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Classifying All Single Travelling Wave Atom Solutions to the Modified Konopelchenko-Dubrovsky Equations

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Abstract: By the polynomial complete discrimination system, we classify the classification of all single travelling wave atom solutions to the Modified Konopelchenko-Dubrovsky Equations.
Keywords: Travelling wave solution; complete discrimination system; the Modified Konopelchenko-Dubrovsky Equations.

INTRODUCTION

Nonlinear partial differential equations play an important role in applied mathematics, physics and engineering. A lot of methods, such as the tanh method [1, 2], the bifurcation theory and the method of phase portraits analysis [3], qualitative theory of polynomial differential system [4, 5], Exp-Function method[6] and so on, have been proposed to solve these equations. Recently, Liu [7-9] introduced a simple and efficient method to give the classification of all single travelling wave atom solutions to some equations [10]. If a nonlinear equation can be directly reduced to the integral form as follows:

$$\pm(\xi-\xi_0)=\int\frac{\mathrm{d}u}{P_n(u)} \tag{1}$$

Where is $p_n(u)$ an n-th order polynomial, we can derive the classification of all solutions to the right integral in Eq.(1) using complete discrimination system for the n-th order polynomial.

In this paper, we consider the following the Modified Konopelchenko-Dubrovsky Equations [11]:

$$u_t-u_{xxx}-6\eta uu_x+2\lambda^2u^3u_x-3v_y+3\lambda u_xv=0, \tag{2}$$

$$v_x=u_y \tag{3}$$

We reduce the Modified Konopelchenko-Dubrovsky Equations to an integrable ODE, and furthermore use complete discrimination system for polynomial to obtain the classification of all single travelling wave atom solutions.

Classification

Taking the travelling wave transformation $u=u(\xi)$, $v=v(\xi)$, $\xi=x+ky+\omega t$, the equations are reduced to the following ordinary differential forms :

$$\omega u'-u'''-6\eta uu'+2\lambda^2u^3u'-3kv'+3\lambda u'v=0, \tag{4}$$

$$\omega v'=ku'. \tag{5}$$

By integrating Eq. (4) and Eq. (5) once, we have

$$\omega u - u'' - 3\eta u^2 + \frac{\lambda^2}{2}u^4 - 3kv + 3\lambda uv + c_1 = 0, \tag{6}$$

$$v = ku + c_2, \tag{7}$$

Where c_1 and c_2 are two arbitrary constants.

Substituting the expression of v into Eq. (6), we have

$$u'' = \frac{\lambda^2}{2}u^4 + 3(\lambda k - \eta)u^2 + (3\lambda c_2 - 3k^2 + \omega)u - 3kc_2 + c_1. \tag{8}$$

Integrating Eq. (8) once, then it is

$$(u')^2 = \frac{1}{5}\lambda^2 u^5 + (\lambda k - \eta)u^3 + \frac{3\lambda c_2 - 3k^2 + \omega}{2}u^2 + (c_1 - 3kc_2)u + c_0, \tag{9}$$

and the corresponding integral form becomes

$$\pm\frac{\lambda}{\sqrt{5}}(\xi - \xi_0) = \int \frac{\mathrm{d}u}{\sqrt{u^5 + pu^3 + qu^2 + ru + s}}, \tag{10}$$

Where, $p = \frac{5(\lambda k - \eta)}{\lambda^2}$, $q = \frac{5(3\lambda c_2 - 3k^2 + \omega)}{2\lambda^2}$, $r = \frac{5(c_1 - 3kc_2)}{\lambda^2}$, $s = c_0$,
c_0, c_1, c_2 and ξ_0 are integral constants.

