DOI: 10.21276/sjet.2016.4.10.4

Scholars Journal of Engineering and Technology (SJET) **ISSN 2321-435X (Online)**
Sch. J. Eng. Tech., 2016; 4(10):489-499 **ISSN 2347-9523 (Print)**

(An International Publisher for Academic and Scientific Resources)
www.saspublisher.com

Original Research Article

Performance Analysis of Diversity Combining Techniques over Rayleigh Fading SIMO Multicasting Wireless Network

Rosni Sayed[1], Md Asaduzzaman Shobug[2], A. S. M. Badrudduza[3]
[1,2]Department of Electrical and Electronic Engineering, Pabna University of Science & Technology, Bangladesh
[3]Department of Electrical and Electronic Engineering, Rajshahi University of Engineering & Technology, Bangladesh

***Corresponding author**
Rosni Sayed
Email: rosnisayed@gmail.com

Abstract: This paper is concerned with the study of Rayleigh fading SIMO multicasting wireless network. The total system is analyzed with and without diversity. At first, we characterize the multicast channel capacity. Then the closed form expressions of Probability Density Function (PDF), Cumulative Distribution Function (CDF) and Complementary Cumulative Distribution Function (CCDF) of the multicast capacity are derived in the case of without applying any diversity combining technique. Next, we derived these parameters by applying two diversity combining techniques, such as Selection Combining (SC) and Maximal Ratio Combining (MRC). Finally we present a comparison between these two cases, with and without diversity combining techniques.
Keywords: CCDF, CDF, diversity combining techniques, MRC, multicasting wireless network, PDF, SC

INTRODUCTION

Multicasting is more efficient than broadcasting because it allows transmission to multiple destinations using fewer network resources. The term ‘multicasting’ is a scenario in which a transmitter sends a common stream of information message to a group of client receivers although there are several other receivers under its transmission range. For example, video-conferencing. It is one of the important and challenging issues in wireless communication to disseminate data to a group of receivers that are interested in the same content. Although multicasting ensures better utilization of communication resources, channel fading is still a big threat for sound wireless communication. There are two techniques to combat the effect of fading. They are

- Transmitter Power Control
- Diversity Technique

Diversity combining technique is a good alternative to transmitter power control method because in Transmitter Power Control method, the transmitter requires a dynamic range and channel information has to be fed back from the receiver to the transmitter. In receiver diversity the independent fading paths associated with multiple receive antennas are combined to obtain a resultant signal that is then passed through a standard demodulator. The combining can be done in several ways. Diversity Combining Techniques are applied to combine the multiple received signals of a diversity reception device into a single improved signal to get proper diversity benefit [1]. Several Combining Techniques are listed below

* Selection Combining (SC)
* Maximal Ratio Combining (MRC)
* Threshold or Switch and Stay Combining (SSC)

Selection Combining (SC)

In selection combining (SC), the combiner outputs the signal on the branch with the highest SNR. With SC the path output from the combiner has an SNR equal to the maximum SNR of all the branches. Fig. 1 shows the system model for SC diversity technique. Consider the SC in a frequency-flat-slow fading Rayleigh Channel. Assume that some diversity form provides M independent paths (each path is Rayleigh fading) Instantaneous SNR in branch is given as [2]

$$\gamma_i = \frac{E}{N_0}|h_i|^2$$

Where $|h_i|^2$ is the channel (complex) gain, E is the symbol energy and N_0 is the noise spectral power density. The output SNR will be

$$\gamma_{out} = \max(\gamma_i)$$

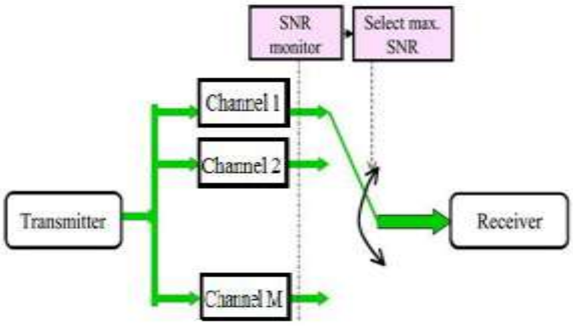

Fig-1: System model for SC diversity technique

Switch and Stay Combining (SSC)

SC diversity combining technique requires a dedicated receiver on each branch to continuously monitor branch SNR for the systems that transmit continuously. A simpler type of combining, called threshold combining, avoids the need for a dedicated receiver on each branch by scanning each of the branches in sequential order and outputting the first signal with SNR above a given threshold γ_t. The system model for SSC technique is shown in Fig. 2.

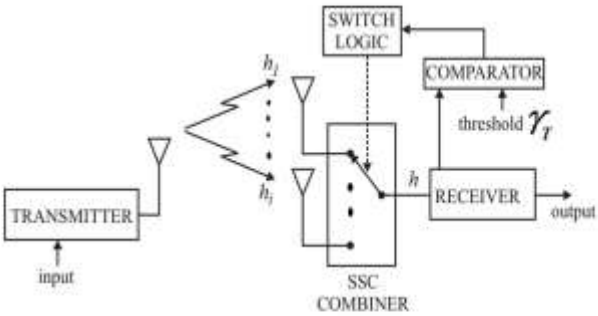

Fig-2: System model for SSC

In this Technique Once a branch is chosen, as long as the SNR on that branch remains above the desired threshold, the combiner outputs that signal. If the SNR on the selected branch falls below the threshold, the combiner switches to another branch. There are several criteria the combiner can use to decide which branch to switch to [3]. The simplest criterion is to switch randomly to another branch. With only two-branch diversity this is equivalent to switching to the other branch when the SNR on the active branch falls below γ_t. This method is called switch and stay combining (SSC). The switching process and SNR associated with SSC is illustrated in Fig.3. Since the SSC does not select the branch with the highest SNR, its performance is between that of no diversity and ideal SC.

