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# On the Non-Homogeneous Octic Equation with Five Unknowns $x^4 - y^4 = T^6(z^2 - w^2)$

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**Abstract:** The non-homogeneous octic equation with five unknowns represented by the diophantine equation $x^4 - y^4 = T^6(z^2 - w^2)$ is analyzed for its patterns of non-zero distinct integral solutions and two different patterns of integral solutions are illustrated. Various interesting relations between the solutions and special numbers, namely, Pyramidal numbers, Pronic numbers, Stella octangular numbers, Gnomonic numbers, polygonal numbers, fourth, fifth and sixth dimensional figurate numbers are exhibited.
**Keywords:** Octic non-homogeneous equation, Pyramidal numbers, Pronic numbers, Fourth, fifth and sixth dimensional figurate numbers.
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## NOTATIONS

$t_{m,n}$ : Polygonal number of rank $n$ with size $m$

$So_n$ : Stella octangular number of rank $n$

$\Pr_n$ : Pronic number of rank $n$

$Gnomic_n$ : Gnomonic number of rank $n$

$P_n^m$ : Pyramidal number of rank n with size m.

$F_{4,n,5}$ : Fourth dimensional pentagonal figurate number of rank n

$F_{4,n,8}$ : Fourth dimensional octagonal figurate number of rank n

$F_{5,n,3}$ : Fifth dimensional triangular figurate number of rank n

$F_{5,n,7}$ : Fifth dimensional heptagonal figurate number of rank n

$F_{6,n,3}$ : Sixth dimensional triangular figurate number of rank n

$F_{6,n,5}$ : Sixth dimensional pentagonal figurate number of rank n

## INTRODUCTION

The theory of diophantine equations offers a rich variety of fascinating problems. In particular, homogeneous and non-homogeneous equations of higher degree have aroused the interest of numerous Mathematicians since antiquity [1-3]. In [4-8] heptic equations with three, four and five unknowns are analyzed. This communication analyses a non-homogeneous octic equation with five unknowns given by $x^4 - y^4 = T^6(z^2 - w^2)$ for determining its infinitely many non-zero integer quintuples $(x, y, z, w, T)$ satisfying the above equation are obtained. Various interesting properties among the values of $x, y, z, w$ and $T$ are presented.

## METHOD OF ANALYSIS

The diophantine equation representing a non-homogeneous octic equation is

$$x^4 - y^4 = T^6(z^2 - w^2) \tag{1}$$

Introducing the linear transformations

$$x = u + v, y = u - v, z = 2ku + v, w = 2ku - v \quad (2)$$

in (1), it leads to

$$u^2 + v^2 = kT^6 \quad (3)$$

**Pattern 1**

Let $$k = \alpha^2 + \beta^2 \quad \alpha, \beta \neq 0 \quad (4)$$

Taking $$T = a^2 + b^2 \quad a, b \neq 0 \quad (5)$$

Substituting (4) in (2) and employing the method of factorization, define

$$u + iv = (\alpha + i\beta)(a + ib)^6 \quad (6)$$

Equating real and imaginary parts in (6),we get

$$\left.\begin{aligned} u &= \alpha f(a,b) - \beta g(a,b) \\ v &= \beta f(a,b) + \alpha g(a,b \end{aligned}\right\} \quad (7)$$

where,

$$\left.\begin{aligned} f(a,b) &= a^6 - 15a^4b^2 + 15a^2b^4 - b^6 \\ g(a,b) &= 6a^5b - 20a^3b^3 + 6ab^5 \end{aligned}\right\} \quad (8)$$

Substituting (7) and (8) in (2), the corresponding integral solutions of (1) are given by

$$x = (\alpha + \beta) f(a,b) + (\alpha - \beta) g(a,b)$$
$$y = (\alpha - \beta) f(a,b) - (\alpha + \beta) g(a,b)$$
$$z = 2k[\alpha f(a,b) - \beta g(a,b)] + [\beta f(a,b) + \alpha g(a,b]$$
$$w = 2k[\alpha f(a,b) - \beta g(a,b)] - [\beta f(a,b) + \alpha g(a,b].$$

For simplicity and clear understanding, we, in what follows present the solutions and their corresponding properties for $k = 5, \alpha = 2, \beta = 1$ as follows

$$x = 3f(a,b) - g(a,b)$$
$$y = f(a,b) - 3g(a,b)$$
$$z = 10[2f(a,b) - g(a,b)] + [f(a,b) + 2g(a,b]$$
$$w = 10[2f(a,b) - g(a,b)] - [f(a,b) + 2g(a,b].$$

