Scholars Journal of Physics, Mathematics and Statistics **ISSN 2393-8056 (Print)**
Abbreviated Key Title: Sch. J. Phys. Math. Stat. **ISSN 2393-8064 (Online)**
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On Uncertain Programming and the Farm Planning Problem

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Article History
Received: 26.02.2018
Accepted: 10.03.2018
Published: 30.03.2018

DOI:
10.21276/sjpms.2018.5.2.4

Abstract: Farmers often encounter the problem of what to produce, how and when to produce it under a given set of resource restrictions. This problem is termed the farm planning problem. In this paper we develop an uncertain farm planning model for maximising expected total farm revenue. The wage-rates and the selling prices for crops are considered to be uncertain variables. In this paper we employ uncertainty theory to model the farm planning problem. The objective is to maximize the expected total farm revenue which is derived from taking the expectation of revenue plus labour earnings minus labour costs minus miscellaneous costs. The notion of inverse uncertainty distribution is used in calculating the expected costs and selling price. After taking the expectation, the objective function reduces to a deterministic function. A numerical example was solved in order to show the applicability of an uncertain farm planning problem.

Keywords: uncertainty theory, farm planning problem, uncertain programming, inverse Uncertainty distribution, deterministic function.

INTRODUCTION

”In developing countries, the agricultural sector’s performance determines overall economic growth, trade expansion, and increased income-earning opportunities,” Gopi *et al*. [1].

Several models have been developed to aid decision making in agriculture. One of the models developed is linear programming (LP) which plays a pivotal role in agricultural modelling. Osburn [2] proved that LP is capable of solving problems involving maximization of profits in farming. One of the problems faced in farming is that of optimizing crop patterns subject to a set of constraints. Singh *et al*. [2] applied LP in finding an optimal crop pattern subject to water level restrictions. Another application of LP in finding optimal crop patterns was done by Mohmoud *et al*. [4] in the Iranian state of Taybad of Khorasn Razavi.

Apart from the problem of finding optimal crop patterns, there is also a problem of calculating crop acreage. The problem of calculating crop acreage was tackled by Hassan *et al.* [5] where he applied LP in calculating crop acreage. According to Hassan *et al*. [5] LP is an appropriate tool in solving crop acreage allocation problems. Mohamad and Said [6] used LP in solving the crop mix problem. In addition, LP was used by Scarpari and Beauclair [7] in agricultural planning of sugar-cane. The problem of optimal crop combinations in small-scale vegetable irrigation farming was solved by Babatunde *et al*. [8] using LP which proved to be a capable technique of solving such problems.

This paper examines a farm planning model where wage-rates and crop prices are considered to be uncertain variables. The model is an extension to the deterministic farm planning problem. In the deterministic farm planning problem, the wage-rates and the crop prices are known in advance whereas for uncertain farm planning problem, beliefs of the farmer are utilised in coming up with the expected wage-rates and selling prices. The farmer’s expertise on coming up with the expected wage-rates and crop prices relies on the fact that these rates and prices are not always fixed. The assumption that the wage-rates are fixed is not necessarily true, since wage reviews affect labour costs. Crop prices fluctuate due to changes in demand, thus assuming these prices as fixed may be false.

Information concerning wage-rates and prices can be gathered from the farmer and this information is helpful in evaluating how the costs and prices behave over time. The fact that wage-rates and prices fluctuate makes them indeterminate. Randomness has been widely used in modelling such problems, but when there is lack of historical data, expert knowledge comes in [9]. Came up with the theory of uncertainty. The theory of uncertainty makes use of expert beliefs which are measured in belief degrees, which range from 0 to 1. Uncertainty theory has found widespread use in the areas of finance, transportation, inventory modelling etcetera. [10] looked at a fixed charge transportation problem and its uncertain programming model and they concluded that uncertainty works well in modelling costs [11]. Tackled a transportation problem with uncertain truck times and unit costs and confirmed the effectiveness of using uncertainty theory in modelling transport problems [12] proposed an uncertain price discrimination model in labour markets and their results showed that indeed uncertainty theory developed by [9] is important in modelling real life situations. In this paper, uncertain programming is used to model a farm planning problem with uncertain wage-rates and crop prices.

The paper consists of 5 sections which are organised as follows: section 1 is the introduction covering the background on the farm planning problem and how uncertainty theory can be used in modelling farm planning. Section 2 introduces basic uncertainty theory concepts used throughout the paper. In section 3, an uncertain farm planning problem is developed. A numerical example is given in section 4. Section 5 presents a brief summary of the validity of the model constructed in section 3.