Denote $F(u) = u^5 + pu^3 + qu^2 + ru + s$, the complete discrimination system for $D_2 = -p$,
$D_3 = 40rp - 12p^3 - 45q^2$,
$D_4 = -4p^3q^2 + 12p^4r + 117pq^2r - 88p^2r^2 - 40qsp^2 - 27q^4 + 160r^3 - 300qrs$
$$D_5 = -1600qsr^3 - 3750pqs^3 + 2000ps^2r^2 - 4p^3q^2r^2 + 16p^3q^3s - 900rs^2p^3 + 825p^2q^2s^2 + 144pq^2r^3 + 2250rq^2s^2 + 16p^4r^3 + 108p^5s^2 - 128r^4p^2 - 27r^2q^4 + 108sq^5 + 256r^5 + 3125s^4 - 72rsqp^4 + 560sqr^2p^2 - 630prsq^3,$$
$$E_2 = 160r^2p^3 + 900q^2r^2 - 48rp^5 + 60rp^2q^2 + 1500pqrs + 16q^2p^4 - 1100qsp^3 + 625s^2p^2 - 3375sq^3, F_2 = 3q^2 - 8rp$$

By the complete discrimination system for polynomial, the classifications of all the single traveling wave solutions to the integral formula (10) can be given as follows:

Case 1 : If $D_5 = 0$, $D_4 = 0$, $D_3 > 0$, then $F(u) = (u-\alpha)^2(u-\beta)^2(u-\gamma)$, α, β, γ are reals numbers, and $\alpha \neq \beta \neq \gamma$. When $w > \gamma$, solutions to u can be given by

$$\pm\frac{\lambda(\alpha - \beta)}{2\sqrt{5}}(\xi - \xi_0) = \sqrt{\gamma - \alpha}\arctan\frac{\sqrt{u-\gamma}}{\sqrt{\gamma - \alpha}} - \sqrt{\gamma - \beta}\arctan\frac{\sqrt{u-\gamma}}{\sqrt{\gamma-\beta}} \quad (\gamma > \alpha, \gamma > \beta) \tag{11}$$

$$\pm\frac{\lambda(\alpha-\beta)}{2\sqrt{5}}(\xi-\xi_0)=\sqrt{\gamma-\alpha}\arctan\frac{\sqrt{u-\gamma}}{\sqrt{\gamma-\alpha}}-\frac{1}{\sqrt{\beta-\gamma}}\ln\left|\frac{\sqrt{u-\gamma}-\sqrt{\beta-\gamma}}{\sqrt{u-\gamma}+\sqrt{\beta-\gamma}}\right| \quad (\gamma>\alpha,\gamma<\beta) \tag{12}$$

$$\pm\frac{\lambda(\alpha-\beta)}{2\sqrt{5}}(\xi-\xi_0)=-\sqrt{\gamma-\beta}\arctan\frac{\sqrt{u-\gamma}}{\sqrt{\gamma-\beta}}+\frac{1}{2\sqrt{\alpha-\gamma}}\ln\left|\frac{\sqrt{u-\gamma}-\sqrt{\alpha-\gamma}}{\sqrt{u-\gamma}+\sqrt{\alpha-\gamma}}\right| \quad (\gamma<\alpha,\gamma>\beta) \tag{13}$$

$$\pm\frac{\lambda(\alpha-\beta)}{2\sqrt{5}}(\xi-\xi_0)=\frac{1}{2\sqrt{\alpha-\gamma}}\ln\left|\frac{\sqrt{u-\gamma}-\sqrt{\alpha-\gamma}}{\sqrt{u-\gamma}+\sqrt{\alpha-\gamma}}\right|-\frac{1}{2\sqrt{\beta-\gamma}}\ln\left|\frac{\sqrt{u-\gamma}-\sqrt{\beta-\gamma}}{\sqrt{u-\gamma}+\sqrt{\beta-\gamma}}\right| \quad (\gamma<\alpha,\gamma<\beta) \tag{14}$$

Case 2: If $D_5=0$, $D_4=0$, $D_3=0$, $D_2\neq 0$, $F_2\neq 0$, then $F(u)=(u-\alpha)^3(u-\beta)^2$, α,β are reals numbers, and $\alpha\neq\beta$. When $w>\alpha$, the solutions to u are

$$\pm\frac{\lambda(\alpha-\beta)}{2\sqrt{5}}(\xi-\xi_0)=-\frac{1}{\sqrt{u-\alpha}}-\sqrt{\alpha-\beta}\arctan\frac{\sqrt{u-\alpha}}{\sqrt{\alpha-\beta}} \quad (\alpha>\beta) \tag{15}$$

$$\pm\frac{\lambda(\alpha-\beta)}{2\sqrt{5}}(\xi-\xi_0)=-\frac{1}{\sqrt{u-\alpha}}-\frac{1}{2\sqrt{\beta-\alpha}}\ln\left|\frac{\sqrt{u-\alpha}-\sqrt{\beta-\alpha}}{\sqrt{u-\alpha}+\sqrt{\beta-\alpha}}\right| \quad (\alpha<\beta) \tag{16}$$

