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Bifurcation of Traveling Wave Solutions for the Bogoyavlenskii- Kadomtsev-Petviashvili Equation

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Abstract: This paper studies each type of traveling waves of the traveling wave system corresponding to the Bogoyavlenskii-Kadomtsev-Petviashvili equation comprehensively and systematically. By transforming its traveling wave system into a dynamical system in R^3, we employ the bifurcation method of dynamical system to investigate its phase space geometry in detail. Finally, by calculating the complicated elliptic integrals, we obtain exact expressions of all traveling wave solutions of the Bogoyavlenskii-Kadomtsev-Petviashvili equation without any loss.
Keywords: Bogoyavlenskii-Kadomtsev-Petviashvili equation, traveling wave solutions, dynamical system, bifurcation.

INTRODUCTION

This paper considers the following Bogoyavlenskii-Kadomtsev-Petviashvili (BKP) equation [1];

$$\left(4u_{xt} + u_{xxxy} + 8u_x u_{xy} + 4u_{xx}u_y\right)_x + \delta \cdot u_{yyy} = 0, \quad \delta = \pm 1.$$

The scalar field $u(x, y, t)$ is an analytic function of the scaled spatial coordinates x, y and temporal coordinate t and presents the amplitude of the relevant wave. BKP equation (1.1) can be used to describe the propagation of nonlinear waves in fluid, plasma, biology and electrical networks [2,3,4,5]. It is a reduction of the KP hierarchy [2], in the specific application, if the surface tension dominates over the gravitational force, then $\delta = -1$ and (1.1) is called BKP-I, whereas if the gravitational force is dominant then $\delta = 1$ and (1.1) is called BKP-II. Therefore, (1.1) has an extremely wide range of applications in physics and other nonlinear sciences.

In fact, the BKP equation (1.1) is widely concerned and many efforts have been devoted to its exact solutions. In 2004, the tanh method was used in [6] to obtain the soliton-like solutions and periodic form solutions of (1.1). In 2011, some exact solutions of (1.1) were derived from two existing simple traveling wave solutions by the finite symmetry transformation groups [7]. In 2012, by applying a direct symmetry method, some new explicit solutions were obtained for the BKP equation, which include trigonometric function solutions and periodic solutions [8]. In 2015, with the use of Hirota method, solitary-wave solutions for (1.1) were derived [9]. In 2017, through the Bell polynomials, the one- and two-kink-soliton solutions of the BKP equation were got in [10]. Recently, in [11], the binary Bell polynomials method was employed to get the multiple wave solutions including the kink periodic solitary wave and bright-dark lump wave solutions.

Though there have been so many profound results about exact solutions of (1.1), there are few studies on its traveling wave solutions, especially the unbounded traveling wave solution. Calculating the traveling wave solutions of a nonlinear partial differential equation (NPDE) is of great help in understanding the nonlinear physical phenomena and wave propagation described by the NPDE. In this paper, our aim is to use the bifurcation method of dynamical system to comprehensively and systematically study each type of traveling waves of the BKP equation (1.1) and give all the exact expressions of them without any loss. The bifurcation method of dynamical system can not only clearly explain how these solutions evolve when the parameters change, but also study the dynamical behavior of solutions. In recent decades, this method has been widely and effectively applied to many different equations [12-15].

Bifurcation analysis of traveling wave system of the BKP equation

Letting $u(x, y, t) = u(\xi) = u(x + ay - ct)$, where $a \neq 0$ represents the wave number in the y direction and $c \neq 0$ is the wave velocity, we transform (1.1) into its corresponding traveling wave system as follows

$$(a^3\delta - 4c)\cdot u''' + a\cdot u''''' + 12a\cdot u'u''' + 12a\cdot (u'')^2 = 0, \tag{2.1}$$

where $'$ denotes the derivative with respect to ξ. Integrating (2.1) twice, we get

$$(a^3\delta - 4c)\cdot u' + a\cdot u''' + 6a\cdot (u')^2 = e,$$

where parameter e is the integral constant. Letting $u' = v$, we have

$$\begin{cases} u' = v, & (2.2)\\ v'' = -6\cdot v^2 - \left(a^2\delta - \dfrac{4c}{a}\right)\cdot v + \dfrac{e}{a}. & (2.3)\end{cases}$$