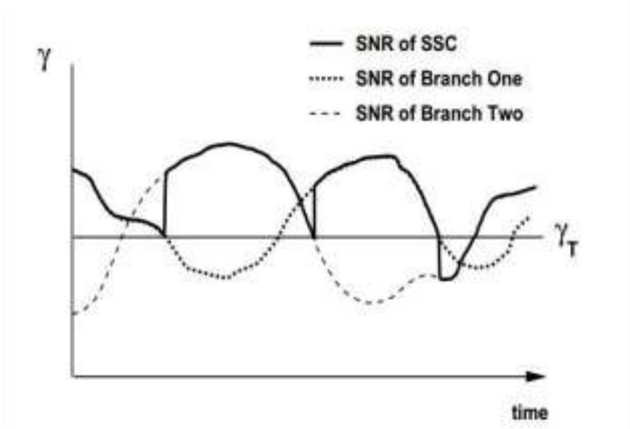

Fig-3: SNR of SSC Technique

Maximal Ratio Combining (MRC)

Maximal Ratio Combining (MRC) the output is a weighted sum of all branches. Fig. 4 shows the system model for MRC technique.

The output SNR will be

$$\gamma_{out} = \frac{1}{N_0} \frac{(\sum_{i=1}^{M} w_i h_i)^2}{\sum_{i=1}^{M} w_i{}^2}$$

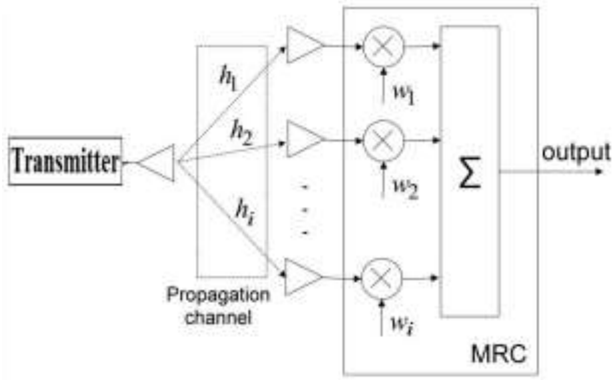

Fig-4: System model for MRC

SYSTEM MODEL AND PROBLEM FORMULATION

In this paper, we consider a Rayleigh fading SIMO multicasting scenario as shown in Fig. 5. Here a single antenna transmitter, denoted by T_x communicates with M receivers. Each receiver is equipped with n_R antennas.

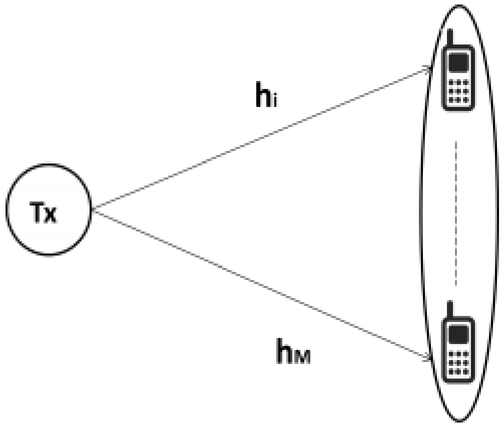

Fig-5: System Model

The direct channel coefficient between the source and the ith receiver is h_i, where, $i = 1, 2, 3 \ldots\ldots, M$. Let x denotes the transmitted signal. The received signals at the ith receiver given by (1)

$$\boldsymbol{y_i} = \boldsymbol{h}_i x + \boldsymbol{z}_i \, , \qquad (1)$$

where $\boldsymbol{z_i} \sim \widetilde{\mathcal{N}}(0, N_{m_0} I_{n_R})$ is the Gaussian noise, imposed on the ith receiver. $\widetilde{\mathcal{N}}(0, N_{m_0} I_{n_R})$ means Gaussian distribution with zero mean and $N_{m_0} I_{n_R}$ variance, where I_{n_R} is an identity matrix of $n_R \times n_R$.

CAPACITY OF RAYLEIGH FADING SIMO MULTICASTING NETWORK

The capacity limit of a communication channel dictates that the amount of maximum data transmission over the channel considering a very small error probability. Capacity was first pioneered by Shannon by maximizing mutual information over all possible input distributions [4].

From (1) the received signal at the i^{th} user is given by

$$\boldsymbol{y_i} = \boldsymbol{h}_i x + \boldsymbol{z}_i$$

Mutual information at i^{th} user is

$$I(x; \boldsymbol{y_i}) = h(\boldsymbol{y_i}) - h(\boldsymbol{z_i}) \qquad (2)$$