Case 3: If $D_5=0$, $D_4=0$, $D_3=0$, $D_2\neq 0$, $F_2=0$, then $F(u)=(u-\alpha)^4(u-\beta)$. α,β are reals numbers, and $\alpha\neq\beta$. When $w>\alpha$, the solutions to u are

$$\pm\frac{\lambda(\alpha-\beta)}{2\sqrt{5}}(\xi-\xi_0)=-\frac{\sqrt{u-\beta}}{2(u-\alpha)}-\frac{1}{2\sqrt{\alpha-\beta}}\arctan\frac{\sqrt{u-\gamma}}{\sqrt{\beta-\alpha}} \quad (\alpha<\beta) \tag{17}$$

$$\pm\frac{\lambda(\alpha-\beta)}{2\sqrt{5}}(\xi-\xi_0)=-\frac{\sqrt{u-\alpha}}{2\sqrt{u-\alpha}}-\frac{1}{4\sqrt{\alpha-\beta}}\ln\left|\frac{\sqrt{u-\beta}-\sqrt{\alpha-\beta}}{\sqrt{u-\beta}+\sqrt{\alpha-\beta}}\right| \quad (\alpha>\beta) \tag{18}$$

Case 4: If $D_5=0$, $D_4=0$, $D_3=0$, $D_2=0$, then $F(u)=(u-\alpha)^5$. α is a real number, when $w>\alpha$, the solutions u can be given by

$$\pm\frac{\lambda(\alpha-\beta)}{2\sqrt{5}}(\xi-\xi_0)=-\frac{2}{3}(u-\alpha)^{-\frac{2}{3}} \tag{19}$$

Case 5: If $D_5=0$, $D_4=0$, $D_3<0$, $E_2\neq 0$, then $F(u)=(u-\alpha)(u^2+ru+m)^2$, α is a real number,, and $r^2-4m<0$. When $w>\alpha$, the solutions to u are

$$\pm\frac{\lambda(\alpha-\beta)}{2\sqrt{5}}(\xi-\xi_0)=-\frac{2}{\rho\sqrt{4m-r^2}}\left(\cos\varphi\arctan\frac{2\rho\sin\varphi\sqrt{u-\alpha}}{u-\alpha-\rho^2}+\frac{\sin\varphi}{2}\ln\frac{u-\alpha-\rho^2-2\rho\cos\varphi\sqrt{u-\alpha}}{u-\alpha-\rho^2+2\rho\cos\varphi\sqrt{u-\alpha}}\right), \quad (20)$$

here $\rho=(\alpha^2+r\alpha+m)^{\frac{1}{4}}$, $\varphi=\frac{1}{2}\arctan\frac{\sqrt{4m-r^2}}{-2\alpha-r}$.

Case 6: If $D_5=0$, $D_4>0$, then $F(u)=(u-\alpha)^2(u-\alpha_1)(u-\alpha_2)(u-\alpha_3)$, $\alpha,\alpha_1,\alpha_2,\alpha_3$ are reals numbers, and $\alpha_1>\alpha_2>\alpha_3$. When $w>\alpha$, the solutions to u are

$$\pm\frac{\lambda(\alpha-\beta)}{2\sqrt{5}}(\xi-\xi_0)=-\frac{2}{(\alpha-\alpha_2)\sqrt{\alpha_2-\alpha_3}}\left\{F(\varphi,l)-\frac{\alpha_1-\alpha_2}{\alpha_1-\alpha}\prod\left(\varphi,\frac{\alpha_1-\alpha_2}{\alpha_1-\alpha},l\right)\right\}. \quad (21)$$

Here $\alpha\neq\alpha_1$, $\alpha\neq\alpha_2$, $\alpha\neq\alpha_3$, $F(\varphi,l)=\int_0^{\varphi}\frac{\mathrm{d}\varphi}{\sqrt{1-l^2\sin^2\varphi}}$,

$\prod(\varphi,n,l)=\int\frac{\mathrm{d}\varphi}{(1+n\sin^2\varphi)\sqrt{1-l^2\sin^2\varphi}}$.

