Scholars Journal of Physics, Mathematics and Statistics **ISSN 2393-8056 (Print)**
Sch. J. Phys. Math. Stat. 2014; 1(2):53-60 **ISSN 2393-8064 (Online)**

(An International Publisher for Academic and Scientific Resources)

Secondary Critical Path in a Fuzzy Project Network

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Abstract: In a fuzzy project network, the higher risk path is called possible critical path. Secondary critical path is one of the possible critical path in fuzzy project network and it plays an important role in the project scheduling, risk analysis and time-cost trade-off problems in fuzzy environment. In this paper, triangular fuzzy numbers are used to express the operation times for all activities in fuzzy project network. We propose a method to find secondary critical path(s) in a fuzzy project network using the fuzzy free float. RM approach is used to rank fuzzy numbers to find fuzzy critical path. Two examples are given to describe the method.
Keywords: Fuzzy Numbers; Possible Critical Paths; Fuzzy Project Network; Fuzzy free float; Project Management

INTRODUCTION

Critical Path Method (CPM) is available for modeling the execution of a project and has been applied to several Industrial Fields [1]. Activity durations in CPM are deterministic and known, but activities in a network are often uncertain and variable. To consider the consequences of variability, a statistical approach was introduced by Malcolm et al. [2] on the Project Evaluation and Review Technique (PERT). PERT has successfully been used in business management factory production, logistic support etc.,[3-5]. However the activity duration time often is an uncertain value so that the result of PERT computation cannot properly match the real-world situation. Fuzzy Set Theory [6] is a powerful tool to deal with real life situations. In [7], Nasution proposed how to compute total floats and find critical paths in a project network. Kaufmann and Gupta [8] devoted a chapter of their book to the critical path method in which activity times are represented by triangular fuzzy numbers. Chen et al.[9] incorporated time window constraint and time schedule constraint into the traditional activity network. They developed a linear time algorithm for finding the critical path in an activity network with these time constraints. Liang and Han [10] presented an algorithm to perform fuzzy critical path analysis for fuzzy project network problems. Sireesha and Shankar [11] presented a new method based on fuzzy set theory for solving fuzzy project scheduling in fuzzy environment. Assuming that the duration of activities are triangular fuzzy numbers, they computed total float time of each activity and fuzzy critical path without computing forward and backward pass computations. Kumar and Kaur [12] proposed a new method to find the fuzzy optimal solution of fully fuzzy critical path problems. Kumar and Kaur [13] proposed a new method that modifies the existing one. They presented the advantages of the proposed method by solving a specific fuzzy critical path problem. Rao and Shankar [18] proposed a new method to find fuzzy critical path using Lexicographic ordering.

In Fuzzy Project Network , the higher risk path is called possible critical path. There is an increasing demand that the decision-maker needs more “Possible Critical Paths” to decrease the decision risk for project management [14]. The free float and other concepts of floats are often used in time-cost trade-off problems and resource allocating optimization problems. But there is no literature focusing attention on solving the secondary critical path(s) in fuzzy project network by using fuzzy free float. The secondary critical paths are of great significance in fuzzy project network. In this paper, we propose a new method to find secondary Critical Path(s) in a fuzzy project network using fuzzy free floats.

In Section 2, we briefly review basic definitions of fuzzy sets, fuzzy arithmetic operations ,notations, and formulae. In Section 3, we present comparison of fuzzy numbers using RM approach [15]. In Section 4, we present a Fuzzy CPM using RM approach and presented a new method using Fuzzy free float to find secondary critical path(s) in a fuzzy project network. In section 5, two numerical examples are presented to illustrate the concept of finding secondary critical paths using fuzzy free float .

FUZZY CONCEPTS

In this section, basic definitions of fuzzy sets and fuzzy numbers, fuzzy arithmetic operations, mathematical notations, and formulae are reviewed.

Basic definitions and fuzzy arithmetic operations

The basic definitions on fuzzy set theory and fuzzy arithmetic operations are reviewed [16, 17].

Definition 1. The characteristic function μ_A of a crisp set $A \subseteq X$ assigns a value either 0 or 1 to each member in X. This function can be generalized to a function $\mu_{\tilde{A}}$ such that the value assigned to the element of the universal set X fall within a specified range i.e.. $\mu_{\tilde{A}} : X \to [0,1]$. The assigned value indicate the membership grade of the element in the set A. The function $\mu_{\tilde{A}}$ is called the membership function and the set $\tilde{A}$ ={(x, $\mu_{\tilde{A}}$ (x) : x $\in$X)} defined by $\mu_{\tilde{A}}(x)$ for each x $\in$X is called a fuzzy set.