Here $h(.)$ denotes entropy. Let the variance of x is given by $Q_x = \mathbb{E}\,(\mathrm{xx}^{\dagger}) = \mathrm{P}$, where $\mathbb{E}(.)$and $(.)^{\dagger}$ denote the expectation and conjugate transpose operations, respectively.
Now, co-variance of received signal can be derived as

$$\begin{aligned}
R_{y_i} &= \mathbb{E}\,(\boldsymbol{y_i}, \boldsymbol{y_i}^{\dagger}) \\
&= \mathbb{E}\{(\boldsymbol{h}_i x + \boldsymbol{z_i})(\boldsymbol{h}_i x + \boldsymbol{z}_i)^{+}\} \\
&= \mathbb{E}\{(\boldsymbol{h}_i x + \boldsymbol{z}_i)(x^{+} \boldsymbol{h}_i^{+} + \boldsymbol{z}_i^{+})\} \\
&= \mathbb{E}(\boldsymbol{h}_i x\, x^{+} \boldsymbol{h}_i^{+} + \boldsymbol{z}_i \boldsymbol{z}_i^{+}) \\
&= \boldsymbol{h}_i \mathbb{E}(x\, x^{+}) \boldsymbol{h}_i^{+} + \mathbb{E}(\boldsymbol{z}_i \boldsymbol{z}_i^{+}) \\
&= \boldsymbol{h}_i P \boldsymbol{h}_i^{+} + N_{m_o} \boldsymbol{I}_{n_R} \qquad (3)
\end{aligned}$$

Similarly, covariance of noise signal is given by,

$$R_{z_i} = \mathbb{E}(\boldsymbol{z}_i \boldsymbol{z}_i^{+}) = N_{m_o} \boldsymbol{I}_{n_R}$$

Hence, the entropy of $\boldsymbol{y_i}$ is given by

$$\begin{aligned}
h(y_i) &= \log_2 \det\left(\pi e R_{y_i}\right) \\
&= \log_2 det[\pi e(\boldsymbol{h}_i P \boldsymbol{h}_i^{+} + N_{m_o} \boldsymbol{I}_{n_R})] \qquad (4)
\end{aligned}$$

Similarly, the entropy of $\boldsymbol{z}_i$is given by

$$h(\boldsymbol{z}_i) = \log_2 det\left(\pi e R_{\boldsymbol{z}_i}\right) = \log_2 det\left(\pi e N_{m_o} \boldsymbol{I}_{n_R}\right) \qquad (5)$$

Hence the mutual information at the i^{th} user is given by

$$I(x; y_i) = \log_2 \frac{\det(\pi e R_{y_i})}{\det(\pi e R_{z_i})} = \log_2 \frac{\det[\pi e(\boldsymbol{h}_i P \boldsymbol{h}_i^{+} + N_{m_o} \boldsymbol{I}_{nR})]}{\det(N_{m_o} \boldsymbol{I}_{nR})}$$

$$= \log_2 \frac{\det[\pi e(\boldsymbol{h}_i P \boldsymbol{h}_i^{+} + N_{m_o} \boldsymbol{I}_{nR})]}{\det(N_{m_o} \boldsymbol{I}_{nR})} = \log_2 \frac{\det[\boldsymbol{n}_{m_o}(\frac{P}{N_{m_o}} ||\boldsymbol{h}_i||^2 + \boldsymbol{I}_{nR})]}{\det(N_{m_o} \boldsymbol{I}_{nR})}$$

$$= \log_2(1 + \frac{P}{N_{m_o}} ||\boldsymbol{h}_i||^2) \tag{6}$$

Multicast capacity of i^{th} user is given by

$$C_m = \log_2\left(1 + \theta_1 \min_{1 \le i \le M} \gamma_{m_i}\right), \tag{7}$$

where $\theta_1 = \frac{P}{N_{m_o}}$ and $\min\limits_{1<i<M} \gamma_{m_i}$ is the minimum instantaneous SNR among all the users.

PDF OF CAPACITY

The PDF of capacity is important parameter to justify a channel quality because it helps to determine the capacity which brings the most benefit. The PDF of γ_{m_i} is given by [5]

$$f(\gamma_{m_i}) = \frac{1}{\gamma_{m_o}} e^{\frac{-\gamma_{m_i}}{\gamma_{m_o}}} \tag{8}$$

Here, γ_{m_o} is average SNR per symbol at the user. Distribution of the minimum instantaneous SNR of the users can be expressed as [5]

$$f_{d_{min}}(\gamma_{m_i}) = M f_{\gamma_{m_i}}(\gamma_{m_i}) \left[1 - F_{\gamma_{m_i}}(\gamma_{m_i})\right]^{M-1} \tag{9}$$

The cumulative distribution function (CDF) of γ_{m_i} can be derived as

$$F_{\gamma_{m_i}}(\gamma_{m_i}) = \int_0^{\gamma_{m_i}} f_{\gamma_{m_i}}(\gamma_{m_i})\, d\gamma_{m_i} \tag{10}$$

Now from (10),

$$F_{\gamma_{m_i}}(\gamma_{m_i}) = 1 - e^{\frac{-\gamma_{m_i}}{\gamma_{m_o}}} \tag{11}$$

Hence,

$$f_{d_{min}}^{MRC}(\gamma_{m_i}) = M \frac{e^{-M\frac{\gamma_{m_i}}{\gamma_{m_o}}}}{\gamma_{m_o}} \tag{12}$$

Proposition 1: Let the probability density function of x is denoted by f(x). Then the probability density function of $\mathrm{C} = \log_e(1 + \theta x)$ is given by

$$q(c) = \frac{e^c}{\theta} f(\frac{e^c - 1}{\theta})$$

Proof:

