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Research Article

Global Solutions to Damped Fisher Equation by Renormalization Group Method Based on the Classical Theory of Envelope

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Abstract: By the Kunihiro's renormalization group method based on the classical theory of envelope, the global approximate solution to damped Fisher equation is obtained.
Keywords: renormalization group; envelope theory; global approximate solutions; damped Fisher equation.

INTRODUCTION

Renormalization group(RG)[1] was initially proposed by Gell-Mann and Low to handle the divergences in the perturbation theory arising in the quantum field theory[2-6], and then many scientists found that the RG method was also powerful in other fields. Recently, Goldenfeld and his colleagues initially applied the RG to nonlinear partial differential equations, such as Barenblatt's equation[7], modified porous medium equation[8], and turbulent-energy-balance equation[9].

Their method is simple and has a lot of applications including singular and reductive perturbation problems in a unified way, and they anticipated that RG method would be useful to perturbed problems. After that, many scientists' work proved it is true. Though Goldenfel's presentation of the method is quiet heuristic, heavily relying on the RG prescription in QFT and statistical physics, namely the mathematical foundation of their method has not been established. In fact, why RG equations even in QFT improve naive perturbation has not been elucidated. In paper[10-11], Teiji Kunihiro formulated the renormalization group method based on the theory of envelop and give a mathematical foundation to this method, then he pointed out that the RG equation can be regarded as the envelop equation, and he also proved why the RG equation can give globally improved solutions to ODEs and PDEs. His method is a more direct way to perturbed problems comparing with traditional RG method.

In this paper, we apply Kunihiro's RG method based on the theory of envelope to obtain the global approximate solutions correct up to $O(\varepsilon^2)$ the damped Fisher equation[12]

$$\varepsilon u_{tt} + u_t = u_{xx} + u(1-u) \tag{1}$$

The outline of this paper is as follows. In sections 2, we use Kunihiro's RG method to obtain the global approximate solutions to the damped Fisher equation. In section 4, we give a conclusion.

APPLICATION TO DAMPED FISHER EQUATION

In this section, we use the Kunihiro's RG method to solve the damped Fisher equation(1). By introducing the inner variables $\sqrt{\varepsilon}X = x$ and $\varepsilon T = t$ and setting $U(X,T) = u(x,t)$, we have

$$\frac{\partial^2 U}{\partial T^2} + \frac{\partial U}{\partial T} = \frac{\partial^2 U}{\partial X^2} + \varepsilon U(1-U) \ , \tag{2}$$

Expanding U as $U = U_0 + \varepsilon U_1 + \ldots$ gives

$$U_{0TT} + U_{0T} = U_{0XX} \ , \tag{3}$$

and

$$U_{1TT} + U_{1T} = U_{1XX} + U_0(1-U_0) \tag{4}$$

and so on. The 0-th order solution is

$$U_0(X,T) = Ae^{\frac{-1+\sqrt{1-4\lambda}}{2}T+i\sqrt{\lambda}X} + c.c. \tag{5}$$

where c.c. denotes the complex conjugate, λ is the eigen value of Eq.(2), and A is a complex constant. If $\lambda \neq 0$, we take formula (5) into Eq.(4), we can get

$$U_{1TT} + U_{1T} = U_{1XX} + Ae^{\frac{-1+\sqrt{1-4\lambda}}{2}T+i\sqrt{\lambda}X} - A^2e^{-1+\sqrt{1-4\lambda}T+2i\sqrt{\lambda}X}c.c., \tag{6}$$

whose solution is given by ,

$$U_1 = \frac{Ai(X-X_0)}{2\sqrt{\lambda}}e^{\frac{sT}{2}+i\sqrt{\lambda}X} + \frac{A^2}{s}e^{ST+2i\sqrt{\lambda}X} + \frac{2|A^2|}{4\lambda+s}e^{sT} + c.c. \tag{7}$$

where $s = -1+\sqrt{1-4\lambda}$. Then we have a family of surfaces $S_{x_0}(X,T)$ is represented by

$$S_{X_0}(X,T) = U(X,T) = Ae^{\frac{-1+\sqrt{1-4\lambda}}{2}T+i\sqrt{\lambda}X} + \varepsilon\left(\frac{Ai(X-X_0)}{2\sqrt{\lambda}}e^{\frac{sT}{2}+i\sqrt{\lambda}X} + \frac{A^2}{s}e^{ST+2i\sqrt{\lambda}X} + \frac{2|A^2|}{4\lambda+s}e^{sT}\right) + c.c \tag{8}$$

parametrized by X_0. We can see it clear that U diverges when X approaches to infinite, so we use the Kunihiro's RG method to eliminate the secular term and get the global solution. Now let us obtain the envelope E of the family of the surfaces of (8). Assuming that A is dependent on X_0, that is

$$A = A(X_0) \tag{9}$$

According to the standard procedure to obtain the envelope, we solve the equation

$$\frac{\partial U}{\partial X_0} = 0 \tag{10}$$

with the point of tangency at $x_0=x$, and then we have

$$\frac{dA}{dX} = \varepsilon\frac{Ai}{2\sqrt{\lambda}}, \tag{11}$$

whose solution is given by

$$A = \overline{A}e^{\frac{i\varepsilon X}{2\sqrt{\lambda}}}, \tag{12}$$

where $\overline{A}$ is a complex constant. Therefore the envelope is represented by

$$U(X,T) = \overline{A}e^{sT+i(\sqrt{\lambda}+\frac{\varepsilon}{2\sqrt{\lambda}})X} + \varepsilon\left(\frac{\overline{A}e^{\frac{i\varepsilon X}{2\sqrt{\lambda}}}i(X-X_0)}{2\sqrt{\lambda}}e^{\frac{sT}{2}+i\sqrt{\lambda}X}\right. \\ \left. + \frac{\overline{A}e^{\frac{i\varepsilon X}{\sqrt{\lambda}}}}{s}e^{ST+2i\sqrt{\lambda}X} + \frac{2|\overline{A}^2|}{4\lambda+s}e^{sT}\right) + c.c, \tag{13}$$

In terms of original variables x and t, we have

$$u(x,t) = \overline{A}e^{\frac{st}{2\varepsilon}+i(\sqrt{\frac{\lambda}{\varepsilon}}+\sqrt{\frac{\varepsilon}{4\lambda}})x} + \varepsilon\left[\frac{\overline{A}^2}{s}e^{\frac{st}{\varepsilon}+i(2\sqrt{\frac{\lambda}{\varepsilon}}+\sqrt{\frac{\varepsilon}{\lambda}})x} + \frac{2|\overline{A}\wedge 2|}{4\lambda+s}e^{st+i\sqrt{\frac{\varepsilon}{\lambda}}x}\right] + c.c. \tag{14}$$

It is clear to see that we have eliminated the secular term with the RG method and a global approximate solution is obtained.

CONCLUSION

In this paper, we apply the Kunihiro's RG method based on the theory of envelope to the damped Fisher equation, the result shows that the RG method is simple to eliminate the secular term and give the global approximate solution.

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