Definition 2. A fuzzy set $\tilde{A}$, defined on the universal set of real number R, is said to be a fuzzy number if its membership function has the following characteristics:

(i) $\tilde{A}$ is convex i.e., $\mu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geq \min\ (\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2))$ $\forall$ $x_1, x_2 \in R$, $\forall \lambda \in [0,1]$,

(ii) $\tilde{A}$ is normal i.e.,$\exists$ $x_0 \in$ R such that $\mu_{\tilde{A}}(x_0) = 1$,

(iii) $\mu_{\tilde{A}}(x)$ is piecewise continuous.

Definition 3. A fuzzy number $\tilde{A}$ is called non-negative fuzzy number if $\mu_{\tilde{A}}(x) = 0\ \forall\, x < 0$.

Definition4. A fuzzy number $\tilde{A}$ = (a,b,c) is said to be a triangular fuzzy number if its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} 0\ , & -\infty < x \leq a \\ \dfrac{x-a}{b-a}, & a \leq x < b \\ \dfrac{c-x}{c-b}, & b \leq x < c \\ 0, & c \leq x < \infty \end{cases}$$

Definition 5. A triangular fuzzy number $\tilde{A}$ =(a,b,c) is said to be zero triangular fuzzy number if a = 0,b = 0,c = 0

Definition 6. A triangular fuzzy number $\tilde{A}$ =(a,b,c) is said to be non-negative triangular fuzzy number if $a \geq 0$.

Fuzzy arithmetic operations :

For real triangular fuzzy numbers, $\tilde{A} = (a_1, a_2, a_3)$ and $\tilde{B} = (b_1, b_2, b_3)$

(i) $\tilde{A} \oplus \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$ **(addition of fuzzy numbers)**

(ii) $\tilde{A}\ \Theta\, \tilde{B} = (a_1 - b_3, a_2 - b_2, a_3 - b_1)$ **(Subtraction of fuzzy numbers)**

Mathematical notations and formulae

$\tilde{T}_{i,j}$: the fuzzy activity time of activity (i,j)

$\tilde{E}^{s}_{i,j}$: the fuzzy earliest start time of activity (i,j)

$\tilde{E}^{f}_{i,j}$: the fuzzy earliest finish time of activity (i,j)

$\tilde{L}^{s}_{i,j}$: the fuzzy latest start time of activity (i,j)

$\tilde{L}^{f}_{i,j}$: the fuzzy latest finish time of activity (i,j)

$\tilde{E}^{s}_{i}$: the earliest possible time of realization of node(i)

$\tilde{L}^{f}_{i}$: the latest possible time of realization of node (i)

$$\tilde{E}^{f}_{i,j} = \tilde{E}^{s}_{i,j} \oplus \tilde{T}_{i,j}$$

$$\tilde{L}^{f}_{i,j} = \tilde{L}^{s}_{i,j} + \tilde{T}_{i,j}$$

$$\tilde{E}^{s}_{i,j} = \tilde{E}^{s}_{i}$$

$$\tilde{L}^{f}_{i,j} = \tilde{L}^{f}_{j}$$

Total float = $\tilde{T}^{f}_{i,j} = \tilde{L}^{s}_{i,j} - \tilde{E}^{s}_{i,j} = \tilde{L}^{f}_{i,j} - \tilde{E}^{f}_{i,j}$

Free float = $\tilde{F}^{f}_{i,j} = \tilde{E}^{s}_{j} - \tilde{E}^{s}_{i} - \tilde{T}_{i,j}$

COMPARISON OF TRIANGULAR FUZZY NUMBERS USING RM APPROACH

In this section, the comparison of triangular fuzzy numbers using RM approach is reviewed [15].