We have, $C = \log_e(1 + \theta x)$. The probability density function of C, can be written as,

$$q(c) = \int \delta(C - \log_e(1 + \theta x)) f(x) dx$$

The following mathematical facts have been used for this proof ;

i) $\delta(f(x)) = \sum_l \frac{\delta(x - x_l)}{\left|\frac{df}{dx}\right|_{x_l}}$, where x_l are the zeros of $f(x)$, i.e. $f(x_l) = 0$;

ii) $\int_a^b \delta(x - x_1)\delta(x - x_2) dx = \delta(x_1 - x_2)$ for $a < x_1, x_2 < b$; and

iii) $\int_V f(x)\delta(x - x_o) dx = \begin{cases} f(x_o), & x_o \epsilon V \\ 0, & otherwise \end{cases}$

Assuming $\kappa = 1 + \theta x$, we have $f(\kappa) = C - \log_e \kappa$ and $f'(\kappa) = -\frac{1}{\kappa}$. Now from $f(\kappa) = 0, \kappa = e^C and\ f'(k)|_{\kappa = e^C} = -\frac{1}{e^C}$. Using the above mathematical facts, we have

$$q(c) = \int e^C \delta(1+\theta x - e^C) f(x)dx$$
$$= e^c \int \delta\left(\theta\left(x - \frac{e^C - 1}{\theta}\right)\right) f(x)dx$$
$$= \frac{e^c}{\theta} \int \delta\left(x - \frac{e^{C}-1}{\theta}\right) f(x)dx,$$
Since, $\delta(dz) = \frac{1}{|d|}\delta(z)$
$$= \frac{e^c}{\theta} f\left(\frac{e^C - 1}{\theta}\right),$$
Where, $\delta(.)$ is a delta function.

Using proposition 1, the PDF of $C_m$can be determined as
$$q(C_m) = M \frac{e^{-M\frac{(e^{C_m}-1)}{\gamma_{m_o}}}}{\gamma_{m_o}} \tag{13}$$

MRC Diversity

The pdf of instantaneous received SNR γ_{m_i} at the MRC output of the i[th] user can be expressed as [1]
$$f_{\gamma_{m_i}}^{MRC}(\gamma_{m_i}) = \frac{\gamma_{m_i}^{n_R-1}}{(n_R-1)!\gamma_{m_o}^{n_R}} e^{-\frac{\gamma_{m_i}}{\gamma_{m_o}}} \tag{14}$$

Here, γ_{m_o} is average SNRs per symbol at the user. Using (10), the cumulative distribution function (CDF) of γ_{m_i} can be derived as
$$F_{\gamma_{m_i}}^{MRC}(\gamma_{m_i}) = \int_0^{\gamma_{m_i}} f_{\gamma_{m_i}}^{MRC}(\gamma_{m_i})\, d\gamma_{m_i}$$

Using the identity, [6, eq. 3.351.1 & eq. 3.351.2]
$$F_{\gamma_{m_i}}^{MRC}(\gamma_{m_i}) = \frac{1}{(n_R-1)!\gamma_{m_o}^{n_R}} \left[\frac{(n_R-1)!}{\left(\frac{1}{\gamma_{m_o}}\right)^{n_R}} - \left(\frac{1}{\gamma_{m_i}}\right)^{-n_R} \times \frac{1}{(n_R-1)!}\Gamma(n_R, \frac{\gamma_{m_i}}{\gamma_{m_o}})\right]$$
$$= 1 - \frac{\Gamma\left(n_R, \frac{\gamma_{m_i}}{\gamma_{m_o}}\right)}{(n_R-1)!} \tag{15}$$

Now putting the value of (14) and (15) into (9) the pdf of the minimum instantaneous SNR is given by
$$f_{d_{min}}^{MRC}(\gamma_{m_i}) = M \frac{\gamma_{m_i}^{n_R-1}}{(n_R-1)!\gamma_{m_o}^{n_R}} e^{-\frac{\gamma_{m_i}}{\gamma_{m_o}}} \left[1 - (1 - \frac{\Gamma\left(n_R, \frac{\gamma_{m_i}}{\gamma_{m_o}}\right)}{(n_R-1)!})\right]^{M-1} \tag{16}$$

Using the proposition 1, the PDF of the capacity can be derived as
$$q^{MRC}(C_m) = \frac{e^{C_m}}{\theta_1} f_{d_{min}}^{MRC}\left(\frac{e^{C_m} - 1}{\theta_1}\right)$$
$$= \frac{e^{C_m}}{\theta_1} M \frac{\left(\frac{e^{C_m}-1}{\theta_1}\right)^{n_R-1}}{(n_R-1)!^M \gamma_{m_o}^{n_R}} e^{-\frac{e^{C_m}-1}{\gamma_{m_o}\theta_1}} \left[\Gamma\left(n_R, \frac{e^{C_m}-1}{\gamma_{m_o}\theta_1}\right)\right]^{M-1}$$
$$= e^{C_m} M \frac{(e^{C_m}-1)^{n_R-1}}{(n_R-1)!^M \gamma_{m_o}^{n_R}} e^{-\frac{e^{C_m}-1}{\gamma_{m_o}}} \left[\Gamma\left(n_R, \frac{e^{C_m}-1}{\gamma_{m_o}}\right)\right]^{M-1} \tag{17}$$
considering $\theta = 1$ for simplicity.