Preliminaries

The theory of uncertain programming was incepted by Liu [9]. Uncertain programming is a type of mathematical programming involving uncertain variables [13]. Argues that since an uncertain objective function cannot be directly minimized, we may minimize its expected value[14]. Suggested that in order to obtain a decision with minimum expected objective value subject to a set of chance constraints, the uncertain programming model should be

$$MinimizeE[f(x,\varepsilon)]$$
$$Subject\ to$$
$$M\{g_i(x,\varepsilon)\leq 0\}\geq \alpha_j, \qquad j=1,2,\dots,p$$

where x is a deterministic variable, ε is an uncertain variable and α_j is the confidence level with which $M\{g_i(x,\varepsilon)\leq 0\}$ holds. According to [13] the uncertainty distribution φ of an uncertain variable ε is defined as

$$\varphi(x)=M\{\varepsilon\leq x\},$$

and if ε is an uncertain variable with regular uncertainty distribution$\varphi(x)$, then the function $\varphi^{-1}(\alpha)$ is called the inverse uncertainty distribution of ε [13]. The expected value of an uncertain variable is given by

$$E[\varepsilon]=\int_{-\infty}^{\infty} xd\,\varphi(x)=\int_0^1 \varphi^{-1}(\alpha)d\,\alpha.$$

For a given function $f(\varepsilon)$ of an uncertain variable, we have

$$E[f(\varepsilon)]=\int_0^1 f\left(\varphi^{-1}(\alpha)\right)d\alpha.$$

The following theorem plays a very pivotal role in taking the expectation of a function of Uncertain variables.

Theorem 2.1 [15] let ε and τ is independent uncertain variables with finite expected Values. Then for any real numbers a and b, we have

$$E[a\varepsilon+b\tau]=aE[\varepsilon]+bE[\tau].$$

Uncertain Farm Planning Design Model

The expected total uncertain net farm revenue is given by

$$E\left[\sum_{\forall c} X_c\, Y_c P_c+\sum_{\forall t} V_t{}^H\, r^H-\sum_{\forall t} V_t{}^T\, r^T-\sum_{\forall c} X_c M_c{}^c\right],$$

where c is the index for the crops, t is the time in months, X_c is the cropping activity, Y_c is the yield of crop c in tonnes per hectare, P_c is the uncertain price of crop c in dollars per tonne, $V_t{}^H$ is labour hired out, r^H is the uncertain hire-out wage rate, $V_t{}^T$ is the temporary labour use, r^T is the uncertain temporary wage-rate and the miscellaneous costs are given by $M_c{}^c$. Expected net farm revenue has four components namely revenue from sales of farm produce, $\sum_{\forall c} X_c\, Y_c P_c$ income received from hiring out labour$\sum_{\forall t} V_t{}^H\, r^H$, labour costs $\sum_{\forall t} V_t{}^T\, r^T$ and miscellaneous costs $\sum_{\forall c} X_c M_c{}^c$.

In order to meet the labour requirements, family labour plus temporary hired labour must be greater or equal to the labour required for cropping activities, that is

$$\sum_{\forall c} X_c\, V_{tc}\leq V_t{}^F+V_t{}^T,$$

where $V_t{}^F$ is the family labour and V_{tc} is the labour required for crop c. Land allocated for cropping activities must be less or equal to the total land available. The land balance constraint is

$$\sum_{\forall c} X_c\, l_{tc}\leq Z,$$

where l_{tc} is the fraction of month t that crop c occupies a piece of land and Z is the total land available in hectares. The quantities X_c, V_t^F, V_t^H and V_t^T should be greater or equal to zero. Let r^T, r^H and P_c be independent uncertain variables with uncertainty distributions $\varphi(x)$, $\gamma(x)$ and $\beta(x)$ respectively. Furthermore, suppose r^T, r^H and P_c have inverse uncertainty distributions $\varphi^{-1}(\alpha), \gamma^{-1}(\alpha)$ and $\beta^{-1}(\alpha)$ respectively. An uncertain farm planning problem takes the form

$$Maximise\ E\left[\sum_{\forall c} X_c\, Y_c P_c + \sum_{\forall t} V_t^H\, r^H - \sum_{\forall t} V_t^T\, r^T - \sum_{\forall c} X_c M_c{}^c\right]$$

Subject to

$$\sum_{\forall c} X_c\, V_{tc} \leq V_t^F + V_t^T,\ \ c = 1,2,\ldots,n,\ \ t = 1,2,\ldots,12$$
$$\sum_{\forall c} X_c\, l_{tc} \leq Z,\ \ c = 1,2,\ldots,n,\ \ t = 1,2,\ldots,12$$
$$X_c \geq 0,\ c = 1,2,\ldots,n$$
$$V_t^F \geq 0,\ V_t^H \geq 0,\ \ V_t^T \geq 0,\ \ t = 1,2,\ldots,12$$