Since (2.3) does not contain function u, we can start with our analysis of (2.3) firstly. Then, rewrite (2.3) to the following equivalent system

$$\begin{cases} v' = y,\\ y' = -6\cdot v^2 - \left(a^2\delta - \dfrac{4c}{a}\right)\cdot v + \dfrac{e}{a}, \end{cases} \tag{2.4}$$

which has the first integral

$$H(v,y) = \frac{1}{2}\cdot y^2 + 2\cdot v^3 + \frac{1}{2}\left(a^2\delta - \frac{4c}{a}\right)\cdot v^2 - \frac{e}{a}\cdot v. \tag{2.5}$$

When $(a^2\delta - \frac{4c}{a})^2 + 24\cdot\frac{e}{a} > 0$, (2.4) has two equilibria $A\left(-\frac{1}{12}\left(a^2\delta - \frac{4c}{a}\right) - \frac{1}{12}\sqrt{\left(a^2\delta - \frac{4c}{a}\right)^2 + 24\cdot\frac{e}{a}}, 0\right)$ and $B\left(-\frac{1}{12}\left(a^2\delta - \frac{4c}{a}\right) + \frac{1}{12}\sqrt{\left(a^2\delta - \frac{4c}{a}\right)^2 + 24\cdot\frac{e}{a}}, 0\right)$. Letting $M(v,y)$ denotes the coefficient matrix of the linearized system of (2.4) at point (v,y), then

$$M(A) = \begin{bmatrix} 0 & 1\\ \sqrt{\left(a^2\delta - \dfrac{4c}{a}\right)^2 + 24\cdot\dfrac{e}{a}} & 0 \end{bmatrix},$$

$$M(B) = \begin{bmatrix} 0 & 1\\ -\sqrt{\left(a^2\delta - \dfrac{4c}{a}\right)^2 + 24\cdot\dfrac{e}{a}} & 0 \end{bmatrix}.$$

By the properties of the Hamilton system [16], the equilibrium A is a saddle, while B is a center.

When $(a^2\delta - \frac{4c}{a})^2 + 24\cdot\frac{e}{a} = 0$, (2.4) has a unique degenerated equilibrium $C(\frac{c}{3a} - \frac{a^2\delta}{12}, 0)$, its coefficient matrix is

$$M(C) = \begin{bmatrix} 0 & 1\\ 0 & 0 \end{bmatrix}.$$

According to the qualitative theory of differential equation [17], we know that C is a cusp.

When $(a^2\delta - \frac{4c}{a})^2 + 24\cdot\frac{e}{a} < 0$, there is no equilibrium of (2.4).

Based on the above analysis, we get the following results:

Case 1. When $(a^2\delta - \frac{4c}{a})^2 + 24\cdot\frac{e}{a} > 0$, there exists a homoclinic orbit γ which connects the saddle A. Center B is surrounded by a family of periodic orbits

$$\Gamma(h) = \left\{ H(v,y) = h, h \in \left(\frac{1}{432}\cdot\left(a^2\delta - \frac{4c}{a}\right)^3 + \frac{1}{12}\cdot\left(a^2\delta - \frac{4c}{a}\right)\cdot\frac{e}{a} - \frac{1}{432}\left(\left(a^2\delta - \frac{4c}{a}\right)^2 + 24\cdot\frac{e}{a}\right)^{\frac{3}{2}}, \right.\right.$$
$$\left.\left. \frac{1}{432}\cdot\left(a^2\delta - \frac{4c}{a}\right)^3 + \frac{1}{12}\cdot\left(a^2\delta - \frac{4c}{a}\right)\cdot\frac{e}{a} + \frac{1}{432}\left(\left(a^2\delta - \frac{4c}{a}\right)^2 + 24\cdot\frac{e}{a}\right)^{\frac{3}{2}} \right) \right\},$$