SC Diversity

The pdf of instantaneous received SNR γ_{m_i} at the SC output of users can be expressed as [1]
$$f_{\gamma_{m_i}}^{SC}(\gamma_{m_i}) = \frac{n_R}{\gamma_{m_o}} \left[1 - e^{-\frac{\gamma_{m_i}}{\gamma_{m_o}}}\right]^{n_R-1} e^{-\frac{\gamma_{m_i}}{\gamma_{m_o}}} \tag{18}$$

The cumulative distribution functions (CDF) of γ_{m_i} can be derived as

$$F^{SC}_{\gamma_{m_i}}(\gamma_{m_i}) = \int_0^{\gamma_{m_i}} f^{SC}_{\gamma_{m_i}}(\gamma_{m_i})\, d\gamma_{m_i}$$

$$= \int_0^{\gamma_{m_i}} \frac{n_R}{\gamma_{m_o}}\left[1-e^{-\frac{\gamma_{m_i}}{\gamma_{m_o}}}\right]^{n_R-1} e^{-\frac{\gamma_{m_i}}{\gamma_{m_o}}}\, d\gamma_{m_i}$$

Let, $1-e^{-\frac{\gamma_{m_i}}{\gamma_{m_o}}} = z$

$$\Rightarrow \frac{1}{\gamma_{m_o}} e^{-\frac{\gamma_{m_i}}{\gamma_{m_o}}} d\gamma_{m_i} = dz$$

For, $\gamma_{m_i} = 0, z = 0$ and for $\gamma_{m_i} = \gamma_{m_i}, z = 1-e^{-\frac{\gamma_{m_i}}{\gamma_{m_o}}}$.

$$\therefore\ F^{SC}_{\gamma_{m_i}}(\gamma_{m_i}) = \int_0^{1-e^{-\frac{\gamma_{m_i}}{\gamma_{m_o}}}} n_R z^{n_R-1}\, dz$$

$$= \left[\frac{n_R z^{n_R}}{n_R}\right]_0^{1-e^{-\frac{\gamma_{m_i}}{\gamma_{m_o}}}}$$

$$= \left(1-e^{-\frac{\gamma_{m_i}}{\gamma_{m_o}}}\right)^{n_R} \tag{19}$$

Using (18) and (19) into (9) we get,

$$f^{SC}_{d_{min}}(\gamma_{m_i}) = M\frac{n_R}{\gamma_{m_o}}\left[1-e^{-\frac{\gamma_{m_i}}{\gamma_{m_o}}}\right]^{n_R-1} e^{-\frac{\gamma_{m_i}}{\gamma_{m_o}}}\left[1-\left(1-e^{-\frac{\gamma_{m_i}}{\gamma_{m_o}}}\right)^{n_R}\right]^{M-1} \tag{20}$$

Using proposition 1, the pdf of C_m can be determined as

$$q^{SC}(C_m) = \frac{e^{C_m}}{\theta_1} f^{SC}_{d_{min}}\left(\frac{e^{C_m}-1}{\theta_1}\right)$$

$$= e^{C_m} M\frac{n_R}{\gamma_{m_o}}\left[1-e^{-\frac{e^{C_m-1}}{\gamma_{m_o}}}\right]^{n_R-1} e^{-\frac{e^{C_m-1}}{\gamma_{m_o}}}\left[1-\left(1-e^{-\frac{e^{C_m-1}}{\gamma_{m_o}}}\right)^{n_R}\right]^{M-1} \tag{21}$$

CDF OF CAPACITY

The CDF of capacity is the other name of the outage probability. It determines the probability of the channel capacity under a certain rate, which results in a poor communication system. Therefore, the CDF of multicast capacity or the outage probability is given by

$$CDF = \int_0^R q(C_m)dC_m\ , \tag{22}$$

where R is the target rate of capacity. The CDF of capacity without applying diversity is

$$CDF_C = \int_0^R M\frac{e^{\frac{-(e^{C_m-1})}{\gamma_0}}}{\gamma_0}\left(1-e^{\frac{-(e^{C_m-1})}{\gamma_0}}\right)^{M-1} dC_m$$

$$CDF_C = \sum_{t=0}^{(M-1)}\frac{M}{\gamma_0}\binom{M-1}{t}(-1)^t\int_0^R e^{\frac{-(t+1)(e^{C_m-1})}{\gamma_0}} dC_m$$

Let, $e^{C_m}-1 = z, e^{C_m}dC_m = dz$,

when $C_m = 0, z = 0$ and when $C_m = R, z = e^R - 1$

$$CDF_C = \sum_{t=0}^{(M-1)}\frac{M}{\gamma_0}\binom{M-1}{t}(-1)^t\int_0^{e^R-1} e^{\frac{-(t+1)}{\gamma_0}z} dz$$

$$CDF_C = \sum_{t=0}^{(M-1)}\frac{M}{t+1}\binom{M-1}{t}(-1)^t\left(1-e^{\frac{-(t+1)(e^R-1)}{\gamma_0}}\right) \tag{23}$$

MRC Diversity

Using (23), the CDF of the capacity for MRC diversity is given by

$CDF_C{}^{MRC} = \int_0^R e^{C_m} M \frac{(e^{C_m-1})^{n_R-1}}{(n_R-1)!^M \gamma_{m_o}^{n_R}} e^{-\frac{e^{C_m-1}}{\gamma_{m_o}}} \left[\Gamma\left(n_R, \frac{e^{C_m-1}}{\gamma_{m_o}}\right)\right]^{M-1} dC_m$

Using the identity, $\Gamma(n,x) = (n-1)!\, e^{-x} \sum_{m=0}^{n-1} \frac{x^m}{m!}$ [6, eq. (8.352.7)]

$CDF_C{}^{MRC} = \int_0^R e^{C_m} M \frac{(e^{C_m-1})^{n_R-1}}{(n_R-1)!^M \gamma_{m_o}^{n_R}} e^{-\frac{e^{C_m-1}}{\gamma_{m_o}}} \quad (n_R-1)!^{M-1} \times$

