

## Solving Fully Fuzzy Critical Path Analysis in Project Networks Using Linear Programming Problems

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**Abstract:** A new method for finding fuzzy optimal solution, the maximum total completion fuzzy time and fuzzy critical path for the given fully fuzzy critical path (FFCP) problems using crisp linear programming (LP) problem is proposed. In this proposed method, all the parameters are represented by triangular fuzzy number. The fuzzy optimal solution of the FFCP problems obtained by the proposed method, do not contain any negative part of the values of the fuzzy decision variables. This paper will present with great clarity of the proposed method and illustrate its application to FFCP problems occurring in real life situations.

**Keywords:** Fully fuzzy critical path problem, linear programming problem, fuzzy linear programming problem, Bound and Decomposition method

### 1. INTRODUCTION

Critical path method (CPM) techniques have become widely recognized as valuable tools for the planning and scheduling of large projects. CPM is the one from the start of the project to finish of project where the slack times are all zeros. The purpose of the CPM is to identify critical activities on the critical path so that resources may be concentrated on these activities in order to reduce project length time. The successful implementation of CPM requires the availability of clear determined time duration for each of the activity. However, in practical situations this requirement is usually hard to fulfill, since many of activities will be executed for the first time. To deal with such real life situations, Zadeh [1] introduced the concept of fuzzy set. Since there is always uncertainty about the time duration of activities in the network planning, due to which fuzzy critical path method (FCPM) was proposed since the late 1970s.

For finding the fuzzy critical path, several approaches are proposed over the past years. Gazdik[2] have developed a fuzzy network of unknown project to estimate the activity durations and used fuzzy algebraic operations to find the duration of the project and its critical path. A chapter of the book Kaufmann and Gupta [3] devoted to the CPM in which activity times are represented by triangular fuzzy numbers. Cahon and Lee [4] have developed a new approach to calculate the fuzzy completion project time. Nasution [5] have proposed a method to compute total floats and the critical paths in a fuzzy project network. Yao and Lin [6] have introduced a method for ranking fuzzy numbers without the need for any assumptions and have used both positive and negative fuzzy values to define ordering which is then applied to fuzzy project network. Dubois et al [7] have extended the fuzzy arithmetic operational model to calculate the latest starting time of each activity in a fuzzy project network. Chen[8] have proposed an approach based on the extension principle and linear programming formulation to critical path analysis in fuzzy project networks. Chen and Hsueh [9] have presented a simple approach to solve the CPM problems with fuzzy activity times (being fuzzy numbers) on the basis of the linear programming formulation and the fuzzy number ranking method that are more realistic than crisp ones. Yakhchali and Ghodspour [10] have introduced the problems of determining possible values of earliest and latest starting times of an activity in networks with minimal time lags and imprecise durations that are represented by means of interval or fuzzy numbers. Amit Kumar and Parmpreet Kaur [11] have proposed a method to find fuzzy optimal solution for FFCP with a new representation of triangular fuzzy numbers with the help of fuzzy linear programming model. Ravi Shankar et al.[12] have introduced a new defuzzification formula for trapezoidal fuzzy number and apply to the float time (slack time) for each activity in the fuzzy project network to find the critical path. Usha Madhuri et al.[13] have proposed a new fuzzy linear programming model is proposed to find fuzzy critical path and fuzzy completion time of a fuzzy project.

**2. PRELIMINARIES**

We need the following mathematical orientated definitions of fuzzy set, fuzzy number and membership function and also, definitions of basic arithmetic operation on fuzzy triangular numbers which can be found in Zadeh [1] and Kaufmann and Gupta [14].

Definition 2.1 Let A be a classical set and  $\mu_A(x)$  be a membership function from A to [0,1]. A fuzzy set  $\tilde{A}$  with the membership function  $\mu_{\tilde{A}}(x)$  is defined by

$$\tilde{A} = \{ (x, \mu_A(x)) : x \in A \text{ and } \mu_A(x) \in [0,1] \}.$$

Definition 2.2 A real fuzzy number  $\tilde{a} = (a_1, a_2, a_3)$  is a fuzzy subset from the real line R with the membership function  $\mu_{\tilde{a}}(a)$  satisfying the following conditions:

- (i)  $\mu_{\tilde{a}}(a)$  is a continuous mapping from R to the closed interval [0, 1],
- (ii)  $\mu_{\tilde{a}}(a) = 0$  for every  $a \in (-\infty, a_1]$ ,
- (iii)  $\mu_{\tilde{a}}(a)$  is strictly increasing and continuous on  $[a_1, a_2]$ ,
- (iv)  $\mu_{\tilde{a}}(a) = 1$  for every  $a \in [a_2, a_3]$ ,
- (v)  $\mu_{\tilde{a}}(a)$  is strictly decreasing and continuous on  $[a_2, a_3]$  and
- (vi)  $\mu_{\tilde{a}}(a) = 0$  for every  $a \in [a_3, +\infty]$ .