where $\Gamma(h)$ tends to B as $h \to \frac{1}{432}\cdot\left(a^2\delta - \frac{4c}{a}\right)^3 + \frac{1}{12}\cdot\left(a^2\delta - \frac{4c}{a}\right)\cdot\frac{e}{a} - \frac{1}{432}\left(\left(a^2\delta - \frac{4c}{a}\right)^2 + 24\cdot\frac{e}{a}\right)^{\frac{3}{2}}$ and tends to γ as $h \to \frac{1}{432}\cdot\left(a^2\delta - \frac{4c}{a}\right)^3 + \frac{1}{12}\cdot\left(a^2\delta - \frac{4c}{a}\right)\cdot\frac{e}{a} + \frac{1}{432}\left(\left(a^2\delta - \frac{4c}{a}\right)^2 + 24\cdot\frac{e}{a}\right)^{\frac{3}{2}}$. Except the homoclinic orbit and periodic orbits, other orbits of (2.4) are unbounded, as shown in Figure 1(a).

Case 2. When $(a^2\delta-\frac{4c}{a})^2+24\cdot\frac{e}{a}\le 0$, there only exist unbounded orbits of (2.4), as shown in Figure 1(b-c).

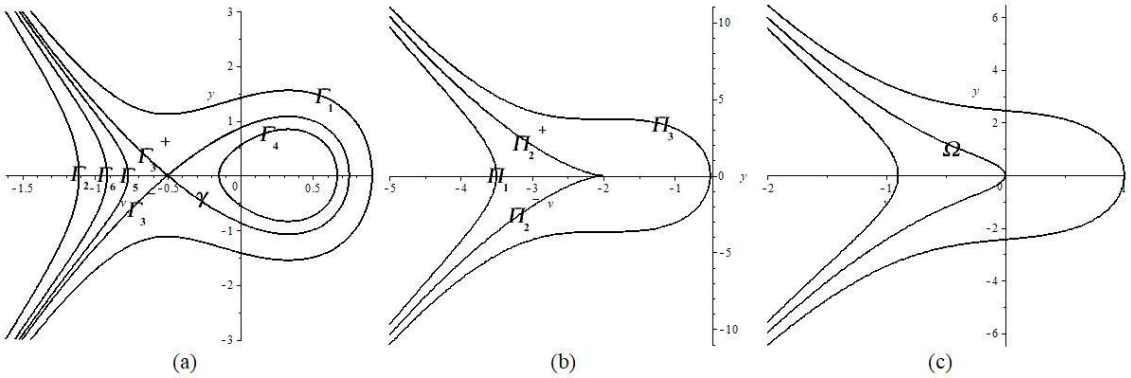


Fig-1: The phase portraits of (2.4) in different parameter bifurcation sets

All explicit traveling wave solutions of (1.1)

In this section, we seek explicit expressions of all traveling wave solutions of (1.1), which needs us to combine the energy function (2.5) to investigate each type of orbits of (2.4) in different parameter bifurcation sets, including bounded and unbounded ones.

Case 1. When $(a^2\delta-\frac{4c}{a})^2+24\cdot\frac{e}{a}>0$, we need to consider seven types of orbits, as orbits Γ_1, Γ_2, γ, $\Gamma_3^+(\Gamma_3^-)$, Γ_4, Γ_5 and Γ_6 shown in Figure 1(a).

