) + \nabla_{k_{nm}^l} u(k_{nm}^l;x,y,t)\cdot\delta k_{nm}^l + o(\|k_{nm}^l\|) \tag{5-10}$$

Then, objective function $F[\delta k_{nm}^l]$ reducible to:

$$F[\delta k_{nm}^l] = \sum_{j=1}^{J}\sum_{i=1}^{I}[u_\eta(x_j,y_j,iT) - u(k_{nm}^l;x_j,y_j,iT) - \nabla_{k_{nm}^l} u(k_{nm}^l;x,y,t)\cdot\delta k_{nm}^l]^2 + \alpha S[\delta k_{nm}^l]$$ Assumption

$$b_j(i) = u_\eta(x_j,y_j,iT) - u(k_{nm}^l;x_j,y_j,iT) \tag{11}$$

$$\alpha_j^T(i) = \nabla_{k_{nm}^l} u(k_{nm}^l;x,y,t) \tag{12}$$

Then

$$F[\delta k_{nm}^l]=\sum_{j=1}^{J}\sum_{i=1}^{I}[b_j(i)-\alpha_j^T(i)\cdot\delta k_{nm}^l]^2+\alpha S[\delta k_{nm}^l]$$

If stability functional $S[\delta k_{nm}^l]$ as $(\delta k_{nm}^l)^T\delta k_{nm}^l$, then $F[\delta k_{nm}^l]$ is positive definite quadratic form δk_{nm}^l,its minimum determined from the following equations:

$$\sum_{i=1}^{I}[A^T(i)A(i)+\alpha E]\delta k_{nm}^l=\sum_{i=1}^{I}A^T(i)B(i) \tag{13}$$

where

$$A(i)=\begin{bmatrix}a_1^T(i)\\ a_2^T(i)\\ \vdots\\ a_J^T(i)\end{bmatrix}$$

$$B(i)=\begin{bmatrix}b_1(i)\\ b_2(i)\\ \vdots\\ b_J(i)\end{bmatrix}$$

E is unit matrix, order $nm\times nm$

Discussed Calculation Method of $\nabla_{k_{nm}^l}u(k_{nm}^l;x,y,t)$. To ensure accuracy of the inverse problem, use difference for definite solution problems (1)~(3)

Discrete differential equations as following:

$$\begin{aligned}u_{i,j}^{k+1}&=u_{i,j}^k+\Delta t[-(\frac{2k}{\Delta x^2}+\frac{2k}{\Delta y^2}+\frac{k_x}{\Delta x}+\frac{k_y}{\Delta y})u_{i,j}^k+\cdots\\&+k(u_{i+1,j}^k+u_{i-1,j}^k+u_{i,j+1}^k+u_{i,j-1}^k)+k_xu_{i+1,j}^k+k_yu_{i,j+1}^k+F_{i,j}+P_{i,j}+S_{i,j}]\end{aligned} \tag{14}$$

Where $u_{i,j}^k$ is column vector constructively as $u(k_{nm}^l;x,y,t)$ in Ω, τ is time step,Select appropriate τ ,let $i\tau$ by sampling period points, iT $(i=0,1,2,\cdots,I)$.

For equation (14) solve component $k_v(v=1,2,\cdots,nm)$ of k_{nm}^l, after differential

$$\begin{aligned}\frac{\partial u_{i,j}^{k+1}}{\partial k}&=\frac{\partial u_{i,j}^k}{\partial k}+\Delta t[-(\frac{2k}{\Delta x^2}+\frac{2k}{\Delta y^2}+\frac{k_x}{\Delta x}+\frac{k_y}{\Delta y})u_{i,j}^k-(\frac{2}{\Delta x^2}+\frac{2}{\Delta y^2})u_{i,j}^k+\cdots\\&+(u_{i+1,j}^k+u_{i-1,j}^k+u_{i,j+1}^k+u_{i,j-1}^k)+k(\frac{u_{i+1,j}^k}{\partial k}+\frac{u_{i-1,j}^k}{\partial k}+\frac{u_{i,j+1}^k}{\partial k}+\frac{u_{i,j-1}^k}{\partial k})]\end{aligned} \tag{16}$$

then, for fixed n,m, iterative process of definite solution inverse problems (1)~(3) as following:

- Firstly, as given unknown $k(x,y)$ and initial guess value $k_{nm}(x,y)$, note k_{nm}^l;
- Applied numerical solution method for definite solution problems by equation(14) to solve $u(k_{nm}^l;x_j,y_j,iT)$ 。Solved equation (15), ascertain $\nabla_{k_{nm}^l}u(k_{nm}^l;x,y,t)$;
- Numerical solution of linear equations (13), get δk_{nm}^l;
- Form equation (9) get one new initial guess value δk_{nm}^l.