Let $\tilde{A} = (a_1, a_2, a_3)$ and $\tilde{B} = (b_1, b_2, b_3)$ be two generalized triangular fuzzy numbers. Then (i) $\tilde{A} > \tilde{B}$ if **RM** $(\tilde{A} \Theta \tilde{B}) >$ **RM** $(\tilde{B} \Theta \tilde{B}) = (0,0,0)$

(ii) $\tilde{A} < \tilde{B}$ if **RM** $(\tilde{A} \Theta \tilde{B}) <$ **RM** $(\tilde{B} \Theta \tilde{B}) = (0,0,0)$

(iii) $\tilde{A} \cong \tilde{B}$ if **RM** $(\tilde{A} \Theta \tilde{B}) =$ **RM** $(\tilde{B} \Theta \tilde{B}) = (0,0,0)$

Method to find value of RM $(\tilde{A} \Theta \tilde{B})$

Let $\tilde{A} = (a_1, a_2, a_3)$ and $\tilde{B} = (b_1, b_2, b_3)$ be two generalized triangular fuzzy numbers. Then use the following steps to find the values of **RM** $(\tilde{A} \Theta \tilde{B})$

Step1: find $\tilde{A} \Theta \tilde{B} = (a_1 - b_3, a_2 - b_2, a_3 - b_1)$

Step2: find **R** $(\tilde{A} \Theta \tilde{B}) = \dfrac{(a_1 - b_3) + 2(a_2 - b_2) + (a_3 - b_1)}{4}$

Step3: If $\mathbf{R}(\tilde{A}\,\Theta\,\tilde{B}) \neq 0$, then **RM** $(\tilde{A}\,\Theta\,\tilde{B}) = (\tilde{A}\Theta\,\tilde{B}) = \mathbf{R}(\tilde{A}\,\Theta\,\tilde{B})$

otherwise **RM** $(\tilde{A}\,\Theta\,\tilde{B}) = \text{mode}(\tilde{A}\,\Theta\,\tilde{B}) = b_1 - b_2.$

Example1: Let $\tilde{A}$ = (0.1,0.3,0.5) and $\tilde{B}$ =(0.1,0.3,0.5) be two generalized triangular fuzzy numbers. Then $\tilde{A}$ and $\tilde{B}$ can be compared by using the following steps.

Step 1: $\tilde{A}\,\Theta\,\tilde{B}$ = (-0.4, 0, 0.4)

Step 2: $\mathbf{R}(\tilde{A}\,\Theta\,\tilde{B})$=0.

Step 3: Since $\mathbf{R}(\tilde{A}\,\Theta\,\tilde{B})$=0, so

RM $(\tilde{A}\,\Theta\,\tilde{B}) = \text{mode}(\tilde{A}\,\Theta\,\tilde{B}) = 0 \Rightarrow$ **RM** $(\tilde{A}\,\Theta\,\tilde{B})$=0.

Then using properties of comparison of triangular numbers, $\tilde{A} \cong \tilde{B}$.

Example 2: Let $\tilde{A}$ = (0.2, 0.4, 0.8) and $\tilde{B}$ =(0.1,0.3,0.4) be two generalized triangular fuzzy numbers. Then $\tilde{A}$ and $\tilde{B}$ can be compared by using the following steps.

Step 1: $\tilde{A}\,\Theta\,\tilde{B}$ = (-0.2, 0.1, 0.7)

Step 2: $\mathbf{R}(\tilde{A}\,\Theta\,\tilde{B})$=0.175.

Step 3: Since $\mathbf{R}(\tilde{A}\,\Theta\,\tilde{B}) \neq 0$, so **RM** $(\tilde{A}\,\Theta\,\tilde{B}) = \mathbf{R}(\tilde{A}\,\Theta\,\tilde{B})$=0.175. Then Using properties of comparison of triangular fuzzy numbers, $\tilde{A} > \tilde{B}$.

Example 3: Let $\tilde{A}$ = (-0.8,-0.6,-0.2) and $\tilde{B}$ =(-0.4,-0.3,-0.1) be two generalized triangular fuzzy numbers. Then $\tilde{A}$ and $\tilde{B}$ can be compared by using the following steps.

Step1: $\tilde{A}\,\Theta\,\tilde{B}$ = (-0.7, -0.3, 0.2)

Step2: $\mathbf{R}(\tilde{A}\,\Theta\,\tilde{B})$= - 0.2

Step3: Since $\mathbf{R}(\tilde{A}\,\Theta\,\tilde{B}) \neq 0$, so **RM** $(\tilde{A}\,\Theta\,\tilde{B}) = \mathbf{R}(\tilde{A}\,\Theta\,\tilde{B})$= -0.2. Then Using properties of comparison of triangular fuzzy numbers, $\tilde{A} < \tilde{B}$.

FUZZY CPM USING RM APPROACH AND PROPOSED SECONDARY CPM

Consider a fuzzy project network with n events. The following method describes fuzzy CPM using RM approach.