Firstly, we consider the unbounded orbit Γ_1 whose energy is higher than the energy of saddle A. From (2.4), its expression can be determined by the following integrals

$$\int_{-\infty}^{v}\frac{dv}{\sqrt{(r_1-v)\cdot\left[v^2+\left(\frac{1}{4}\left(a^2\delta-\frac{4c}{a}\right)+r_1\right)\cdot v+r_1^2+\frac{1}{4}\left(a^2\delta-\frac{4c}{a}\right)\cdot r_1-\frac{e}{2a}\right]}}=\int_0^{\xi}2d\xi\,,\ \ \xi>0,$$

$$-\int_{v}^{-\infty}\frac{dv}{\sqrt{(r_1-v)\cdot\left[v^2+\left(\frac{1}{4}\left(a^2\delta-\frac{4c}{a}\right)+r_1\right)\cdot v+r_1^2+\frac{1}{4}\left(a^2\delta-\frac{4c}{a}\right)\cdot r_1-\frac{e}{2a}\right]}}=\int_{\xi}^{0}2d\xi\,,\ \ \xi<0,$$

where the real parameter r_1 satisfies the relation $-\infty<v<r_1$ and $r_1>-\frac{1}{12}\left(a^2\delta-\frac{4c}{a}\right)+\frac{1}{6}\sqrt{\left(a^2\delta-\frac{4c}{a}\right)^2+24\cdot\frac{e}{a}}$. By calculating elliptic integral, we obtain the first type of traveling wave solution of (2.4) as follows

$$v_1(\xi)=r_1+\sqrt{3r_1^2+\frac{1}{2}\left(a^2\delta-\frac{4c}{a}\right)\cdot r_1-\frac{e}{2a}}-\frac{2\sqrt{3r_1^2+\frac{1}{2}\left(a^2\delta-\frac{4c}{a}\right)\cdot r_1-\frac{e}{2a}}}{1-cn\left(2\sqrt[4]{3r_1^2+\frac{1}{2}\left(a^2\delta-\frac{4c}{a}\right)\cdot r_1-\frac{e}{2a}}\cdot|\xi|\right)},$$

where $0<|\xi|<\xi_1$ and $\xi_1=\frac{2}{\sqrt[4]{3r_1^2+\frac{1}{2}\left(a^2\delta-\frac{4c}{a}\right)\cdot r_1-\frac{e}{2a}}}\int_0^{\frac{\pi}{2}}\frac{d\vartheta}{\sqrt{1-\frac{\sqrt{3r_1^2+\frac{1}{2}\left(a^2\delta-\frac{4c}{a}\right)\cdot r_1-\frac{e}{2a}}+\frac{1}{8}\left(a^2\delta-\frac{4c}{a}\right)+\frac{3}{2}r_1}{2\sqrt{3r_1^2+\frac{1}{2}\left(a^2\delta-\frac{4c}{a}\right)\cdot r_1-\frac{e}{2a}}}\cdot\sin^2\vartheta}}$. From (2.2), we need to integrate $v_1(\xi)$ once again to get the first type of traveling wave solutions of (1.1)

$$u_1(\xi)=\left(r_1-\sqrt{3r_1^2+\frac{1}{2}\left(a^2\delta-\frac{4c}{a}\right)\cdot r_1-\frac{e}{2a}}\right)\cdot\xi+\sqrt[4]{3r_1^2+\frac{1}{2}\left(a^2\delta-\frac{4c}{a}\right)\cdot r_1-\frac{e}{2a}}$$
$$\cdot\left[E\left(2\sqrt[4]{3r_1^2+\frac{1}{2}\left(a^2\delta-\frac{4c}{a}\right)\cdot r_1-\frac{e}{2a}}\cdot\xi\right)\right.$$
$$\left.+\frac{sn\left(2\sqrt[4]{3r_1^2+\frac{1}{2}\left(a^2\delta-\frac{4c}{a}\right)\cdot r_1-\frac{e}{2a}}\cdot\xi\right)\cdot dn\left(2\sqrt[4]{3r_1^2+\frac{1}{2}\left(a^2\delta-\frac{4c}{a}\right)\cdot r_1-\frac{e}{2a}}\cdot\xi\right)}{1-cn\left(2\sqrt[4]{3r_1^2+\frac{1}{2}\left(a^2\delta-\frac{4c}{a}\right)\cdot r_1-\frac{e}{2a}}\cdot\xi\right)}\right],$$

where $0<|\xi|<\xi_1$.