$e^{-\frac{M-1}{\gamma_{m_o}}(e^{C_m-1})} \sum_{v=0}^{(n_R-1)(M-1)} \beta(n_R, M1) \left(\frac{e^{C_m-1}}{\gamma_{m_o}}\right)^v dC_m$

Where $\beta(n_R, M-1)$ is the coefficient of $(e^{C_m-1})^v$ in the expansion of $\left[\sum_{v=0}^{n_R-1} \frac{1}{v!}\left(\frac{e^{C_m-1}}{\gamma_{m_o}}\right)^v\right]^{M-1}$.

let, $e^{C_m-1} = z, e^{C_m} dC_m = dz$

when $C_m = 0, z = 0$ and when $C_m = R, z = e^R - 1$

$CDF_C{}^{MRC} = \sum_{v=0}^{(n_R-1)(M-1)} M \frac{\beta(n_R, M-1)}{(n_R-1)! \gamma_{m_o}^{n_R+v}} \int_0^{e^R-1} z^{n_R+v-1} e^{-\frac{M}{\gamma_{m_o}} z} dz$

$$CDF_C{}^{MRC} = \sum_{v=0}^{(n_R-1)(M-1)} M \frac{\beta(n_R, M-1)}{(n_R-1)! \gamma_{m_o}^{n_R+v}} \frac{(n_R+v-1)!}{\left(\frac{M}{\gamma_{m_o}}\right)^{n_R+v}} e^{-\frac{M}{\gamma_{m_o}}(e^R-1)} \sum_{k=0}^{n_R+v-1} \frac{(n_R+v-1)!}{k!} \times \frac{(e^R-1)^k}{\left(\frac{M}{\gamma_{m_o}}\right)^{n_R+v-k}} \quad (24)$$

SC Diversity

Using (22), the CDF of the capacity for SC diversity is given by

$CDF_C{}^{SC} = \int_0^R e^{C_m} M \frac{n_R}{\gamma_{m_o}} \left[1 - e^{-\frac{e^{C_m-1}}{\gamma_{m_o}}}\right]^{n_R-1} e^{-\frac{e^{C_m-1}}{\gamma_{m_o}}} \left[1 - \left(1 - e^{-\frac{e^{C_m-1}}{\gamma_{m_o}}}\right)^{n_R}\right]^{M-1} dC_m$

Using the identity, $(a+x)^n = \sum_{k=0}^{n} \binom{n}{k} x^k a^{n-k}$ of [6, eq. (1.111)] we have

$$CDF_C{}^{SC} = \sum_{l=0}^{n_{R-1}} \sum_{w=0}^{M-1} \sum_{u=0}^{w.n_R} \frac{Mn_R}{\gamma_{m_o}} (-1)^{l+u+w} \binom{n_R-1}{l} \binom{M-1}{w} \binom{n_R w}{u} \int_0^R e^{C_m} e^{-(k+l+1)\frac{e^{C_m-1}}{\gamma_{m_o}}} dC_m$$

Let, $e^{C_m-1} = z, e^{C_m} dC_m = dz$

When $C_m = 0, z = 0$

And when $C_m = R, z = e^R - 1$

$$CDF_C{}^{SC} = \sum_{l=0}^{n_{R-1}} \sum_{w=0}^{M-1} \sum_{u=0}^{w.n_R} \frac{Mn_R}{\gamma_{m_o}} (-1)^{l+u+w} \binom{n_R-1}{l} \binom{M-1}{w} \binom{n_R w}{u} \int_0^{e^R-1} e^{-\frac{(k+l+1)}{\gamma_{m_o}} z} dz$$

$$= \sum_{l=0}^{n_{R-1}} \sum_{w=0}^{M-1} \sum_{u=0}^{w.n_R} \frac{Mn_R}{\gamma_{m_o}} (-1)^{l+u+w} \binom{n_R-1}{l} \binom{M-1}{w} \binom{n_R w}{u} \frac{\gamma_{m_0}}{k+l+1} \left(1 - e^{\frac{k+l+1}{\gamma_{m_0}}(1-e^R)}\right) \quad (25)$$

CCDF OF CAPACITY

The CCDF of capacity determines the probability of the capacity being greater than the target rate given by

$$CCDF = \int_R^\infty q(C_m) dC_m \quad (26)$$

The CCDF of capacity without applying diversity is

$CCDF_C = \int_R^\infty M \frac{e^{\frac{-(e^{C_m-1})}{\gamma_0}}}{\gamma_0} \left(1 - e^{\frac{-(e^{C_m-1})}{\gamma_0}}\right)^{M-1} dC_m$

$CCDF_C = \sum_{t=0}^{(M-1)} \frac{M}{\gamma_0} \binom{M-1}{t} (-1)^t \int_R^\infty e^{\frac{-(t+1)(e^{C_m-1})}{\gamma_0}} dC_m$

Let, $e^{C_m} - 1 = z, e^{C_m} dC_m = dz,$

when $C_m = R, z = e^R - 1$ and when $C_m = \infty, z = \infty$.

