Scholars Journal of Physics, Mathematics and Statistics
Sch. J. Phys. Math. Stat. 2015; 2(3A):323-324

ISSN 2393-8056 (Print)
ISSN 2393-8064 (Online)

The Digital Features of the Function of Order Statistics

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Abstract: In this paper, we deduce the digital features of the function of order statistics by the probability density function of order statistics and the properties of the Euler integral.
Keywords: Order statistics, Distribution function, Expectation, The variance.

The probability density function of order statistics

By the literature [1] [2],we have the following lemma.

Lemma

Let the distribution function of population X is $F(x)$ and the corresponding probability density function is $f(x)$.Thus the probability density function of $X_{(k)}$ is

$$g_k(x)=\frac{n!}{(k-1)!(n-k)!}[F(x)]^{k-1}[1-F(x)]^{n-k}f(x) \qquad (1)$$

Theorem Let $X_1, X_2, \cdots, X_n$ independent and identically distribution and their common distribution function is $F(x)$.The order statistics of $X_1, X_2, \cdots, X_n$ is $X_{(1)} \le X_{(2)} \le \cdots \le X_{(n)}$. If $F(x)$ is continuous function, then

$$E[F(X_i)]=\frac{i}{n+1}, \quad D[F(X_{(i)})]=\frac{i[n-i+1]}{(n+1)^2(n+2)} \qquad (2)$$

Proof $E[F(X_i)]=\int_{-\infty}^{+\infty}F[x_{(i)}]\cdot g_k(x)dx$

$$=\int_{-\infty}^{+\infty}F(x)\cdot\frac{n!}{(i-1)!(n-i)!}F(x)^{i-1}[1-F(x)]^{n-i}f(x)dx$$

$$=\int_0^1 F(x)\cdot\frac{n!}{(i-1)!(n-i)!}F(x)^{i-1}[1-F(x)]^{n-i}dF(x)$$

$$=\frac{n!}{(i-1)!(n-i)!}\cdot B(i+1,n-i+1)$$

$$=\frac{n!}{(i-1)!(n-i)!}\frac{\Gamma(i+1)\Gamma(n-i+1)}{\Gamma(n+2)}$$

$$=\frac{n!}{(i-1)!(n-i)!}\frac{i!(n-i)!}{(n+1)!}$$

$$=\frac{i}{n+1}$$

$$E[F^2(X_i)]=\int_{-\infty}^{+\infty}F^2(x)\cdot g_k(x)dx$$

$$=\int_{-\infty}^{+\infty}\frac{n!}{(i-1)!(n-i)!}F^2(x)\cdot F(x)^{i-1}[1-F(x)]^{n-i}f(x)dx$$

$$= \frac{n!}{(i-1)!(n-i)!}\int_0^1 F(x)^{i+1}[1-F(x)]^{n-i}\,d[F(x)]$$

$$= \frac{n!}{(i-1)!(n-i)!}B(i+2, n-i+1)$$

$$= \frac{n!}{(i-1)!(n-i)!}\frac{\Gamma(i+2)\Gamma(n-i+1)}{\Gamma(n+3)}$$

$$= \frac{n!}{(i-1)!(n-i)!}\frac{(i+1)!(n-i)!}{(n+2)!}$$

$$= \frac{i(i+1)!}{(n+1)(n+2)}$$

$$D[F(X_{(i)})] = E[F^2(x_{(i)})] - \{E[F(x_{(i)}))]\}^2$$

$$= \frac{i(i+1)}{(n+1)(n+2)} - \frac{i^2}{(n+1)^2}$$

$$= \frac{i[n-i+1]}{(n+1)^2(n+2)}$$

CONCLUSIONS

Using the probability density function of order statistics and the nature of the euler integral, we deduced the digital characteristics of the function of order statistic.

$$E[F(X_i)] = \frac{i}{n+1}, \quad D[F(X_{(i)})] = \frac{i[n-i+1]}{(n+1)^2(n+2)}$$

ACKNOWLEDGEMENTS

I would like to thank the referees and the editor for their valuable suggestions.

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