**Properties**

1. Each of the following is a nasty number.

   (a) $3[T(1,1) - x(1,1) + y(1,1)]$

   (b) $3[T(ka,ka)]$

2. $z(a,1) - w(a,1) - 480F_{6,a,5} - 48F_{5,a,7} + 528F_{4,a,8} - 40SO_a - 2t_{4,a} + 41Gnomic_a \equiv 0 \pmod{43}$

3. $x(1,b)) + y(1,b) + 2880F_{6,b,3} - 17280F_{5,b,7} + 1824F_{4,b,5} - 174SO_b - 1432\Pr_b + 319Gnomic_b \equiv 0 \pmod 5$

4. $x(a,1) + T(4a,4a) - 48F_{6,a,5} + 1920F_{5,a,3} - 40t_{4,a^2} - 740P_a^5 - 342\Pr_a - 2t_{4,a} \equiv 0 \pmod 2$

**Case 1:** $\beta = 0\,,\ k = \alpha$ (9)

For this choice, the corresponding integer solutions of (1) are represented by

$$x = \alpha\left[f(a,b) + g(a,b)\right]$$
$$y = \alpha\left[f(a,b) - g(a,b)\right]$$
$$z = \alpha\left[2kf(a,b) + g(a,b)\right]$$
$$w = \alpha\left[2kf(a,b) - g(a,b)\right]$$
$$T = a^2 + b^2$$

**Case 2:** Assume $k = (i\alpha)(-i\alpha)$

By using similar calculations of pattern 1, the corresponding integral solutions of (1) are given by

$$x = \alpha\left[f(a,b) - g(a,b)\right]$$
$$y = -\alpha\left[f(a,b) + g(a,b)\right]$$
$$z = \alpha\left[f(a,b) - 2kg(a,b)\right]$$
$$w = -\alpha\left[f(a,b) + 2kg(a,b)\right]$$
$$T = a^2 + b^2$$

**Pattern 2**

It is worth to note that (3) can also be written as

$$u^2 + v^2 = kT^6 * 1 \tag{10}$$

write 1 as

$$1 = \frac{(1+i)^{2n}(1-i)^{2n}}{2^{2n}} \tag{11}$$

Using (3), (4) and (11) in (10),and employing the method of factorization ,define

$$u + iv = (\alpha + i\beta)(a + ib)^6\,\frac{(1+i)^{2n}}{2^n}$$
$$= (\alpha + i\beta)(a + ib)^6\left[\cos\frac{n\pi}{2} + i\sin\frac{n\pi}{2}\right] \tag{12}$$

Equating real and imaginary parts of (12) and using (2),the corresponding integral solutions of (1) are given by

$$x = \cos\frac{n\pi}{2}\left[(\alpha+\beta)f(a,b) + (\alpha-\beta)g(a,b)\right] + \sin\frac{n\pi}{2}\left[(\alpha-\beta)f(a,b) - (\alpha+\beta)g(a,b)\right]$$

$$y = \cos\frac{n\pi}{2}\left[(\alpha-\beta)f(a,b) - (\alpha+\beta)g(a,b)\right] - \sin\frac{n\pi}{2}\left[(\alpha+\beta)f(a,b) + (\alpha-\beta)g(a,b)\right]$$

$$z = 2k\left[\cos\frac{n\pi}{2}(\alpha f(a,b) - \beta g(a,b) - \sin\frac{n\pi}{2}(\alpha g(a,b) + \beta f(a,b)\right] +$$
$$\left[\cos\frac{n\pi}{2}(\beta f(a,b) + \alpha g(a,b) + \sin\frac{n\pi}{2}(\alpha f(a,b) - \beta g(a,b)\right]$$

$$w = 2k\left[\cos\frac{n\pi}{2}(\alpha f(a,b) - \beta g(a,b) - \sin\frac{n\pi}{2}(\alpha g(a,b) + \beta f(a,b)\right] -$$
$$\left[\cos\frac{n\pi}{2}(\beta f(a,b) + \alpha g(a,b) + \sin\frac{n\pi}{2}(\alpha f(a,b) - \beta g(a,b)\right]$$

$$T = a^2 + b^2$$

## CONCLUSION

In this paper, we have presented two different patterns of integer solutions to the octic equation (1). However, in (10) the integer 1 can also be represented by

$$1=\frac{\left((m^2-n^2)+i2mn\right)\left((m^2-n^2)-i2mn\right)}{(m^2+n^2)^2},\quad m,n>0$$

which leads to a different solution pattern to (1).To conclude one may search for other pattern of solutions and their corresponding to the considered octic equation with five unknowns.

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