Step 1: Earliest times $\tilde{E}_i^s$'s (i =1,2,..n) of each event i are calculated as follows :

Let $\tilde{E}_1^s$ =(0,0,0) and calculate $\tilde{E}_j^s$, j = 2,3,…,n by using

$$\tilde{E}_j^s = \max\left\{\left(\tilde{E}_1^s \oplus \tilde{T}_{i,j}\right) / j \neq 1, j = 2,3,...n\right\}$$ and RM approach of fuzzy ranking

Step 2 : Latest times $\tilde{L}_i^f$'s (j=1,2,..n) of each event j are calculated as follows :

Let $\tilde{L}_n^f = \tilde{E}_n^s$ and calculate $\tilde{L}_j^f$, j = n-1, n-2,…, 2, 1 by using

$$\tilde{L}_j^f = \min\left\{\left(\tilde{L}_k^f \Theta \tilde{T}_{jk}\right) / j \neq n, k > j, j = n-1, n-2,...,2,1\right\}$$ and RM approach of fuzzy ranking.

Step 3 : Calculate the total float of each activity (i,j) using $\tilde{T}_{ij} = \tilde{L}_j \; \Theta \; (\tilde{E}_i \oplus \tilde{t}_{ij})$,
$1 \leq i < j \leq n$.

Step 5 : Find all possible paths in the fuzzy project network and let P be the set of all possible paths in the network. Calculate fuzzy completion time of each path

$P_k \in P$ in the project network by C(P_k)= $\sum_{\substack{1\leq i<j\leq n \\ i,j\in P_K}} \tilde{T}_{ij}$.

Step 6 : Find the fuzzy critical path P_f where C(P_f) = min {C(P_k) / $P_k \in$ P} using RM approach of ranking fuzzy numbers.

Secondary critical path using fuzzy free float

The following procedure describes the method to find second critical path using fuzzy free float :

Step 7 : Find out all of the critical nodes (the nodes in the critical path obtained in step 6) and compute the free float value of the non-critical activities immediately preceding the critical nodes obtained in the path and choose the minimum one.

Step 8: Find the preceding path for critical node i.
The preceding path can be defined as the path formed by a set of activities whose free float is minimum from source node (1) to node (i)

Step 9 : Obtain the secondary critical path by adding preceding path obtained from step 8 for the critical node i to the succeeding path (The path from node i to terminal node).

NUMERICAL EXAMPLES

In this section, the secondary path for two different fuzzy project networks are discussed using two examples. In example 1, we can have non-critical nodes in the fuzzy project network and we can find secondary critical path whereas in example 2, all the nodes in the project network are critical nodes. In example 2, the method of finding secondary critical path is not required.

Example 1: .Let us consider a fuzzy project network shown Fig. 1. The duration of fuzzy activity times, which are assumed as triangular fuzzy variables are presented in Table-1.

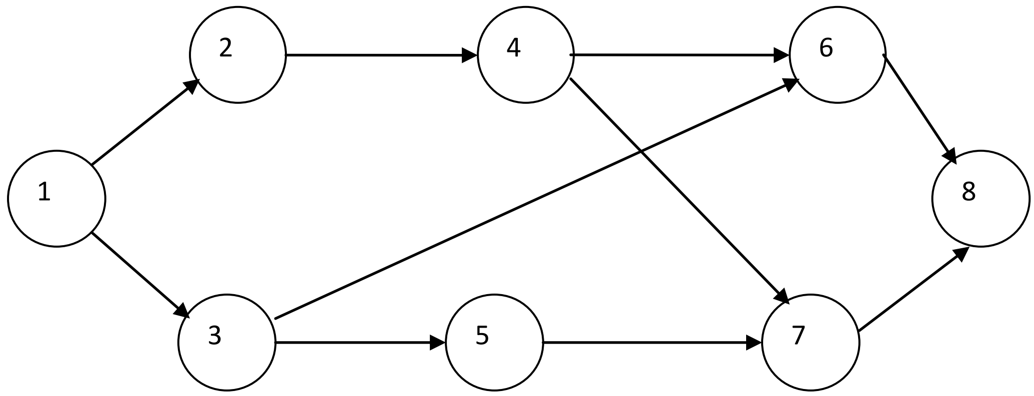

Fig.1 Fuzzy Project network

Table-1: Fuzzy activity times

Activity	Fuzzy Activity time
1-2	(2,3,4)
1-3	(1,3,4)
2-4	(1,3,5)
3-5	(1,2,3)
3-6	(2,5,7)
4-6	(3,4,6)
4-7	(3,4,5)
5-7	(1,4,5)
6-8	(2,5,6)
7-8	(3,4,7)

Using the RM approach of the fuzzy critical path method(CPM) in section 4,we get the values of earliest, latest and float times of fuzzy project network and are presented in Table-2.

