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Graph Labelling of A Topological Space Graph

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Abstract: Rosa, A [5] introduced the concept of labelling of a graph. The main aim of this paper is to study the concept of labelling of a graph by using topological space graph. Also we proved some fundamental results on this concept. .
Keywords: Graph, Topological graph, Topological space graph, labelling of a graph.

INTRODUCTION

Given a non-empty set X and a topology τ on X, we define a graph whose vertices are members of τ and arcs are defined among vertices if one node is included in another one which is known as "Topological space graph". The vertex u of a topological space graph $G_X(\tau)$ is labelled with a positive integer n where n-cardinality of and the edge $<u,v>$ of a topological space graph $G_X(\tau)$ is labelled with a positive integer n, where n=cardinality of $u \cup v$. This construction is of interest in the context of labelling of a graph analysis.

Topological spaces. A topology T on a set X is a family of subsets of X with the following properties:

(1) Any union of elements of T is in T.
(2) Any finite intersection of elements of T is in T.
(3) ϕ and X are in T .

Then (X, T) is called a topological space.

PRELIMINARIES: NOTATION, DEFINITIONS, AND SOME SIMPLE FACTS

In this section we have given the preliminary definitions and some examples from the existing literature**.**

Definition: A graph G is a triple consisting of a vertex set V(G), an edge set E(G), and a relation that associates with each edge two vertices (not necessarily distinct) called its end points.

Example (Königsberg bridge problem): The city of Königsberg (now Kaliningrad) used to have seven bridges across the river, linking the banks with two islands. The people living in Königsberg had a game where they would try to walk across each bridge once and only once. You can chose where to start.

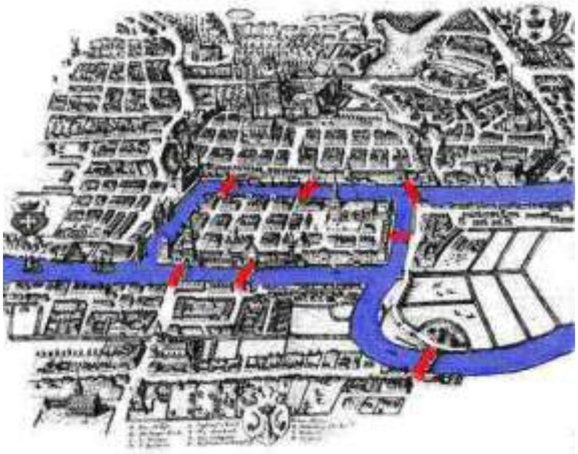

Diagram - 1

Solution:

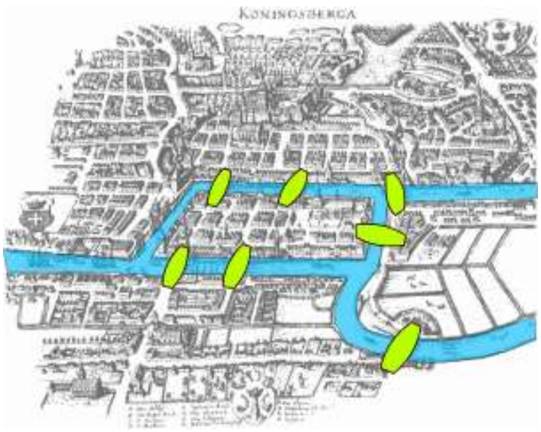

Diagram – 2

Back in the 18th century in the Prussian city of Königsberg, a river ran through the city and seven bridges crossed the forks of the river. The river and the bridges are highlighted in the picture to the right[1].

As a weekend amusement, townsfolk would see if they could find a route that would take them across every bridge once and return them to where they started.

Leonard Euler (pronounced OY-lur), one of the most prolific mathematicians ever, looked at this problem in 1735, laying the foundation for graph theory as a field in mathematics. To analyze this problem, Euler introduced edges representing the bridges:

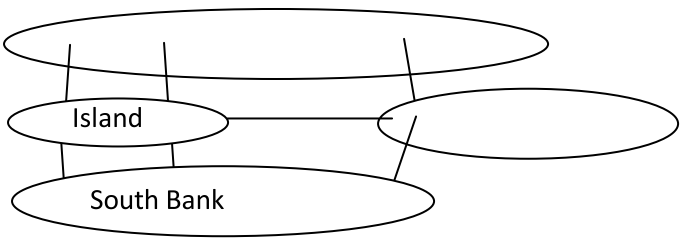

Diagram – 3

Since the size of each land mass it is not relevant to the question of bridge crossings, each can be shrunk down to a vertex representing the location:

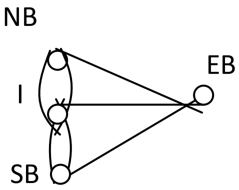

Diagram – 4

Notice that in this graph there are *two* edges connecting the north bank and island, corresponding to the two bridges in the original drawing. Depending upon the interpretation of edges and vertices appropriate to a scenario, it is entirely possible and reasonable to have more than one edge connecting two vertices.

