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On Quasi -r-Normal Spaces

Jeyanthi V*[1], Janaki C[2]
[1]Asst .Prof, Depart. Mathematics, Sree Narayana Guru College, Coimbatore -105. Tamilnadu, India
[2]Asst.Prof, Depart. Mathematics, L.R.G. Govt. Arts College for Women, Tirupur-4, Tamilnadu, India

***Corresponding Author:**
Jeyanthi V
Email: jeyanthi_sngc@yahoo.com

Abstract: In this paper, we introduce the concept of quasi-r-normal spaces in topological spaces by using regular open sets in topological spaces and obtain some characterizations and preservation theorems for πgr-closed sets.
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INTRODUCTION

In 1968, Zaitsev [8] introduced the concept of quasi normal space in topological spaces and obtained several properties of such a space. Sadeq Ali Saad et al.,[7] introduced the concept of quasi p-normal spaces by using p-open sets and obtained its characterization.

In this paper, we use πgr-open sets to obtain the characterization of quasi-r-normal spaces.

PRELIMINARIES

Throughout this paper, spaces (X,τ) and (Y,σ)(or simply X or Y) always mean topological spaces on which no separation axioms are assumed unless explicitly stated. Let A be a subset of a space X. The closure of A and the interior of A are denoted by Cl(A) and Int(A) respectively.

Definition 2.1: A subset A of a topological space X is said to be

1. regular open[5]if A = int (cl(A)).
2. π-open [8]if A is the finite union of regular open sets.

The complement of the above defined open sets are their respective closed sets in X.
Closure(r-closure) of A is the intersection of all closed (regular closed) sets containing A and Interior of A is the union of all open sets contained in A.

Definition 2.2: A subset A of X is called

1. g-closed[2] if cl $(A)\subset U$ whenever $A\subset U$ and U is open in X.
2. g*r-closed[5] if $rcl(A)\subset U$ whenever $A\subset U$ and U is open.
3. πg-closed[1] if $cl(A)\subset$ whenever $A\subset U$ and U is π-open.
4. πgr-closed[3] if $rcl(A)\subset U$ whenever $A\subset U$ and U is π-open .

Definition 2.4: A topological space X is said to be
1. a normal space [7]if for every pair of disjoint closed subsets H and K , there exists disjoint open sets U and V of X such that $H\subset U$ and $K\subset V$.
2. a quasi normal[7] if for every pair of disjoint π-closed subsets H and K , there exists disjoint open sets U and V of X such that $H\subset U$ and $K\subset V$.
3**.** mildly normal[] if for every pair of disjoint regular closed subsets H and K , there exists disjoint open sets U and V of X such that $H\subset U$ and $K\subset V$.

Theorem 2.5: A subset A of a topological space X is πgr-open iff $F\subset rint(A)$ whenever F is π-closed and $F\subset A$.

Definition 2.6: A function f: X→Y is said to be

1. Almost closed [4](g*r-closed, πgr-closed)if f(F) is regular-closed (g*r-closed, πgr-closed) in Y for every closed set F of X.
2. rc-preserving [4](almost g*r-closed, almost πgr-closed)if f(F) is regular closed(g*r- closed, πgr-closed) in Y for every regular closed set F of X.
3. continuous[2](resp. almost continuous[6],π-continuous[1])if $f^{-1}(F)$ is closed (resp. regular closed, π-closed) in X for every closed set F of Y.
4. πgr-continuous [3] if $f^{-1}(F)$ is πgr-closed in X for every closed set F of Y.

QUASI R-NORMAL SPACES.

Definition 3.1: A topological space X is said to be r-normal if for every pair of disjoint closed subsets H and K , there exists disjoint regular open sets U and V of X such that H⊂U and K⊂V.

Definition 3.2: A topological space X is said to be quasi -r-normal (quasi regular normal)if for every pair of disjoint π-closed subsets H and K , there exists disjoint regular open sets U and V of X such that H⊂U and K⊂V.

Theorem 3.3:

The following are equivalent for a space X.

a) X is quasi r-normal.
b) For any disjoint π-closed sets H and K, there exists disjoint g*r-open sets U and V such that H⊂U and K⊂V.
c) For any disjoint π-closed sets H and K, there exists disjoint πgr-open sets U and V such that H⊂U and K⊂V.
d) For any π-closed set H any π-open set V containing H, there exists an g*r-open set U of X such that H ⊂ U ⊂ rcl(U) ⊂ V.
e) For any π-closed set H and π-open set V containing H , there exists an πgr-open set U of X such that H ⊂ U ⊂ rcl(U) ⊂ V.

Proof: (a)⇒(b): Let X be quasi- r- normal. Let H and K be disjoint π-closed sets in X. By assumption, there exists disjoint regular open sets U, V such that H⊂U and K⊂V. Since every regular open set is g*r-open, U, V are g*r-open sets such that H⊂U and K⊂V.
(b)⇒(c): Obvious.
(c)⇒(d): Let H be any π-closed set and V be any π-open set containing H. By assumption, there exists πgr-open sets U and W such that H⊂U and X−V⊂W. By theorem 2.5, we get A⊂rint(U),X−V⊂ rint(W) and rcl(U)∩rint(W) =ϕ. Hence H⊂U⊂rcl(U) ⊂ X− rint(W) ⊂V.
(d)⇒(e): Obvious.
(e)⇒(a): Let H, K be two disjoint π-closed sets of X.Then H⊂X – K and X – K are π-open. By assumption there exists πgr-open set G of X such that H⊂G⊂ rcl(G) ⊂X – K . Put U = r int (G), V =X – rcl(G). Then U and V are disjoint regular open sets of X such that H⊂U and K⊂V.