Table-2: Fuzzy earliest, Latest and total float times

Activities	**Activity time**	**ES**	**EF**	**LS**	**LF**	**TF**
1-2	(2,3,4)	(0,0,0)	(2,3,0)	(-13,0,13)	(-9,3,15)	(-13,0,13)
1-3	(1,3,4)	(0,0,0)	(1,3,4)	(-11,2,15)	(-7,5,16)	(-11,2,15)
2-4	(1,3,5)	(2,3,4)	(3,6,9)	(-9,3,15)	(-4,6,16)	(-13,0,13)
3-5	(1,2,3)	(1,3,4)	(2,5,7)	(-7,5,16)	(-4,7,17)	(-11,2,15)
3-6	(2,5,7)	(1,3,4)	(3,8,11)	(-5,5,17)	(2,10,19)	(-9,2,16)
4-6	(3,4,6)	(3,6,9)	(6,10,15)	(-4,6,16)	(2,10,19)	(-13,0,13)
4-7	(3,4,5)	(3,6,9)	(6,10,14)	(-4,7,15)	(1,11,18)	(-13,1,12)
5-7	(1,4,5)	(2,5,7)	(3,9,12)	(-4,7,17)	(1,11,18)	(-11,2,15)
6-8	(2,5,6)	(6,10,15)	(8,15,21)	(2,10,19)	(8,15,21)	(-13,0,13)
7-8	(3,4,7)	(6,10,14)	(9,14,21)	(1,11,18)	(8,15,21)	(-13,1,12)

Calculated critical path of the fuzzy project network is 1-2-4-6-8. It is obtained using step 1 to step 6 of section 4.

Method to find secondary critical path of the fuzzy project network is explained as follows :

Secondary critical path of the fuzzy project network is obtained using step 7 to step 9 of section 4.

Step 7: critical nodes are 1,2,4,6,8 now compute free float for non-critical activities immediately preceding the critical nodes.

Non-critical nodes are 3,5,7

Critical nodes are 1,2,4,6,8

Non-critical nodes immediately preceding the critical nodes are 3,7

Free float of activity 3-6 $(FF_{36}) = ES_6 - ES_3 - T_{36}$
$= (6,10,15) - (1,3,4) - (2,5,7)$
$= (-5,2,12)$

Free float of activity 7-8 $(FF_{78}) = ES_8 - ES_7 - T_{78}$
$= (8,15,21) - (6,10,14) - (3,4,7)$
$= (-13,1,12)$

Minimum of FF_{36} and FF_{78} is FF_{78}

Step 8: The preceding path from source node 1 to node 7

i.e., $1 \rightarrow 2 \rightarrow 4 \rightarrow 7$

Step 9 : The secondary critical path is $1 \rightarrow 2 \rightarrow 4 \rightarrow 7 \rightarrow 8$.

Example 2: Let us consider a fuzzy project network shown in Fig.2. The duration of fuzzy activity times, which are assumed as triangular fuzzy variables are presented in Table-3, respectively.

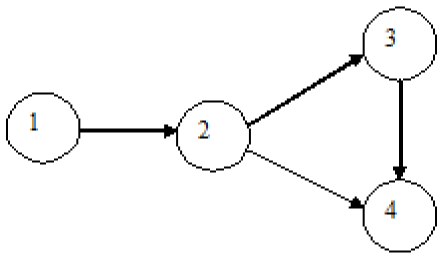

Fig.2 Fuzzy Project network

Table-3 : Fuzzy activity times

Activity	Fuzzy Activity time
1-2	(3,4,5)
2-3	(2.8,3,3.2)
2-4	(4,5,6)
3-4	(1.8,2,2.2)

Using the RM approach of the fuzzy critical path method(CPM) in section4,we get the values of earliest, latest and float times of the fuzzy project network and are mentioned in Table-4.