The following lemma collects some obvious observations whose verification (partly indicated in the lemma) is left to the reader:

2.3 Lemma : Let u, v be any two subsets of a universal set X . Then
(i) $u \cup v = u \text{ or } v \iff u \cap v = v \text{ or } u.$
(ii) $u \subseteq v \iff u \cup v = v \iff u \cap v = u.$
(iii) $\emptyset \subseteq u \; \& \; u \subseteq X.$

THE GRAPH LABELLING ON $G_X(\tau)$

To state our main results on a arbitrary topological space τ on a non-empty set X.

Definitions (i) Let X be a non-empty set, τ be an arbitrary topology on X. A graph $G_X(\tau)$ whose vertex set is τ and two vertices u, v of $G_X(\tau)$ are adjacent if and only if $u \cup v = u \text{ or } v$. This graph is called "**Topological space graph"** .It is denoted by $G_X(\tau)$.

(ii) Let X be the set of vertices and ***E*** the set of ***edges*** of G. For each x ∈ X, ***d(x)*** is the number of edges containing x. We call ***d(x)*** the degree of the vertex. Sometimes it is also known as **valency** of the vertex x.

Definition: A **graph labeling** is the assignment of labels, traditionally represented by integers, to the edges or vertices, or both, of a graph

Result . In any topological space graph $G_X(\tau)$, the vertex labelled value is at least 0 and at most $O(X)$

Proof : $\phi, X \in V(G_X(\tau)) \Rightarrow \phi \subseteq v \subseteq X, \forall v \in V(G_X(\tau)) \Rightarrow |\phi| \le |v| \le |X|$

$\Rightarrow 0 \le |v| \le |X|, \forall v \in V(G_X(\tau))$.

Hence, the vertex labelled value is atleast 0 and atmost $O(X)$ in $G_X(\tau)$.

Result : In any topological space graph $G_X(\tau)$, the edge labelled value is at least one and atmost $O(X)$

Proof : $< u, v >$ in an edge in $G_X(\tau)$
$\Rightarrow u \cup v = u \text{ or } v$
$\Rightarrow \phi \cup u \subseteq u \cup v \subseteq X \cup u$ or $\phi \cup v \subseteq u \cup v \subseteq X \cup v$
$\Rightarrow |\phi \cup u| \le |u \cup v| \le |X \cup u|$ or $|\phi \cup v| \le |u \cup v| \le |X \cup v|$
$\Rightarrow |u| \le |u \cup v| \le |X \cup u| \text{ or } |v| \le |u \cup v| \le |X \cup v|$

$\Rightarrow 1 \leq |u| \leq |u \cup v| \leq |X|$ or $1 \leq |u| \leq |u \cup v| \leq |X|$ $(\because \forall v \notin \phi \in \tau \Rightarrow |v| \geq 1)$

$\Rightarrow 1 \leq |u \cup v| \leq |X|$

$\Rightarrow 1 \leq |< u, v >| \leq 1$ for every $< u, v >$ is $G_X(\tau)$.

Result: The edge labelling $< u, v >$ is $G_X(\tau)$ is the max of labelled values end nodes u, v.

Proof : Labelling of $< u, v >$ in $G_X(\tau)$ is $|u \cup v|$

$\Rightarrow |u \cup v| = |u|$ or $|v|$

Since $< u, v >$ is an edge in $G_X(\tau)$

$\Rightarrow u \cup v = u$ or $v \Rightarrow |u \cup v| = Max\{|u|, |v|\}, (\because u \cup v \supseteq u, v)$

$\Rightarrow$ labelling of $< u.v >$ is the Max of labelled values end nodes u, v.

Result : In any topological space graph, the sum of the labelling values of the edges which are incident at ϕ is less than or equal to the sum of the labelling values of the edges which are incident at other nodes.

Proof : From known result, we have that , $\deg(\phi) = \deg(X)$ in $G_X(\tau)$

Also we know that ϕ is adjacent to all $v \in \tau$ and X is adjacent to all $v \in \tau$.

Thus $< \phi, u >$ is an edge in $G_X(\tau), \forall v \in \tau$ and $< x, u >$ is an edge in $G_X(\tau), \forall v \in \tau$

Let $\tau = \{\phi = v_1, v_2 ......, v_n = X\}$

Then $< \phi, v_2 >, < \phi, v_3 >, ... < \phi, v_n >$ and $< X, v_2 >, < X, v_3 >, ... < X, v_n >$ are edges in $G_X(\tau)$.

Thus, $|\phi \cup v_2| + |\phi \cup v_3| + ... + |\phi \cup v_{n-1}| + |\phi \cup v_n| \leq |X \cup \phi| + |X \cup v_2| + ... + |X \cup v_{n-1}|$

(since $\phi \cup v = u, \forall u \in \tau, X \cup u = X, \forall u \in \tau$)

$\Rightarrow |v_2| + |v_3| + ... |v_{n-1}| + |v_n = X| \leq |X| + |X| + ... + |X|$ $\quad (n-1)$ times

$\Rightarrow |v_2| + (v_3) + ... + |v_{n-1}| + 1 \times 1 \leq (n-1)|X|$.

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