Definition 2.3A fuzzy number  $\tilde{a}$  is a triangular fuzzy number denoted by  $(a_1, a_2, a_3)$  where  $a_1, a_2$  and  $a_3$  are real numbers and its membership function  $\mu_{\tilde{a}}(x)$  is given below:

$$\mu_{\tilde{a}}(x) = \begin{cases} (x - a_1)/(a_2 - a_1) & \text{for } a_1 \leq x \leq a_2 \\ (a_3 - x)/(a_3 - a_2) & \text{for } a_2 \leq x \leq a_3 \\ 0 & \text{otherwise} \end{cases}$$

where  $a_1 \leq a_2 \leq a_3$ .

Definition 2.4 Let  $(a_1, a_2, a_3)$  and  $(b_1, b_2, b_3)$  be two triangular fuzzy numbers. Then

- (i)  $(a_1, a_2, a_3) \oplus (b_1, b_2, b_3) = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$ .
- (ii)  $(a_1, a_2, a_3) \ominus (b_1, b_2, b_3) = (a_1 - b_3, a_2 - b_2, a_3 - b_1)$ .
- (iii)  $k(a_1, a_2, a_3) = (ka_1, ka_2, ka_3)$ , for  $k \geq 0$ .
- (iv)  $k(a_1, a_2, a_3) = (ka_3, ka_2, ka_1)$ , for  $k < 0$ .
- (v)  $(a_1, a_2, a_3) \otimes (b_1, b_2, b_3) = \begin{cases} (a_1 b_1, a_2 b_2, a_3 b_3), & a_1 \geq 0, \\ (a_1 b_3, a_2 b_2, a_3 b_3), & a_1 < 0, a_3 \geq 0, \\ (a_1 b_3, a_2 b_2, a_3 b_1), & a_3 < 0. \end{cases}$

Let  $F(R)$  be the set of all real triangular fuzzy numbers.

Definition 2.5 Let  $\tilde{A} = (a_1, a_2, a_3)$  and  $\tilde{B} = (b_1, b_2, b_3)$  be in  $F(R)$ , then

- (i)  $\tilde{A} \approx \tilde{B}$  iff  $a_i = b_i, i = 1, 2, 3$ ; (ii)  $\tilde{A} \preceq \tilde{B}$  iff  $a_i \leq b_i, i = 1, 2, 3$
- (iii)  $\tilde{A} \succeq \tilde{B}$  iff  $a_i \geq b_i, i = 1, 2, 3$  and  $\tilde{A} \succeq \tilde{0}$  iff  $a_i \geq 0, i = 1, 2, 3$ .

Definition 2.6 Let  $\tilde{A} = (a_1, a_2, a_3)$  be in  $F(R)$ .  $\tilde{A}$  is said to be positive if  $a_i \geq 0, \forall i = 1$  to 3.

Definition 2.7 Let  $\tilde{A} = (a_1, a_2, a_3)$  be in  $F(R)$ .  $\tilde{A}$  is said to be integer if  $a_i \geq 0, \forall i = 1$  to 3 are integers.

Consider the following crisp linear programming problems with  $m$  inequality/equality constraints and  $n$  variables may be formulated as follows:

$$\begin{aligned} &\text{Maximize } Z = c^T X \\ &\text{subject to } A \otimes X \{ \leq, =, \geq \} b, \\ & \quad \quad \quad x \geq 0, \end{aligned}$$

where  $c^T = (c_j)_{1 \times n}$  is a nonnegative crisp vector,  $A = (a_{ij})_{m \times n}$  is a nonnegative real crisp matrix and  $X = (x_j)_{n \times 1}$  and  $B = (b_i)_{m \times 1}$  are nonnegative real vectors for all  $1 \leq i \leq m$  and  $1 \leq j \leq n$ .