$CCDF_C = \sum_{t=0}^{(K-1)} \frac{K}{\gamma_0} \binom{K-1}{t} (-1)^t \int_{e^R-1}^\infty e^{\frac{-(t+1)}{\gamma_0} z} dz$

$$CCDF_C = \sum_{t=0}^{(K-1)} \frac{K}{t+1}\binom{K-1}{t}(-1)^t\left(e^{\frac{-(t+1)(e^R-1)}{\gamma_0}}\right) \quad (27)$$

MRC Diversity

Using (26), the CCDF of the capacity for MRC diversity is given by

$$CCDF_C{}^{MRC} = \int_R^{\infty} e^{C_m} M \frac{(e^{C_m-1})^{n_R-1}}{(n_R-1)!^M \gamma_{m_o}^{n_R}} e^{-\frac{e^{C_m-1}}{\gamma_{m_o}}}\left[\Gamma\left(n_R, \frac{e^{C_m-1}}{\gamma_{m_o}}\right)\right] dC_m$$

Using the identity,

$\Gamma(n,x) = (n-1)!\, e^{-x} \sum_{m=0}^{n-1} \frac{x^m}{m!}$ [6, eq. (8.352.7)]

$$CCDF_C{}^{MRC} = \int_R^{\infty} e^{C_m} M \frac{(e^{C_m-1})^{n_R-1}}{(n_R-1)!^M \gamma_{m_o}^{n_R}} e^{-\frac{e^{C_m-1}}{\gamma_{m_o}}} (n_R-1)!^{M-1}$$

$$\times e^{-\frac{M-1}{\gamma_{m_o}}(e^{C_m-1})} \sum_{v=0}^{(n_R-1)(M-1)} \beta(n_R, M-1)\left(\frac{e^{C_m-1}}{\gamma_{m_o}}\right)^v dC_m$$

Where $\beta(n_R, M-1)$ is the coefficient of $(e^{C_m-1})^v$ in the expansion of $\left[\sum_{v=0}^{n_R-1}\frac{1}{v!}\left(\frac{e^{C_m-1}}{\gamma_{m_o}}\right)^v\right]^{M-1}$.

Let, $e^{C_m-1} = z, e^{C_m} dC_m = dz$

when $C_m = R, z = e^R - 1$ and when $C_m = \infty, z = \infty$

$$CCDF_C{}^{MRC} = \sum_{v=0}^{(n_R-1)(M-1)} M \frac{\beta(n_R, M-1)}{(n_R-1)!\gamma_{m_o}^{n_R+v}} \int_{e^R-1}^{\infty} z^{n_R+v-1} e^{-\frac{M}{\gamma_{m_o}}z} dz$$

$$CCDF_C{}^{MRC} = \sum_{v=0}^{(n_R-1)(M-1)} M \frac{\beta(n_R, M-1)}{(n_R-1)!\gamma_{m_o}^{n_R+v}}\left[\frac{(n_R+v-1)!}{\left(\frac{M}{\gamma_{m_o}}\right)^{n_R+v}} - e^{-\frac{M}{\gamma_{m_o}}(e^R-1)} \sum_{k=0}^{n_R+v-1} \frac{(n_R+v-1)!}{k!} \frac{(e^R-1)^k}{\left(\frac{M}{\gamma_{m_o}}\right)^{n_R+v-k}}\right] \quad (28)$$

SC Diversity

Using (26), the CCDF of the capacity for SC diversity is given by

$$CCDF_C{}^{SC} = \int_R^{\infty} e^{C_m} M \frac{n_R}{\gamma_{m_o}}\left[1 - e^{-\frac{e^{C_m-1}}{\gamma_{m_o}}}\right]^{n_R-1} e^{-\frac{e^{C_m-1}}{\gamma_{m_o}}} \times \left[1 - \left(1 - e^{-\frac{e^{C_m-1}}{\gamma_{m_o}}}\right)^{n_R}\right]^{M-1} dC_m$$

Using the identity, $(a+x)^n = \sum_{k=0}^{n}\binom{n}{k}x^k a^{n-k}$ of [6, eq. (1.111)], we have

$$CCDF_C{}^{SC} = \sum_{l=0}^{n_{R-1}}\sum_{w=0}^{M-1}\sum_{u=0}^{w.n_R} \frac{Mn_R}{\gamma_{m_o}}(-1)^{l+u+w}\binom{n_R-1}{l}\binom{M-1}{w}\binom{n_R w}{u}\int_R^{\infty} e^{C_m} e^{-(k+l+1)\frac{e^{C_m-1}}{\gamma_{m_o}}} dC_m$$

Let, $e^{C_m-1} = z, e^{C_m} dC_m = dz$

when $C_m = R, z = e^R - 1$ and when $C_m = \infty, z = \infty$

$$CCDF_C{}^{SC} = \sum_{l=0}^{n_{R-1}}\sum_{w=0}^{M-1}\sum_{u=0}^{w.n_R} \frac{Mn_R}{\gamma_{m_o}}(-1)^{l+u+w}\binom{n_R-1}{l}\binom{M-1}{w}\binom{n_R w}{u}\int_{e^R-1}^{\infty} e^{-\frac{(k+l+1)}{\gamma_{m_o}}z} dz$$

$$= \sum_{l=0}^{n_{R-1}}\sum_{w=0}^{M-1}\sum_{u=0}^{w.n_R} \frac{Mn_R}{\gamma_{m_o}}(-1)^{l+u+w}\binom{n_R-1}{l}\binom{M-1}{w}\binom{n_R w}{u}\frac{\gamma_{m_0}}{k+l+1} \quad (29)$$

NUMERICAL RESULTS

Fig. 6 depicts the simulation of PDF as a function of average SNR. It shows that the PDF of capacity increases after using diversity combining techniques. The PDF in case of MRC diversity is greater than the case of SC diversity. Fig. 7 shows the simulation of CDF as a function of average SNR. It shows that the CDF of capacity decreases after using diversity combining techniques. The CDF in case of MRC diversity is lower than the case of SC diversity. Fig. 8 depicts the simulation of CCDF as a function of average SNR. It shows that the CCDF of capacity increases after using diversity combining techniques. The CCDF in case of MRC diversity is greater than the case of SC diversity.