Distributing the expectation, we have

$$Maximise\ \left[\sum_{\forall c} X_c\, Y_c E[P_c] + \sum_{\forall t} V_t^H\, E[r^H] - \sum_{\forall t} V_t^T E[\, r^T] - \sum_{\forall c} X_c M_c{}^c\right]$$

Subject to

$$\sum_{\forall c} X_c\, V_{tc} \leq V_t^F + V_t^T,\ c = 1,2,\ldots,n,\ t = 1,2,\ldots,12$$
$$\sum_{\forall c} X_c\, l_{tc} \leq Z,\ \ c = 1,2,\ldots,n,\ \ t = 1,2,\ldots,12$$
$$X_c \geq 0,\ c = 1,2,\ldots,n$$
$$V_t^F \geq 0,\ V_t^H \geq 0,\ \ V_t^T \geq 0,\ \ t = 1,2,\ldots,12$$

Applying the concept of expectation, the uncertain farm planning model takes the form

$$Maximise\ \sum_{\forall c} X_c\, Y_c \int_0^1 \varphi^{-1}(\alpha) + \sum_{\forall t} V_t^H \int_0^1 \gamma^{-1}(\alpha) - \sum_{\forall t} V_t^T \int_0^1 \beta^{-1}(\alpha) - \sum_{\forall c} X_c M_c{}^c$$

Subject to

$$\sum_{\forall c} X_c\, V_{tc} \leq V_t^F + V_t^T,\ c = 1,2,\ldots,n,\ \ t = 1,2,\ldots,12,$$
$$\sum_{\forall c} X_c\, l_{tc} \leq Z,\ \ c = 1,2,\ldots,n,\ \ t = 1,2,\ldots,12$$
$$X_c \geq 0,\ c = 1,2,\ldots,n$$
$$V_t^F \geq 0,\ V_t^H \geq 0,\ \ V_t^T \geq 0,\ \ t = 1,2,\ldots,12$$

which is a deterministic linear programming problem.

Numerical examples

A numerical example is given, in order to show the validity of the model. An experienced communal farmer is asked about the information pertaining to his beliefs on how likely the crop prices and wage rates would be. The farmer cannot tell the exact prices and wages since planning is done in advance. However, since the farmer is an expert in the farming industry, uncertain data can be obtained by means of experience evaluation. Assuming that all uncertain variables under consideration are normally distributed, and then we have

$N(ep_c, \sigma_{pc}),\ c = 1,2,\ldots,n,$
$N(e_r H, \sigma_r H),$
$N(e_r T, \sigma_r T),$

and the uncertain farm planning model becomes

$$Maximise\ \sum_{\forall c} X_c\, Y_c ep_c + \sum_{\forall t} V_t^H\, e_r H - \sum_{\forall t} V_t^T\, e_r T - \sum_{\forall c} X_c M_c{}^c$$

Subject to

$$\sum_{\forall c} X_c\, V_{tc} \leq V_t^F + V_t^T,\ c = 1,2,\ldots,n,\ \ t = 1,2,\ldots,12,$$
$$\sum_{\forall c} X_c\, l_{tc} \leq Z,\ \ c = 1,2,\ldots,n,\ \ t = 1,2,\ldots,12$$
$$X_c \geq 0,\ c = 1,2,\ldots,n$$
$$V_t^F \geq 0,\ V_t^H \geq 0,\ \ V_t^T \geq 0,\ \ t = 1,2,\ldots,12$$

The parameters ep_c, $e_r H$ and $e_r T$ are the expected crop prices, expected hiring out labour rate and expected temporary labour rate respectively. The data associated to the normal distributions is shown in tables 1-3.

Table-1: Parameters of normal distribution for the labour hiring-out rate

Uncertain variable	$(e_r H, \sigma_r H)$
Hiring out labour rate	(2.8, 0.2)

The crops under consideration have varying requirements in terms of land and labour. In this example, five hectares are being utilised at a farm under consideration.