### 3. LP FORMULATION OF CRISP CRITICAL PATH AND FULLY FUZZY CRITICAL PATH PROBLEMS

The CPM is a network-based method designed to support in the planning, scheduling and control of the project. Its objective is to construct the time scheduling for the project. Two basic results provided by CPM are the total duration time needed to complete the project and the critical path. One of the efficient approaches for finding the critical paths and total duration time of project networks is LP. The LP formulation assumes that a unit flow enters the project network at the start node and leaves at the finish node. In this section the LP formulation of CCP problems is reviewed and also the LP formulation of FFCP problems is proposed.

The linear programming model discussed in the book written by Taha[15] is reviewed in this section which can be found in Amit Kumar and Parmpreet Kaur [11]. Consider a project network  $G = (N, A)$  consisting of a finite set  $N = \{1, 2, \dots, n\}$  of  $n$  nodes (events) and  $A$  is the set of activities  $(i, j)$ . Denote  $t_{ij}$  as the time period of activity  $(i, j)$ .

The LP formulation of crisp critical path problems is as follows:

$$\begin{aligned} &\text{Maximize} \\ & \sum_{(i, j) \in A} t_{ij} x_{ij} \\ & \text{Subject to} \\ & \sum_{j: (i, j) \in A} x_{ij} = 1, \\ & \sum_{i: (i, j) \in A} x_{ij} = \sum_{j: (j, k) \in A} x_{ij}, \quad i \neq 1, k \neq n, \\ & \sum_{i: (i, n) \in A} x_{in} = 1, \end{aligned}$$

$x_{ij}$  is the decision variable denoting the amount of flow in activity  $\forall (i, j) \in A$ ,  $t_{ij}$  is the time duration of activity  $(i, j)$  and the constraints represents the conservation of flow at each node.

There are several real life problems in which a decision maker may be uncertain about the precise values of activity time. Suppose time parameters  $t_{ij}$  and  $x_{ij}$ ,  $\forall (i, j) \in A$  are imprecise and are represented by fuzzy numbers  $\tilde{t}_{ij}$  and  $\tilde{x}_{ij}$ ,  $\forall (i, j) \in A$  respectively. Then the FFCP problems may be formulated into the fuzzy linear programming (FLP) problem:

$$\begin{aligned} &\text{Maximize} \\ & \sum_{(i, j) \in A} \tilde{t}_{ij} \otimes \tilde{x}_{ij} \\ & \text{Subject to} \\ & \sum_{j: (i, j) \in A} \tilde{x}_{ij} = \tilde{1}, \\ & \sum_{i: (i, j) \in A} \tilde{x}_{ij} = \sum_{j: (j, k) \in A} \tilde{x}_{ij}, \quad i \neq 1, k \neq n, \\ & \sum_{i: (i, n) \in A} \tilde{x}_{in} = \tilde{1}, \end{aligned}$$

$\tilde{x}_{ij}$  is a non-negative real number  $\forall (i, j) \in A$ .

Suppose all the parameters  $\tilde{t}_{ij}$  and  $\tilde{x}_{ij}$  are represented by  $(a, b, c)$  type triangular fuzzy numbers  $(t_{ij}, t'_{ij}, t''_{ij})$  and  $(x_{ij}, y_{ij}, z_{ij})$  are respectively then the LP formulation of FFCP problems, proposed in the Section 2, may be written as:

Maximize

$$\sum_{(i, j) \in A} (t_{ij}, t'_{ij}, t''_{ij}) \otimes (x_{ij}, y_{ij}, z_{ij})$$

Subject to

$$\sum_{j: (i, j) \in A} (x_{1j}, y_{1j}, z_{1j}) \approx (1, 1, 1),$$

$$\sum_{i: (i, j) \in A} (x_{ij}, y_{ij}, z_{ij}) \approx \sum_{j: (j, k) \in A} (x_{jk}, y_{jk}, z_{jk}), \quad i \neq 1, k \neq n,$$

$$\sum_{i: (i, n) \in A} (x_{in}, y_{in}, z_{in}) \approx (1, 1, 1),$$

$(x_{ij}, y_{ij}, z_{ij})$  is a non-negative triangular fuzzy number  $\forall (i, j) \in A$ .

