

Research Article**An Optimization Algorithm for the Layout of Well Drilling**

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Abstract: The author designs a genetic algorithm for the layout of well drilling based on partial order relations and establishes a mathematical model for the algorithm. Based on this, ideal results are obtained. The results of numerical experiments show that the algorithm is highly precise and runs at a rapid speed.

Keywords: genetic algorithm, partial order relation, layout, model.

INTRODUCTION

While prospecting mines, exploration departments need to distribute well locations using grid points with identical vertical and horizontal distances in a region. If the location of a newly designed well coincides with or close to that of an old well, the geological data of the old well can be used instead of drilling the new well, so as to save drilling costs[1-5].

This problem can be converted as: when there are n points of P_i on a plane and the coordinate is expressed as $(x_i, y_i), i = 1, 2, \dots, n$, it indicates that there are n well locations. Moreover, the layout of the new wells is all the nodes in a square grid N . Assume that the side length (vertical and horizontal distance of wells) of each grid is one unit and the whole grid can move arbitrarily on the plane. If the distance between a known point P_i and a grid node Q_i is less than the given error ε (0.05 unit), the geological data of old wells at P_i can be used and there is no need to drill new wells at the node Q_i .

In order to support the decision, exploration departments need to solve the following problems:

- (1) Suppose that the lateral and vertical directions of the grid are fixed (i.e., in EW and SN directions) and distance errors are calculated along lateral and vertical directions. It requires that the differences (absolute values) in both the horizontal and the vertical ordinates between the old well P_i and corresponding node Q_i are no more than ε . The grid N is moved parallel to make the number of available old wells as large as possible (Table 1).
- (2) Based on the problem (1), the situation that the lateral and vertical directions of the grid are not fixed, that is, they can be rotated, is considered.

Table 1: Coordinates of old wells

i	1	2	3	4	5	6	7	8	9	10	11	12
a_i	0.50	1.41	3.00	3.37	3.40	4.72	4.72	5.43	7.57	8.38	8.89	9.50
b_i	2.00	3.50	1.50	3.51	5.50	2.00	6.24	4.10	2.01	4.50	3.41	0.80

THE RELEVANT THEORIES OF PARTIAL ORDER RELATION

Definition 1: Assume that S and T are sets, let $S \times T = \{(s, t) \mid s \in S, t \in T\}$, then $S \times T$ is Cartesian product of S and T .

Definition 2: If $R \subseteq S \times S$, R is a relation in S .

Suppose that $(s_1, s_2) \in S \times S$ and R is a relation in S . If $(s_1, s_2) \in R$, a relation R exists between s_1 and s_2 and denoted as $s_1 R s_2$. Otherwise, there is no relation R between s_1 and s_2 .

Definition 3: When $R \subseteq S \times S$ is a partial order relation, it means that $\forall a, b, c \in S$. The following 3 conditions are tenable.

- 1) aRa ;
- 2) If aRb and bRa , then $a = b$;
- 3) When aRb and bRc , then aRc .

Definition 4: A set S with a partial order relation is called a partial order set and expressed as (S, R) .

Definition 5: Suppose that $A \subseteq (S, R)$, if $\forall a, b \in A$, there is definitely aRb or bRa , which indicates that A is a chain of S .

Definition 6: Assume that $A \subseteq (S, R)$. If $a \in A$ is the minimal element of A , it means that there is no $x \in A - \{a\}$ making the xRa tenable. While, if $a \in A$ is the least element of A , it indicates that when $\forall x \in A$, there is aRx .

Definition 7: Suppose that $A \subseteq (S, R)$. $b \in S$ is known as the lower bound of A and means that when $\forall x \in A$, there is bRx .

Theorem 1: When $A \subseteq (S, R)$, there are minimal elements.

Theorem 2: If each chain of (S, R) has a lower bound, S has minimal elements.

Theorem 3: There are definitely minimal elements in any finite nonempty set of (S, R) .

EVOLUTIONARY COMPUTATION METHODS FOR SOLVING GENERAL LAYOUT PROBLEMS

The core of evolutionary computation is the genetic algorithm[4-6], which is a global optimization algorithm simulating the natural selection and genetic mechanism in the process of biological evolution. Combined with relevant theories of partial order relations, the genetic algorithm is more effective in the theory and application.

Layout problems of well drilling are NP complete problems. Although there are many algorithms for solving these problems, most of them have two shortcomings. On the one hand, the algorithms are essentially exhaustion methods, so the efficiency is low. On the other hand, as the step length of explosion is arbitrarily selected, the algorithms present low subjectivity and accuracy. Using genetic algorithm can avoid the above problems, and the algorithm is robust.

As for the problems 1 (1) and 1 (2), they can be simplified as that for calculating $\max_{x \in D} f(X)$.