Case 7: If $D_5=0$, $D_4=0$, $D_3<0$, $E_2=0$, then $F(u)=(u-\alpha)^3[(u-k_1)^2+m_1^2]$, α,k_1,m_1 are reals numbers. When $w>\alpha$, if $\alpha\neq k_1+m_1$, the solution to u is

$$\pm\frac{\lambda(\alpha-\beta)}{2\sqrt{5}}(\xi-\xi_0)=-\frac{\tan\theta+\cot\theta}{2(s_1\tan\theta-k_1-\alpha)\sqrt{\frac{m_1}{\sin^3 2\theta}}}F(\varphi,k)-\frac{m_1\tan\theta+m_1\cot\theta}{m_1\cot\theta+k_1+\alpha}\times\left[\frac{\tan\theta+k_1+\alpha}{(m_1\cot\theta+k_1-\alpha)}\sqrt{1-l^2\sin^2\varphi}+F(\varphi,l)-E(\varphi,l)\right] \quad (22)$$

If $\alpha\neq k_1+m_1$, the solution to u is

$$\pm\frac{\lambda(\alpha-\beta)}{2\sqrt{5}}(\xi-\xi_0)=\sqrt{\frac{\sin^3 2\theta}{4s^3}}\left[\frac{1}{l}\arcsin(l\sin\varphi)-F(\varphi,l)\right], \quad (23)$$

here $\tan 2\theta=\frac{m_1}{\alpha-k_1}$, $l=\sin\theta$, $0<\theta<\frac{\pi}{2}$, $E(\varphi,l)=\int_0^{\varphi}\sqrt{1-l^2\sin^2\psi}\,d\psi$.

Case 8: If $D_5=0$, $D_4<0$, then $F(u)=(u-\alpha)^2(u-\beta)[(u-k_1)^2+m_1^2]$, α,k_1,m_1 are reals numbers. If $\alpha\neq k_1-m_1\tan\theta$ and $\alpha\neq k_1+m_1\cot\theta$, the solution to u is

$$\pm\frac{\lambda(\alpha-\beta)}{2\sqrt{5}}(\xi-\xi_0)=-\frac{\tan\theta+\cot\theta}{2(m_1\tan\theta-k_1-\alpha)\sqrt{\frac{m_1}{\sin^3 2\theta}}}F(\varphi,k)-\frac{m_1\tan\theta+m_1\cot\theta}{m_1\cot\theta+k_1+\alpha}\times$$

$$\left[\frac{\tan\theta+k_1+\alpha}{(m_1\cot\theta+k_1-\alpha)}\sqrt{1-l^2\sin^2\varphi}+F(\varphi,l)-E(\varphi,l)\right] \tag{24}$$

If $\alpha=k_1-m_1\tan\theta$, the solution to u is

$$\pm\frac{\lambda(\alpha-\beta)}{2\sqrt{5}}(\xi-\xi_0)=\sqrt{\frac{\sin^3 2\theta}{4m^3}}\left[\frac{1}{l}\arcsin(l\sin\varphi)-F(\varphi,l)\right] \tag{25}$$

If $\alpha\neq k_1+m_1\cot\theta$, the solution to u is

$$\pm\frac{\lambda(\alpha-\beta)}{2\sqrt{5}}(\xi-\xi_0)=\sqrt{\frac{\sin^3 2\theta}{4s^3}}\left[F(\varphi,l)-\frac{1}{\sqrt{1-l^2}}\ln\frac{\sqrt{1-l^2\sin^2\varphi}+\sqrt{1-l^2}\sin\varphi}{\cos\varphi}\right], \tag{26}$$

here $\tan 2\theta=\frac{m_1}{\beta-k_1}$, $l=\sin\theta$, $0<\theta<\frac{\pi}{2}$.

Remark: By substituting the expressions of u from Eq. (10) to (26) into the Eq.(7), we gained all the expressions of v. For simplicity, we omitted the expressions of v.

CONCLUSION

By means of the complete discrimination system for polynomial, we obtain the classifications of all single travelling wave atom solutions to the Modified Konopelchenko-Dubrovsky Equations. The solutions are very rich.

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