Then, consider the unbounded orbit Γ_2 whose energy is lower than the energy of center B. Similar to the calculation of the orbit Γ_1, we can give the explicit expression of the second type of traveling wave solution of (2.4)

$$v_2(\xi) = r_2 + \sqrt{3r_2^2 + \frac{1}{2}\left(a^2\delta - \frac{4c}{a}\right)\cdot r_2 - \frac{e}{2a}} - \frac{2\sqrt{3r_2^2 + \frac{1}{2}\left(a^2\delta - \frac{4c}{a}\right)\cdot r_2 - \frac{e}{2a}}}{1 - cn\left(2\sqrt[4]{3r_2^2 + \frac{1}{2}\left(a^2\delta - \frac{4c}{a}\right)\cdot r_2 - \frac{e}{2a}}\cdot|\xi|\right)},$$

and the traveling wave solution of (1.1)

$$u_2(\xi) = \left(r_2 - \sqrt{3r_2^2 + \frac{1}{2}\left(a^2\delta - \frac{4c}{a}\right)\cdot r_2 - \frac{e}{2a}}\right)\cdot\xi + \sqrt[4]{3r_2^2 + \frac{1}{2}\left(a^2\delta - \frac{4c}{a}\right)\cdot r_2 - \frac{e}{2a}}$$
$$\cdot\left[E\left(2\sqrt[4]{3r_2^2 + \frac{1}{2}\left(a^2\delta - \frac{4c}{a}\right)\cdot r_2 - \frac{e}{2a}}\cdot\xi\right)\right.$$
$$\left. + \frac{sn\left(2\sqrt[4]{3r_2^2 + \frac{1}{2}\left(a^2\delta - \frac{4c}{a}\right)\cdot r_2 - \frac{e}{2a}}\cdot\xi\right)\cdot dn\left(2\sqrt[4]{3r_2^2 + \frac{1}{2}\left(a^2\delta - \frac{4c}{a}\right)\cdot r_2 - \frac{e}{2a}}\cdot\xi\right)}{1 - cn\left(2\sqrt[4]{3r_2^2 + \frac{1}{2}\left(a^2\delta - \frac{4c}{a}\right)\cdot r_2 - \frac{e}{2a}}\cdot\xi\right)}\right],$$

where the real parameter r_2 satisfies the relation $r_2 < -\frac{1}{12}\left(a^2\delta - \frac{4c}{a}\right) - \frac{1}{6}\sqrt{\left(a^2\delta - \frac{4c}{a}\right)^2 + 24\cdot\frac{e}{a}}$, $0 < |\xi| < \xi_2$ and $\xi_2 = \frac{2}{\sqrt[4]{3r_2^2 + \frac{1}{2}\left(a^2\delta - \frac{4c}{a}\right)\cdot r_2 - \frac{e}{2a}}}\int_0^{\frac{\pi}{2}} \frac{d\vartheta}{\sqrt{1 - \frac{\sqrt{3r_2^2 + \frac{1}{2}\left(a^2\delta - \frac{4c}{a}\right)\cdot r_2 - \frac{e}{2a}} + \frac{1}{8}\left(a^2\delta - \frac{4c}{a}\right) + \frac{3}{2}r_2}{2\sqrt{3r_2^2 + \frac{1}{2}\left(a^2\delta - \frac{4c}{a}\right)\cdot r_2 - \frac{e}{2a}}}\cdot\sin^2\vartheta}}$.