Where, $D = \{X \in S \subseteq R^n; g_i \leq 0; i = 1, 2, \dots, m\}$. n and m represent the dimension of feasible solutions and the number of constraint inequalities, respectively.

$$\text{Defined as: } h_i(X) = \begin{cases} 0, & g_i(X) \leq 0 \\ g_i(X), & g_i(X) > 0 \end{cases} \quad H(X) = \sum_{i=1}^m h_i(X)$$

$$\text{better}(X_1, X_2) = \begin{cases} \text{true}, & H(X_1) < H(X_2) \\ \text{false}, & H(X_1) > H(X_2) \\ \text{true}, & H(X_1) = H(X_2) \wedge f(X_1) > f(X_2) \\ \text{false}, & H(X_1) = H(X_2) \wedge f(X_1) \leq f(X_2) \end{cases}$$

If the relation \leq in set D is defined as follows, \leq is the partial order relation in set D and expressed as (D, \leq) .

$$\leq = \{(x_1, x_2) \mid \text{better}(x_1, x_2) = \text{true}, (x_1, x_2 \in D)\} \cup \{(x, x) \mid x \in D\}$$

The competing operator e is defined as:

$$e = e(X_1, X_2) = \begin{cases} X_1, & \text{better}(X_1, X_2) = \text{true} \\ X_2, & \text{better}(X_1, X_2) = \text{false} \\ \text{rand}(X_1, X_2), & \end{cases}$$

The function of competing operator is that when individuals $x_1 \leq x_2$ or $x_2 \leq x_1$, the descendants are x_1 or x_2 through evolution, respectively. If the individuals x_1, x_2 have no partial order relation \leq , the descendant is generated randomly from x_1, x_2 .

ANALYSIS AND ESTABLISHMENT OF THE MODEL

Assume that a well in the new grid is located at the origin of the new rectangular coordinate with its grid lines being vertical (coincidence) or horizontal to the horizontal or vertical coordinate. Under such condition, the movement and rotation of the grid is converted to the translation or rotation of the coordinate system. Then, each feasible solution (a

layout) in the Problem 1 (2) can be decomposed as translating the coordinate system by h units horizontally and k units vertically first, followed by a rotation for an angle of θ . By doing so, Problem 1 (1) can be considered as a special case of Problem 1 (2) when $\theta = 0$. Therefore, the layout problems of well drilling are converted to that for calculating the triple $X = (h, k, \theta)$ to make more old wells be used.

Suppose that point $P_i^*(x_i^*, y_i^*)$ is a coordinate of point $P_i(x_i, y_i)$ in the new coordinate system determined by $X = (h, k, \theta)$, thus:

$$\begin{cases} x_i^* = (x_i - h) \cos \theta + (y_i - k) \sin \theta \\ x_i^* = (h - x_i) \sin \theta + (y_i - k) \cos \theta \end{cases}$$

Assume that $f(X) = f(h, k, \theta) = \sum_{i=1}^n f_i(h, k, \theta)$, where

$$f_i = \begin{cases} 1 & \text{if } D_x \leq \varepsilon \ \&\& \ D_y \leq \varepsilon \\ 0 & \text{otherwise} \end{cases}$$

$$D_x = \min\{x_i^* - [x_i^*], [x_i^*] + 1 - x_i^*\}, \quad D_y = \min\{y_i^* - [y_i^*], [y_i^*] + 1 - y_i^*\}$$

$$\text{Max } f(X) = f(h, k, \theta) = \sum_{i=1}^n f_i, (h, k, \theta) \in S \subset R^3,$$

$$S = \left\{ (h, k, \theta) \mid 0 \leq h, k \leq 10 \ \&\& \ -\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2} \right\}$$

The coordinate distance in Problem 1(1) is revised as Euclidean distance (The distance between the available old well and the cross point in the grid), the model is:

$$f_i = \begin{cases} 1 & \text{if } D \leq \varepsilon \\ 0 & \text{otherwise} \end{cases}, \text{ where, } D = \min\{\delta_1, \delta_2, \delta_3, \delta_4\}$$

$$\delta_1 = \sqrt{(x_i^* - [x_i^*])^2 + (y_i^* - [y_i^*])^2}, \quad \delta_2 = \sqrt{([x_i^*] + 1 - x_i^*)^2 + (y_i^* - [y_i^*])^2}$$

$$\delta_3 = \sqrt{(x_i^* - [x_i^*])^2 + ([y_i^*] + 1 - y_i^*)^2}, \quad \delta_4 = \sqrt{([x_i^*] + 1 - x_i^*)^2 + ([y_i^*] + 1 - y_i^*)^2}$$

Hereinto, the individual space S equals to R^3 , and the value of function $f(X) = f(h, k, \theta)$ is the number of available old wells.

ALGORITHM DESIGN

Coding and fitness function

Individuals are coded using real numbers. For example, the code of individual X_i is (h_i, k_i, θ_i) , then a population with a size being N is represented as:

$$P = \{X_1, X_2, \dots, X_N\} = \{(h_1, k_1, \theta_1), (h_2, k_2, \theta_2), \dots, (h_N, k_N, \theta_N)\} \quad X_i = (h_i, k_i, \theta_i) \in S$$

The fitness function is $f(X) = f(h, k, \theta) = \sum_{i=1}^n f_i(h, k, \theta)$. Where, $X = (h, k, \theta) \in S$, and the larger the

$f(X)$, the better the individual is.