Now, since  $(x_{ij}, y_{ij}, z_{ij})$  is a triangular fuzzy number, then

$$x_{ij} \leq y_{ij} \leq t_{ij}, \quad j = 1, 2, \dots, n \tag{3.1}$$

The relation (3.1) is called bounded constraints.

Jayalakshmi and Pandian [16] have proposed a method namely, bound and decomposition method to a fully fuzzy linear programming (FFLP) problem to obtain an optimal fuzzy solution. Now, by using Bound and Decomposition method, the above FFCP problems can be decomposed into three crisp LP problems namely, middle level problem (MLP), upper level problem (ULP) and lower level problem (LLP) as follows:

(MLP) Maximize  $Z_2 = \sum_{j=1}^n$  middle value of  $\left( \sum_{(i, j) \in A} (t_{ij}, t'_{ij}, t''_{ij}) \otimes (x_{ij}, y_{ij}, z_{ij}) \right)$   
subject to

Constraints in the decomposition problem in which at least one decision variable of the (MLP) occurs and all decision variables are non-negative integers.

(ULP) Maximize  $Z_3 = \sum_{j=1}^n$  upper value of  $\left( \sum_{(i, j) \in A} (t_{ij}, t'_{ij}, t''_{ij}) \otimes (x_{ij}, y_{ij}, z_{ij}) \right)$   
subject to

$$\sum_{j=1}^n \text{upper value of } \left( \sum_{(i, j) \in A} (t_{ij}, t'_{ij}, t''_{ij}) \otimes (x_{ij}, y_{ij}, z_{ij}) \right) \geq Z_2^0;$$

Constraints in the decomposition problem in which at least one decision variable of the (ULP) occurs and are not used in (MLP); all variables in the constraints and objective function in (ULP) must satisfy the bounded constraints ; replacing all values of the decision variables which are obtained in (MLP) and all decision variables are nonnegative integers.

and

(LLP) Minimize  $Z_1 = \sum_{j=1}^n$  lower value of  $\left( \sum_{(i, j) \in A} (t_{ij}, t'_{ij}, t''_{ij}) \otimes (x_{ij}, y_{ij}, z_{ij}) \right)$   
subject to

$$\sum_{j=1}^n \text{lower value of } \left( \sum_{(i, j) \in A} (t_{ij}, t'_{ij}, t''_{ij}) \otimes (x_{ij}, y_{ij}, z_{ij}) \right) \leq Z_2^0$$

Constraints in the decomposition problem in which at least one decision variable of the (LLP) occurs which are not used in (MLP) and (ULP) ; all variables in the constraints and objective function in (LLP) must satisfy the

bounded constraints; replacing all values of the decision variables which are obtained in (MLP) and (ULP) and all decision variables are nonnegative integers.

where  $Z_2^{\circ}$  is the optimal objective value of (MLP).

Now, we propose a new algorithm to find the fuzzy optimal solution, the maximum total completion fuzzy time and fuzzy critical path for FFCP problems.

The steps of the proposed method are as follows:

**STEP 1:** Formulate the given FFCP problems into FFLP problems.

**STEP2:** Using Bound and Decomposition method, construct (MLP), (ULP) and (LLP) problems from FFLP obtained in Step 1.

**STEP 3:** Using existing linear programming technique, solve the (MLP) problem, then the (ULP) problem and then, the (LLP) problem in the order only and obtain the values of all real decision variables  $x_{ij}, y_{ij}$  and  $t_{ij}$  and values of all objectives are  $Z_1, Z_2$  and  $Z_3$ . Let the decision variables values be  $x_{ij}^{\circ}, y_{ij}^{\circ}$  and  $t_{ij}^{\circ}, \forall (i, j) \in A$  and objective values be  $Z_1^{\circ}, Z_2^{\circ}$  and  $Z_3^{\circ}$ .

**STEP 4:** The fuzzy optimal solution to the FFLP problems is  $\tilde{x}_{ij}^{\circ} = (x_{ij}^{\circ}, y_{ij}^{\circ}, t_{ij}^{\circ}), \forall (i, j) \in A$  and the maximum fuzzy objective is  $(Z_1^{\circ}, Z_2^{\circ}, Z_3^{\circ})$ .

**STEP 5:** Find the fuzzy critical path by combining all the activities  $(i, j)$  such that  $\tilde{x}_{ij} \approx (1, 1, 1)$  and the corresponding maximum total completion fuzzy time is the maximum fuzzy objective value obtained in Step 4.