CONCLUSION

In this paper, we study the multicast capacity of a Rayleigh fading multicast SIMO network. Here we focus on the analysis of multicast capacity employing SC and MRC diversity combining techniques. Here we compare the proposed system with and without diversity combining techniques in terms of PDF, CDF and CCDF. From the comparison we come to a conclusion that, diversity combining techniques improve the communication quality. Again MRC diversity shows better performance than SC diversity combining technique. So, MRC diversity combining technique is beneficial for multicast communication system.

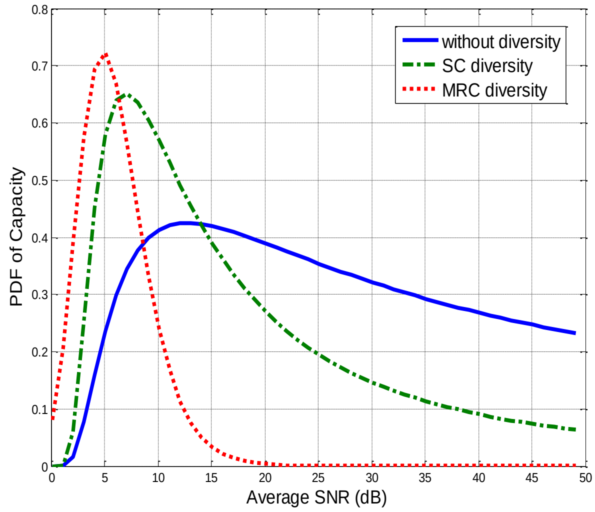

Fig-6: PDF of multicast capacity with M=2, $n_R = 2$

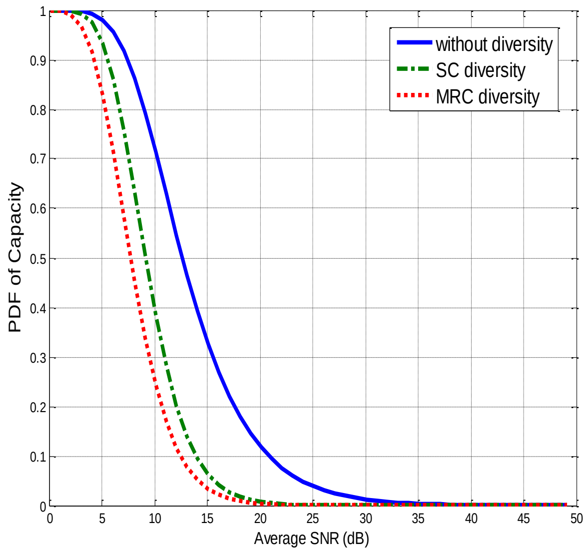

Fig-7: CDF of multicast capacity with M=2, $n_R = 2$

FUTURE WORK

The research work described in this paper has some future scopes, they are-

1. This work can be extended to analyze the SC and MRC receiver performance in other fading channels i.e., Hoyt Fading Channel, Rician Fading Channel, Nakagami-m Fading Channel, Log-normal Fading Channel, Weibull Fading Channel and Beckmann Fading Channel.
2. Some other diversity combining techniques can be applied.
3. This work can be extended in MISO and MIMO channels.

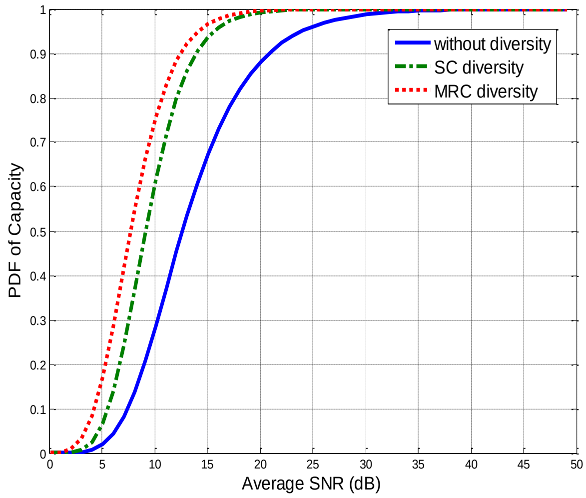

Fig-8: CCDF of multicast capacity with M=2, $n_R = 2$

REFERENCES

1. Goldsmith A. Wireless communications, 7th Edition, New York. Cambridge University Press. 2005.
2. Das D. Diversity Combining Techniques under Employment of Generalized Receiver in Wireless Communication Systems with Rayleigh Fading Channels. in Proceedings of the Global Engineering, Science and Technology Conference Dhaka, Bangladesh. December 2012.
3. Blanco M, Zdunek K. Performance and optimization of switched diversity systems for the detection of signals with rayleigh fading. in IEEE Transactions on Communications. 1993;1887–1895.
4. Haykin S, Moher M. Communication systems. 5th Edition. John Wiley & Sons. Toronica City. 2010.
5. Giti JE, Chowdhury SAH, Ali MM. Enhancing security in wireless multicasting with selective precoding. International Journal of Systems. Control and Communications (IJSCC). 2015;6(4).
6. Gradshteyn IS, Ryzhik IM. Table of Integrals. Series. and Products. 7th ed. San Diego, CA: Academic. 2007.