Crossover and mutation operators

Suppose that X_1, X_2 are parents. Through integrated arithmetic crossover, descendants X_1^*, X_2^* are generated. Hence,

$$\begin{cases} X_1^* = \lambda X_1 + (1 - \lambda) X_2 \\ X_2^* = \lambda X_2 + (1 - \lambda) X_1 \end{cases} \quad (-0.5 \leq \lambda \leq 1.5)$$

The mutation is conducted by replacing X with λX at a probability of p_m , in which $-0.5 \leq \lambda \leq 1.5$.

Algorithm

The termination condition of the following algorithm is an evolutionary algebra.

The initial population $P(0) = \{X_1, X_2, \dots, X_N\}, t := 0$ is generated randomly to calculate $f(X_i), i = 1, 2, \dots, N$.

The optimal individual X_{best} and the worst individual X_{worst} are found from $P(t)$ and expressed as $f_{best} := f(X_{best}), f_{worst} := f(X_{worst})$

While (Termination criteria are not met) do
begin

The random number $m (1 \leq m \leq N)$ is generated.

The random number λ is generated, and $0 \leq \lambda \leq 1$ (or $-0.5 \leq \lambda \leq 1.5$).

Corsrossover: $X^* = \lambda X_{best} + (1 - \lambda) X_m$

Calculating $f(X^*)$

if $e(X^*, X_{worst})$ then $X_{worst} = X^*$

Mutation: The random number $m (1 \leq m \leq N)$ is generated.

If random $[0, 1] < p_m$, then $X_m := \lambda X_m$

$f(X_m)$ is calculated

The optimal individual X_{best} and the worst individual X_{worst} are found from $P(t)$ and expressed as

$f_{best} := f(X_{best}), f_{worst} := f(X_{worst})$

$t := t + 1$

end

The optimal solution is output.

EXPERIMENT

The optimal solutions (Table 2 and Table 3) can be obtained through the repeated operation of the algorithm on the computer with the CPU of P4 2.28 G. The average running time of translation and rotation are 0.5 ms and 0.1 ms, respectively. By using the algorithm, the maximum number of available old wells obtained is 4 (P_2, P_4, P_5 and P_{10}) while merely translating the coordinate axes. While translating and rotating the grid simultaneously, the maximum number is 6, namely, P_1, P_6, P_7, P_8, P_9 and P_{11} . Although the measurement of translation and rotation of the coordinates is not unique, the number of available old wells is identified. In addition, the results obtained by the algorithm are significantly higher than those in previous research [1, 2]. Similar conclusions are obtained by changing the ordinate distance into Euclidean distance (Table 3).

Table-2: Operation results of the algorithm

Experiment 1	h	2.3923	k	3.5402	θ	0	The number of available old wells in the new coordinate system				4
x^*	-	0.6077	0.9777	1.0077	2.3277	2.3277	3.0377	5.1777	5.9877	6.4977	7.1077
	1.8923	0.9823									
y^*	-	-	-	1.9598	-	2.6998	0.5598	-	0.9598	-	-
	1.5402	0.0402	2.0402	0.0302	1.5402			1.5302		0.1302	2.7402
Experiment 2	h	2.5817	k	4.1066	θ	0.7883	The number of available old wells in the new coordinate system				6
x^*	-	-	0.1327	1.5651	0.0137	3.0205	2.0035	2.0301	4.3670	3.9536	2.5328
	2.9616	1.2563	1.5536								
y^*	-	0.4032	-	0.4021	-	-	-	-	-	-	-
	0.0090		2.1344	0.9797	3.0016	0.0123	2.0246	5.0157	3.8346	4.9647	7.2375

Table-3: The operation results of the algorithm while applying Euclidean distance

Experiment 1	h	3.4007	k	2.5291	θ	0	The number of available old wells in the new coordinate system				4	
x^*	- 2.9007	- 1.9907	- 0.4007	- 0.0307	- 0.0007	1.3193	1.3193	2.0293	4.1693	4.9793	5.4893	6.0993
y^*	- 0.5291	0.9709	- 1.0291	0.9809	2.9709	- 0.5291	3.7109	1.5709	- 0.5191	1.9709	0.8809	- 1.7291
Experiment 2	h	0.4525	k	2.011	θ	- 0.7867	The number of available old wells in the new coordinate system				6	
x^*	0.0413	- 0.3782	2.1607	0.9988	- 0.3891	3.0213	0.0191	2.0357	5.0267	3.8356	4.9675	7.2463
y^*	0.0259	1.7295	1.4430	3.1243	4.5508	3.0139	6.0080	4.9996	5.0389	7.3708	6.9622	5.5511

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