The bounded orbit γ, whose energy is equal to the energy of saddle A, is a homocilinic orbit of (2.4) and can be identified by the following integral expressions

$$\int_v^{r_4} \frac{dv}{(v - r_3)\sqrt{r_4 - v}} = \int_\xi^0 2d\xi\,, \quad \xi < 0,$$
$$-\int_{r_4}^v \frac{dv}{(v - r_3)\sqrt{r_4 - v}} = \int_0^\xi 2d\xi\,, \quad \xi > 0,$$

where $r_3 = -\frac{1}{12}\left(a^2\delta - \frac{4c}{a}\right) - \frac{1}{12}\sqrt{\left(a^2\delta - \frac{4c}{a}\right)^2 + 24\cdot\frac{e}{a}}$, $r_4 = -\frac{1}{12}\left(a^2\delta - \frac{4c}{a}\right) + \frac{1}{6}\sqrt{\left(a^2\delta - \frac{4c}{a}\right)^2 + 24\cdot\frac{e}{a}}$ and the relation $r_3 < v < r_4$ holds. The solitary wave solution of (2.4) can be given by

$$v_3(\xi) = r_3 + \frac{4\cdot\exp\left(2\sqrt{r_4 - r_3}\cdot|\xi|\right)}{\left(1 + \exp\left(2\sqrt{r_4 - r_3}\cdot|\xi|\right)\right)^2}, \quad -\infty < \xi < +\infty.$$

Then, the third type of traveling wave solution of (1.1) can be calculated by

$$u_3(\xi) = \int v_3(\xi)\,d\xi = r_3\cdot\xi - \frac{2}{\sqrt{r_4 - r_3}\cdot\left(1 + \exp\left(2\sqrt{r_4 - r_3}\cdot\xi\right)\right)} + C_1, \quad -\infty < \xi < +\infty,$$

where C_1 is an integral constant. In particular, when $r_3 = 0$, the third type of traveling wave of (1.1) degenerates to a kink wave.

Next, we consider the unbounded orbit $\Gamma_3^+(\Gamma_3^-)$ whose energy is also equal to the energy of salddle A. So, the similar calculation can get its expression as follows

$$v_4(\xi) = r_3 - \frac{4\cdot\exp\left(2\sqrt{r_4 - r_3}\cdot|\xi|\right)}{\left(1 - \exp\left(2\sqrt{r_4 - r_3}\cdot|\xi|\right)\right)^2}, \quad |\xi| > 0.$$

Integrating above expression once with respect to ξ, we can get the fourth type of traveling wave solution of (1.1)

$$u_4(\xi) = r_3\cdot\xi - \frac{2}{\sqrt{r_4 - r_3}\cdot\left(1 - \exp\left(2\sqrt{r_4 - r_3}\cdot\xi\right)\right)} + C_2, \quad |\xi| > 0,$$

where C_2 is an integral constant.

Consider the orbit Γ_4 whose energy lies between the energy of saddle A and center B. It is a closed orbit surrounding the center B and can be determined by the following integrals

$$\int_{r_6}^{v} \frac{dv}{\sqrt{(v-r_5)(v-r_6)(r_7-v)}} = \int_0^{\xi} 2d\xi\,, \quad 0<\xi<T,$$
$$-\int_{v}^{r_6} \frac{dv}{\sqrt{(v-r_5)(v-r_6)(r_7-v)}} = \int_{\xi}^{0} 2d\xi\,, \quad -T<\xi<0,$$

where the real parameters r_5, r_6 and r_7 satisfy the relation $r_5<r_6<v<r_7$. Calculating the elliptic integral, we get the following periodic wave solution of (2.4)

$$v_5(\xi) = r_5 + \frac{r_6-r_5}{1-\frac{r_7-r_6}{r_7-r_5}\cdot sn^2(\sqrt{r_6-r_5}\cdot|\xi|)}, \qquad -T<\xi<T.$$