Table-2: Parameters of normal distribution for the rate of hiring temporary labour

Uncertain variable	$(e_r T, \sigma_r T)$
temporary labour rate	(3.9, 0.7)

Table-3: Parameters of the normal distribution for crop prices

Uncertain variable	(ep_c, σ_{pc})
price of wheat	(879.8, 83.5)
price of beans	(1840.8, 95.0)
price of onions	(731.4, 153.3)
price of maize	(272.0, 34.6)
price of tomatoes	(694.1, 50.9)
price of potatoes	(590.2, 44.5)
price of cabbage	(188.8, 29.0)

The crops are rotated on this piece of land all year round depending on favourable conditions for each crop. Labour force provided by each family member is equivalent to labour of one and a half person. The yields of the crops per hectare are wheat 1.5 tonnes, beans 1 tonne, onions 6 tonnes, maize 1.8 tonnes, tomatoes 6 tonnes, cabbage 3 tonnes and potatoes 2 tonnes. Members of the family are the major source of labour. Sometimes the labour requirements exceed those that can be sustained by the family, as such temporary labour is hired. In some cases, work is minimal to an extent that the family hires out its labour. The farmer needs to properly allocate enough land to each crop in preparation of the cropping activity. Also, the farmer needs to create a budget for each cropping season. Since crop rotation is being exercised, there is need for the crops to match the right season. The data concerning crop labour requirements and land allocated to the crops under consideration is given in tables 4-7.

The entries for tables 4 and 5 are fractions of the month when land is occupied by a certain crop and for tables 6 and 7 the entries represent labour in man-days per hectare.

Table-4: Crop land allocation in hectares from January to June

	wheat	beans	onions	maize	tomatoes	cabbage	potatoes
JAN		1	1	1			
FEB		1	1	1			
MAR	0.5	1	1	0.5			
APR	1	1	1				
MAY	1						
JUN	1						

Table-5: Crop land allocation in hectares from July to December

	wheat	beans	onions	maize	tomatoes	cabbage	potatoes
JUL	1				0.75		
AUG	1				1	1	1
SEP	1				1	1	1
OCT				0.25	1	1	1
NOV		0.25	0.51	1	1	1	1
DEC		1	1	1	0.75		

Table-6: Crop labour requirements in man-days per hectare from January to June

	wheat	beans	onions	maize	tomatoes	cabbage	potatoes
JAN		7	43	64			
FEB		7	42	36			
MAR	20	7	42	9			
APR	13	129	160				
MAY	15		20				
JUN	5						

Table-7: Crop labour requirements in man-days per hectare from July to December

	wheat	beans	onions	maize	tomatoes	cabbage	potatoes
JUL	9				137		
AUG	9				121	124	40
SEP	140				98	110	72
OCT				35	58	86	64
NOV		63	90	42	51	54	40
DEC		8	40	60	58		

Table-8: Crop optimal solution summary

	beans	potatoes	total
land use in hectares	5.000	3.750	8.75
output in tonnes	5.000	7.500	12.50
revenue in US dollars	9204.00	4426.50	13630.50

The crop optimal solution is given in table 8. Beans contribute much of the required revenue with a total contribution of $9204.00. Table 9 and 10 show the optimal labour requirements from January to December. In May, June and July, the rest of family labour is devoted to hiring out. For the rest of the months, the family concentrate on the crops at their farm.

Table-9: Optimal labour requirements from January to June

	demand	family	temporary	hire-out
JAN	35.000	25.000	10.000	
FEB	35.000	25.000	10.000	
MAR	35.000	25.000	10.000	
APR	645.000	25.000	620.000	
MAY				25.000
JUN				25.000

Table-10: Optimal labour requirements from July to December

	demand	family	temporary	hire-out
JUL				25.000
AUG	150.000	25.000	125.000	
SEP	270.000	25.000	245.000	
OCT	240.000	25.000	215.000	
NOV	465.000	25.000	440.000	
DEC	40.000	25.000	15.000	

Results show that the total annual labour demanded is 1915 man-days of which 225 man-days are contributed by the family and 1690 man-days come from temporary labour. Temporary labour constitutes the greater portion of the farm labour force from April to November. According to the data in table 11, when the farmer specialises in farming beans and potatoes, he realises net revenue of US$6815.75. This is found using equation 4, which is the objective function. The family spends only 75 day per year on hiring out labour and they generate labour earnings amounting to $210.00, which is almost half of the miscellaneous costs incurred.

Table-11: Net revenue calculation

	Amount
total revenue	13630.50
miscellaneous costs	433.75
labour costs	6591.00
labour earnings	210.00
net revenue	6815.75

The labour costs are slightly below the net revenue generated per year, which is typical of a communal set-up.

CONCLUSIONS

This paper focused on uncertain farm planning problem based on uncertainty theory from which an uncertain farm planning model was developed. The uncertain objective function was reduced to a deterministic function by taking the expectation. A classical method was used in finding the solution to the uncertain farm planning problem. A numerical test was carried out in order to validate the model. The developed uncertain model proved to work well in solving the farm planning problem.

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