Definition 3.4:A topological space X is said to be mildly r-normal if for every pair of disjoint regular closed sets H and K of X, there exists disjoint regular open sets U and V of X such that H⊂U and K⊂V.

Theorem 3.5: The following are equivalent for a space X.

a) X is mildly r-normal.
b) For any disjoint regular closed sets H and K , there exists disjoint g*r-open sets U and V such that H⊂U and K⊂V.
c) For any disjoint regular closed sets H and K, there exists disjoint πgr-open sets U and V such that H⊂U and K⊂V.
d) For any regular closed set H any regular open set V containing H, there exists an g*r-open set U of X such that H ⊂ U ⊂ rcl(U) ⊂ V.
e) For any regular closed set H and each regular open set V containing H , there exists a πgr-open set U of X such that H ⊂ U ⊂ rcl(U) ⊂ V.

Proof: Similar to that of above theorem 3.3.

Theorem 3.6: A surjection f: X→Y is almost πgr-closed iff for each subset S of Y and each regular open set U of X containing $f^{-1}(S)$, there exists a πgr-open set V of Y such that S⊂V and $f^{-1}(V)$ ⊂U.

Proof: Necessity: Suppose that f is almost πgr-closed. Let S be a subset of Y and U a regular open set containing $f^{-1}(S)$. If V = Y – f(X–U), then V is a πgr-open set of Y such that S⊂V and $f^{-1}(V)$⊂U.

Sufficiency: Let F be any regular closed set of X. Then $f^{-1}(Y-f(F)) \subset X - F$ and $X-F$ is regular open in X. There exists a πgr-open set V of Y such that $Y-f(F) \subset V$ and $f^{-1}(V) \subset X-F$. Therefore, we have $Y-V \subset f(F)$ and $F \subset X-f^{-1}(V) \subset f^{-1}(Y-V)$. Hence we obtain $f(F) = Y-V$ and f(F) is πgr-closed in Y which shows that f is almost πgr-closed.

Preservation theorems:

Theorem 3.7: If $f: (X,\tau) \to (Y,\sigma)$ is an almost πgr-continuous π-closed injection and Y is quasi -r-normal, then X is quasi-r-normal.

Proof: Let A and B be any disjoint π-closed sets of X. Since f is a π-closed injection, f(A) and f(B) are disjoint π-closed sets of Y. Since Y is quasi-r-normal, there exists disjoint regular open sets G and H such that $f(A) \subset G$ and $f(B) \subset H$. Since f is almost πgr-continuous, $f^{-1}(G)$ and $f^{-1}(H)$ are disjoint πgr-open sets containing A and B which shows that X is quasi -r-normal.

Lemma 3.8: A surjection $f: (X,\tau) \to (Y,\sigma)$ is rc-preserving iff for each subset S of Y and each regular open set U of X containing $f^{-1}(S)$ there exists a regular open set V of Y such that $S \subset V$ and $f^{-1}(V) \subset U$.

Theorem 3.9: If $f: (X,\tau) \to (Y,\sigma)$ is a π-continuous, rc-preserving surjection and X is quasi-r-normal space, then Y is r-normal.

Proof: Let A and B be any two disjoint closed sets of Y. Then $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint π-closed sets of X. Since X is quasi-r-normal, there exists disjoint regular open sets G and H such that $f^{-1}(A) \subset G$ and $f^{-1}(B) \subset H$.. Set $K = Y-f(X-G)$ and $L = Y-f(X-H)$. By lemma 3.8, K and L are regular open sets of Y such that $A \subset K$, $B \subset L$, $f^{-1}(K) \subset G$, $f^{-1}(L) \subset H$. Since G and H are disjoint and so K and L. Since K and L are regular open, we obtain $A \subset rint(K)$, $B \subset rint(L)$ and $rint(K) \cap rint(L) = \phi$. Therefore Y is r-normal.

Theorem 3.10: Let $f: (X,\tau) \to (Y,\sigma)$ be a π-irresolute, almost πgr-closed surjection .If X is a quasi-r-normal space, then Y is quasi-r-normal.

Proof: Let A and B be any two disjoint π-closed sets of Y. Since f is π-irresolute, $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint π-closed subsets of X. Since X is quasi-r-normal, there exists regular open sets G and H of X such that $f^{-1}(A) \subset G$ and $f^{-1}(B) \subset H$. By theorem 3.7, there exists πgr-open sets K and L of such that $A \subset K$ and $B \subset L$, $f^{-1}(K) \subset G$, $f^{-1}(L) \subset H$. Since G and H are disjoint, so $rint(K) \cap rint(L) = \phi$. Therefore, U is quasi -r-normal.

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