Table-4: Fuzzy earliest, Latest and total float times

Activities	Activity time	ES	EF	LS	LF	TF
1-2	(3,4,5)	(0,0,0)	(3,4,5)	(-4,0,4)	(1,4,7)	(-4,0,4)
2-3	(2.8,3,3.2)	(3,4,5)	(5.8,7,8.2)	(1,4,7)	(4.8,7,9.2)	(-4,0,4)
2-4	(4,5,6)	(3,4,5)	(7,9,11)	(1,4,7)	(7,9,11)	(-3.4,0,3.4)
3-4	(1.8,2,2.2)	(5.8,7,8.2)	(7,9,11)	(4.8,7,9.2)	(7,9,11)	(-4,0,4)

Calculated critical paths of the fuzzy project network are 1-2-3-4 and 1-2-4 . Here, all the nodes (critical nodes) in the project are involved in critical path .In the above project network , non-critical nodes are not existed . There is no need to find secondary critical path.

CONCLUSION

In Fuzzy Project Network , the higher risk path is called possible critical path. There is an increasing demand that the decision-maker needs more "Possible Critical Paths" to decrease the decision risk for project management. To reduce the decision risk in fuzzy project management, we have proposed a new method to find secondary critical path(s) in a fuzzy project network using the fuzzy free float. RM approach is used to rank fuzzy numbers to find fuzzy critical path. Two examples are given to describe the method.

REFERENCES

1. Ahuja HN, Dozzi SP, Abourizk SM; Project Management. New York: Wiley. 1994.
2. Malcolm DG, Roseboom JH, Clark CE, Fazar W; Application of a technique for research and development project evaluation, Operations Research, 1959; 7:646-669.
3. Mon DL, Cheng CH, Lu HC; Application of fuzzy distribution on project management, Fuzzy Sets Systems, 1995;73(3):227-234.
4. Alvares Valdes R, Tamarit JM; Heuristic algorithms for Resource- constrained project scheduling : A review and an empirical analysis, in : R. slowionski and J.Weglarz, Eds.,Advances in Project Scheduling, Amsterdam, The Netherlands: Elsevier, 1989; 114-134.
5. Wang HF, Fu CC; Fuzzy Project Scheduling models under inflation condition, International Journal of Uncertainty, Fuzziness and Knowledge -Based System, 1996; 4(6):497-514.
6. Zadeh LA; Fuzzy Sets, Information and Control, 1965; 8(3):338-353.
7. Nasution SH; Fuzzy Critical Path Method, IEEE Transaction on Systems Man and Cybernetics, 1994; 24(1):48-57.
8. Kaufmann A, Gupta MM; Fuzzy mathematical models in engineering and management science, Amsterdam, The Netherland : Elsevier. 1988.
9. Chen YL, Rinks D, Tang K.; Critical path in an activity network with time constraints, European Journal of Operations Research, 1997; 100: 122-133.
10. Liang GS, Han TC; Fuzzy critical path for project network, Information and Management Science, 2004; 15:29-40.
11. Sireesha V, Shankar NR; A new approach to find total float time and critical path in a fuzzy project network, International Journal of Engineering Science and Technology, 2010; 2: 600-609.
12. Kumar A, Kaur P; A new method for fuzzy critical path analysis in project networks with a new representation of triangular fuzzy numbers, Applications and Applied Mathematics : An International Journal, 2010; 5(10):1442-1466.
13. Kumar A, Kaur P; Exact Optimal solution of fuzzy critical path problems, Applications and Applied Mathematics : An International Journal, 2011; 6:1992-2008.
14. Chen HK, Kao HP; Some issues in fuzzy PERT, Proc.First, Nat. Symp. Fuzzy Set theory Application, Hsinchu, Taiwan, R.O.C., 1993; 36-41.
15. Kumar A, Singh P, Kaur A, Kaur P; RM approach for Ranking of Generalized Trapezoidal Fuzzy Numbers. Fuzzy Information and Engineering, 2010; 37-47.
16. Dubois D, Prade H; Fuzzy Sets and Systems: Theory and Applications, Mathematics in Science and Engineering, 144, academic press, Inc, London. 1980.
17. Kaufmann A, Gupta MM; Introduction to Fuzzy Arithmetics: Theory and Applications, New York, EUA: Van Nostrand Reinhold. 1985.
18. Rao PPB, Shankar NR; Fuzzy Critical Path Method Based on Lexicographic Ordering, Pakistan Journal of Statistics and Operation Research, 2012; 8(1):139-154.