Repeat process (2), (3),until satisfy accuracy.

Numerical Examples

In this, k^* as true value, k as inverse value.With 1.1 times true value as initial predicted value.Regularization parameters taken 1e-4, $u(x,y,t)$ as k^* calculated value. As $u(x,y,t)(1+r\%\xi)$ (ξ is random Number in[-1，1]) simulated with $r\%$ Relative random measurement error.

Measure accuracy of predicted value error as ε， expression as following

$$\varepsilon = \sqrt{\sum_{i=1}^{m}\sum_{j=1}^{n}(k^*(i,j)-k(i,j))^2} \Bigg/ \sqrt{\sum_{i=1}^{m}\sum_{j=1}^{n}k^*(i,j)^2}$$

Diffusion parameter inversion identification parameters under different noise simulation results are as follows:

Table-1:The precision contrast of inverse value and real value

Noise (r%)	0	15
Accuracy (ε)	8.16‰	1.05%

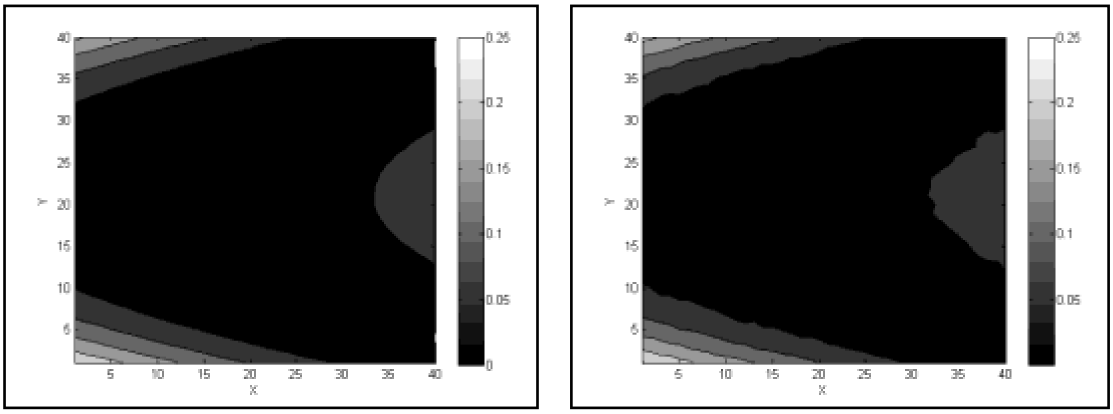

Fig-1: The compared graph of real value and inverse value (free noise)

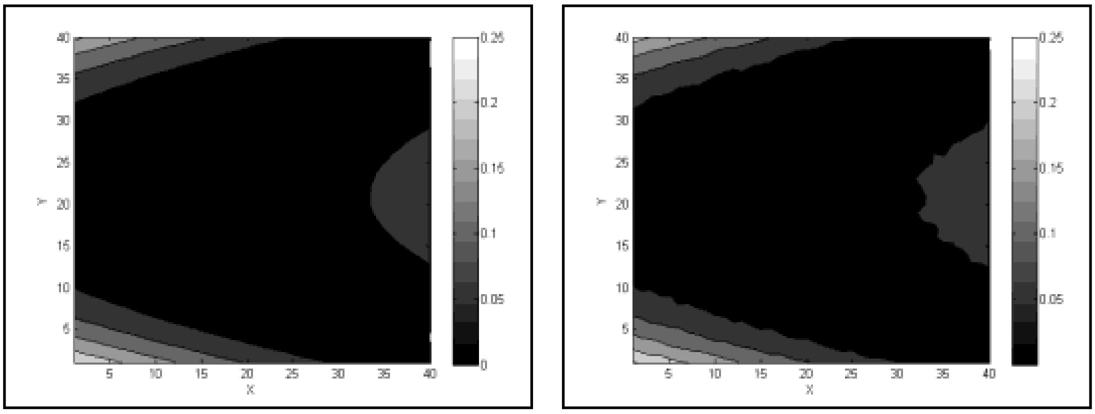

Fig-2: The compared graph of real value and inverse value (noise 15%)

As shown significant results in Figs, this inversion algorithm is feasible and better noise immunity.

RESULTS AND DISCUSSION

This article elaborately introduced the theory and actuality of city flood. And it clarifies the process of rectified model: replace terrain function to diffusion parameter, integrate rainfall function, drain function and seepage function function terms. Then it chooses finite-different method to solve convection-diffusion equations. It applied to this model. Classical optimal inversion method-Best Motion method is used, constructed arithmetic for the related parameter of equation. Apparently, this model could exactly describe city flood. The inverse method is feasible in the process of simulation. Summary, inverse modeling is the effective forecast method of flood-city model and the further development of this theory.

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