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The approximate solutions of the partial differential evolution equation by algebraic dynamics algorithm

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Abstract: We apply a new form of the infinitesimal time translation operator in algebraic dynamical algorithm to KdV-Burgers equation and modified Bousinnesq and give their approximate solutions of at least second order.
Keywords: Algebraic dynamical; KdV-Burgers equation; modified Bousinnesq equation; sinh-Gordon equation

INTRODUCTION

In a series of papers[1-3], Wang and Zhang give the algebraic dynamics algorithm to solve nonlinear evolution equations. Liu[4] obtains an equivalent construction of infinitesimal time translation operator. Since Liu's operator involves only simple partial differential operations, this is rather convenient in practice.

In the present paper, we give three new applications of Liu's construction to the KdV-Burgers equation, the modified Bousinnesq equation and sinh-Gordon equation and give their second order approximate solutions.

CONSTRUCTIONS AND APPLICATIONS

For the general nonlinear evolution equation

$$u_t = H(u, u_x, \ldots), \tag{1}$$

the corresponding infinitesimal time translation operator is given by [4]

$$L = \int u_t(y,t)\frac{\delta}{\delta u}dy + \int u_{yt}(y,t)\frac{\delta}{\delta u_y}dy + \int u_{yyt}(y,t)\frac{\delta}{\delta u_{yy}}dy + \ldots, \tag{2}$$

In fact, when G is a function of u, u_x, u_{xx} the operator L has a more simple form

$$L = u_t\frac{\partial}{\partial u} + u_{xt}\frac{\partial}{\partial u_x} + u_{xxt}\frac{\partial}{\partial u_{xx}} + \ldots, \tag{3}$$

Example 1: Now we apply this operator to the KdV-Burgers equation

$$u_t = \alpha u_{xx} - \beta u_{xxx} - uu_x, \tag{4}$$

$$u(x,0) = F(x). \tag{5}$$

According to our construction, the infinitesimal time translation operator is given by

$$L = (\alpha u_{xx} - \beta u_{xxx} - uu_x)\frac{\partial}{\partial u} + (\alpha u_{xx} - \beta u_{xxx} - uu_x)_x\frac{\partial}{\partial u_x} + (\alpha u_{xx} - \beta u_{xxx} - uu_x)_{xx}\frac{\partial}{\partial u_{xx}} + \ldots, \tag{6}$$

Therefor, we have

$$LF = \alpha F_{xx} - \beta F_{xxx} - FF_x, \tag{7}$$

and the second approximate solution is

$$u(x,t) \approx L^2F = 3\beta F_{xx}^2 - 4\alpha F_xF_{xx} + 5\beta F_xF_{xxx} + 2FF_x^2 - 2\alpha FF_{xxx} + 2\beta FF_{xxxx} + F^2F_{xx} + \alpha^2F_{xxxx} - 2\alpha\beta F_{xxxxx} + \beta^2F_{xxxxxx}. \tag{8}$$

Example 2. The Boussinesq equations read as

$$u_t = -(uv)_x - v_{xxx}, \tag{9}$$

$$u_t = -u_x - vv_x, \tag{10}$$

$$u(x,0) = F(x), v(x,0) = K(x). \tag{11}$$

By Liu's method, the infinitesimal time translation operator is given by

$$L = (-(uv)_x - v_{xxx})\frac{\partial}{\partial u} + (-u_x - vv_x)\frac{\partial}{\partial v} + (-(uv)_x - v_{xxx})_x \frac{\partial}{\partial u_x} + (-u_x - vv_x)_x \frac{\partial}{\partial v_x}$$

$$+(-(uv)_x - v_{xxx})_{xx}\frac{\partial}{\partial u_{xx}} + (-u_x - vv_x)_{xx}\frac{\partial}{\partial v_{xx}} \tag{12}$$

Therefore we give LF, L^2F and so on as follows

$$LF = -(FK)_x - K_{xxx}, \tag{13}$$

$$LK = -F_x - KK_x, \tag{14}$$

and the second approximate solutions are

$$u \approx L^2F = 4KK_xF_x + 2FK_x^2 + 5K_xK_{xxx} + K^2F_{xx} + 2KFK_{xx} + 2KK_{xxxx} + F_x^2 + FF_{xx} + 3K_{xx}^2 + F_{xxxx}. \tag{15}$$

$$v \approx L^2K = K_{xxx} + K^2K_{xx} + 2KF_{xx} + 3F_xK_x + 2KK_x^2 \tag{16}$$

Example 3. Sinh-Gordon equation is as follows

$$u_{tt} = u_{xx} - \sinh u, \tag{17}$$

$$u(x,0) = F(x); u_t(x,0) = K(x). \tag{18}$$

Letting $u_t = v$ gives

$$u_t = v \tag{19}$$

$$v_t = u_{xx} - \sinh u, \tag{20}$$

$$u(x,0) = F(x); v(x,0) = K(x). \tag{21}$$

Its the infinitesimal time translation operator is given by

$$L = v\frac{\delta}{\delta u} + (u_{xx} - \sinh u)\frac{\partial}{\partial u} + v_x\frac{\partial}{\partial u_x} + (u_{xx} - \sinh u)_x\frac{\delta}{\delta v_x} + v_{xx}\frac{\partial}{\partial u_{xx}}$$

$$+(u_{xx} - \sinh u)_{xx}\frac{\partial}{\partial v_{xx}} + \cdots. \tag{22}$$

By the above operator, we have

$$LF = K, \tag{23}$$

$$L^2F = LK = F_{xx} - \sinh F, \tag{24}$$

$$L^3F = L(F_{xx} - \sinh F) = -K\cosh F + K_{xx}, \tag{25}$$

$$L^4F = F_{xxxx} - K^2\sinh F + \cosh F\sinh F - F_x^2\sinh F - 2F_{xx}\cosh F. \tag{26}$$

Therefor, we give the following forth order approximate solution to sinh-Gordon

$$u(x,t) = F(x) + K(x)t + (F_{xx} - \sin F)\frac{t^2}{2} + (K_{xx} - K\cos F)\frac{t^3}{6}$$

$$+\{F_{xxxx} - K^2\sinh F + \cosh F\sinh F - F_x^2\sinh F - 2F_{xx}\cosh F\}\frac{t^4}{24} + \cdots. \tag{27}$$

We use Liu's equivalent construction of infinitesimal time translation operator in the algebraic dynamics for nonlinear evolution equations to give KdVBurgers equation, the modofied Boussinesq equation and sinh-Gordon equation.It is easy to see this method is very simple and direct.

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