**REMARKS 3.1** In the case of FFCP problems involving trapezoidal fuzzy numbers and variables decompose it into four crisp LP problems and then, we solve the middle level problems (second and third problems) first. Then, solve the upper level and lower level problems and then, the fuzzy critical path and maximum total completion fuzzy time is obtained involving trapezoidal fuzzy numbers and variables.

Now, the proposed method for finding fuzzy critical path and maximum total completion fuzzy time involving triangular fuzzy number for FFCP problem is illustrated using the following numerical examples.

**EXAMPLE 3.1** The problem is to find the fuzzy critical path and maximum total completion fuzzy time of the project network, shown in Figure 3.1, in which the fuzzy time duration of each activity is represented by a triangular fuzzy numbers

$$\tilde{t}_{12} \approx (3, 4, 5) ; \tilde{t}_{23} \approx (2.8, 3, 3.2) ; \tilde{t}_{24} \approx (4, 5, 6) ; \tilde{t}_{34} \approx (1.8, 2, 2.2)$$

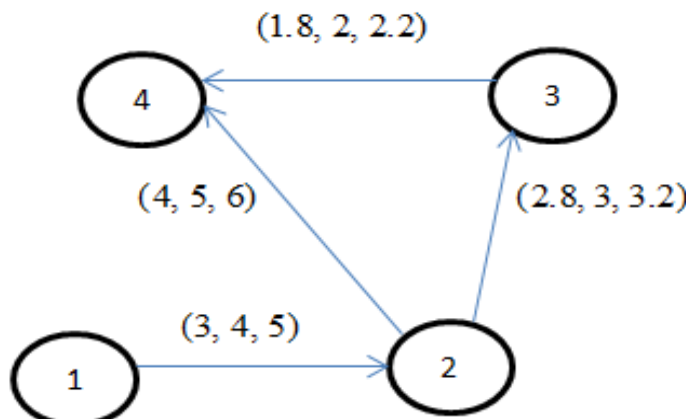


Fig 3.1

Now we can convert the given FFCP problem into linear programming problem as:  
 Maximize  $Z = [(3, 4, 5) \otimes (x_{12}, y_{12}, z_{12}) \oplus (2.8, 3, 3.2) \otimes (x_{23}, y_{23}, z_{23}) \oplus (4, 5, 6) \otimes (x_{24}, y_{24}, z_{24})$   
 $(1.8, 2, 2.2) \otimes (x_{34}, y_{34}, z_{34})]$  subject

to

$$\begin{aligned} (x_{12}, y_{12}, z_{12}) &= (1, 1, 1); \\ (x_{23}, y_{23}, z_{23}) \oplus (x_{24}, y_{24}, z_{24}) &= (x_{12}, y_{12}, z_{12}); \\ (x_{34}, y_{34}, z_{34}) &= (x_{23}, y_{23}, z_{23}); \\ (x_{24}, y_{24}, z_{24}) \oplus (x_{34}, y_{34}, z_{34}) &= (1, 1, 1); \\ (x_{12}, y_{12}, z_{12}), (x_{23}, y_{23}, z_{23}), (x_{24}, y_{24}, z_{24}), (x_{34}, y_{34}, z_{34}) &\text{are non-negative triangular fuzzy numbers.} \end{aligned}$$

Now, using Bound and Decomposition method, the above LP can be decomposed into three crisp LP problems.

Now, the problem (P2) is given below:

Maximize  $Z_2 = 4y_{12} + 3y_{23} + 5y_{24} + 2y_{34}$

Subject to

$$\begin{aligned} y_{12} &= 1; \\ y_{23} + y_{24} &= y_{12}; \\ y_{34} &= y_{23}; \\ y_{24} + y_{34} &= 1; \\ y_{12}, y_{23}, y_{24}, y_{34} &\geq 0. \end{aligned}$$

Solving the problem (P2) by linear programming technique, the optimal solution is  $y_{12} = 1; y_{23} = 1; y_{24} = 0; y_{34} = 1$  and Maximize  $Z_2 = 9$  and the alternate solution is  $y_{12} = 1; y_{23} = 0; y_{24} = 1; y_{34} = 0$  and Maximize  $Z_2 = 9$ .

Now, the problem (P3) is given below:

Maximize  $Z_3 = 5z_{12} + 3.2z_{23} + 6z_{24} + 2.2z_{34}$

Subject to

$$\begin{aligned} 5z_{12} + 3.2z_{23} + 6z_{24} + 2.2z_{34} &\geq Z_2^0 \\ z_{12} = 1; z_{23} + z_{24} - z_{12} = 0; z_{34} - z_{23} = 0; z_{24} + z_{34} &= 1; \\ z_{12} \geq y_{12}; z_{23} \geq y_{23}; \\ z_{24} \geq y_{24}; z_{34} \geq y_{34}; \\ z_{12}, z_{23}, z_{24}, z_{34} \geq 0 \text{ and } y_{12}, y_{23}, y_{24}, y_{34} &\geq 0. \end{aligned}$$

Solving the problem (P3) by linear programming technique with  $y_{12} = 1; y_{23} = 1; y_{24} = 0; y_{34} = 1$  and  $Z_2 = 9$ , the optimal solution is  $z_{12} = 1, z_{23} = 1, z_{24} = 0, z_{34} = 1$  and Maximum  $Z_3 = 10.40$  and its alternate solution is  $z_{12} = 1, z_{23} = 0, z_{24} = 1, z_{34} = 0$  and Maximize  $Z_3 = 11$ .

Now, the problem (P1) is given below:

Maximize  $Z_1 = 3x_{12} + 2.8x_{23} + 4x_{24} + 1.8x_{34}$

Subject to

$$\begin{aligned} 3x_{12} + 2.8x_{23} + 4x_{24} + 1.8x_{34} &\leq Z_2^0 \\ x_{12} = 1; x_{23} + x_{24} - x_{12} = 0; x_{34} - x_{23} = 0; x_{24} + x_{34} &= 1; \end{aligned}$$

$$\begin{aligned}
 &x_{12} \leq y_{12} ; x_{23} \leq y_{23} ; \\
 &x_{24} \leq y_{24} ; x_{34} \leq y_{34} ; \\
 &x_{12}, x_{23}, x_{24}, x_{34} \geq 0 \text{ and } y_{12}, y_{23}, y_{24}, y_{34} \geq 0.
 \end{aligned}$$

Solving the problem (P1) by linear programming technique with  $y_{12} = 1; y_{23} = 1; y_{24} = 0; y_{34} = 1$  and  $Z_2 = 9$ , the optimal solution is  $x_{12} = 1; x_{23} = 1; x_{24} = 0; x_{34} = 1$  and Maximum  $Z_1 = 7.6$  and the alternate solution is  $x_{12} = 1; x_{23} = 0; x_{24} = 1; x_{34} = 0$  and Maximize  $Z_1 = 7$ .

Therefore, the fuzzy optimal solution is  $\tilde{x}_{12} \approx (1, 1, 1)$ ,  $\tilde{x}_{23} \approx (1, 1, 1)$ ,  $\tilde{x}_{24} \approx (0, 0, 0)$ ,  $\tilde{x}_{34} \approx (1, 1, 1)$ , the fuzzy critical path is  $1 \rightarrow 2 \rightarrow 3 \rightarrow 4$  and the maximum total completion fuzzy time is  $(7.6, 9, 10.4)$ .

Using the alternative fuzzy optimal solution  $\tilde{x}_{12} \approx (1, 1, 1)$ ,  $\tilde{x}_{23} \approx (0, 0, 0)$ ,  $\tilde{x}_{24} \approx (1, 1, 1)$ ,  $\tilde{x}_{34} \approx (0, 0, 0)$ , the fuzzy critical path is  $1 \rightarrow 2 \rightarrow 4$  and the maximum total completion fuzzy time is  $(7, 9, 11)$ .

Hence, in this problem, the fuzzy critical paths are  $1 \rightarrow 2 \rightarrow 3 \rightarrow 4$  and  $1 \rightarrow 2 \rightarrow 4$ , the corresponding maximum time completion fuzzy time are  $(7.6, 9, 10.4)$  and  $(7, 9, 11)$  respectively.

#### 4. CONCLUSION

A new method has been proposed to find the fuzzy critical path and fuzzy completion time of a fuzzy project using crisp LP problems. The advantage of the proposed method is that the values of the fuzzy decision variables do not contain any negative part and fuzzy ranking functions were not used. Since, the proposed method is based only on crisp LP problem it is very easy to solve FFCP problems having more number of fuzzy variables with help of existing computer software.

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