From (2.2), the fifth type of traveling wave solution of (1.1) can be expressed by

$$u_5(\xi) = r_5\cdot\xi + \frac{r_7-r_5}{\sqrt{r_6-r_5}}\cdot\left[E\left(\sqrt{r_6-r_5}\cdot\xi\right) + \sqrt{\frac{r_7-r_6}{r_7-r_5}}\cdot cd\left(\sqrt{r_6-r_5}\cdot\xi\right)\right], \qquad -T<\xi<T.$$

The expression of the unbounded orbit Γ_5, whose energy still lies between the energy of saddle A and center B, can be identified by

$$\int_{-\infty}^{v} \frac{dv}{\sqrt{(r_8-v)(r_9-v)(r_{10}-v)}} = \int_0^{\xi} 2d\xi\,, \qquad \xi>0,$$
$$-\int_{v}^{-\infty} \frac{dv}{\sqrt{(r_8-v)(r_9-v)(r_{10}-v)}} = \int_{\xi}^{0} 2d\xi\,, \qquad \xi<0,$$

where the real parameters r_8, r_9 and r_{10} satisfy the relation $-\infty<v<r_8<r_9<r_{10}$. The sixth type of traveling wave solution of (2.4) is

$$v_6(\xi) = r_{10} - \frac{r_{10}-r_8}{sn^2\left(\sqrt{r_{10}-r_8}\cdot|\xi|\right)}, \quad 0<|\xi|<\xi_3,$$

where $\xi_3 = \frac{2}{\sqrt{r_{10}-r_8}}\int_0^{\frac{\pi}{2}} \frac{d\vartheta}{\sqrt{1-\frac{r_{10}-r_9}{r_{10}-r_8}\cdot sin^2\vartheta}}$. And then, we have

$$u_6(\xi) = r_8\cdot\xi + \sqrt{r_{10}-r_8}\cdot\left[E\left(\sqrt{r_{10}-r_8}\cdot\xi\right) + dn\left(\sqrt{r_{10}-r_8}\cdot\xi\right)\cdot cs\left(\sqrt{r_{10}-r_8}\cdot\xi\right)\right], \quad 0<\xi<\xi_3.$$

Now, consider the unbounded orbit Γ_6 whose energy is equal to the energy of center B. By the following two integral expressions

$$\int_{-\infty}^{v} \frac{dv}{(r_{12}-v)\sqrt{r_{11}-v}} = \int_0^{\xi} 2d\xi\,, \qquad \xi>0,$$
$$-\int_{v}^{-\infty} \frac{dv}{(r_{12}-v)\sqrt{r_{11}-v}} = \int_{\xi}^{0} 2d\xi\,, \qquad \xi<0,$$

we get the explicit expression of the seventh type of traveling wave solution of (2.4)

$$v_7(\xi) = r_{11} - (r_{12}-r_{11})\cdot\cot^2\left(\sqrt{r_{12}-r_{11}}\cdot|\xi|\right), \quad 0<|\xi|<\xi_4,$$

where $r_{11} = -\frac{1}{12}\left(a^2\delta-\frac{4c}{a}\right) - \frac{1}{6}\sqrt{\left(a^2\delta-\frac{4c}{a}\right)^2+24\cdot\frac{e}{a}}$, $r_{12} = -\frac{1}{12}\left(a^2\delta-\frac{4c}{a}\right) + \frac{1}{12}\sqrt{\left(a^2\delta-\frac{4c}{a}\right)^2+24\cdot\frac{e}{a}}$ and $\xi_4 = \frac{\pi}{\sqrt{r_{12}-r_{11}}}$. Thus, the seventh type of traveling wave solution of (1.1) has the form

$$u_7(\xi) = r_{12}\cdot\xi + \sqrt{r_{12}-r_{11}}\cdot\cot\left(\sqrt{r_{12}-r_{11}}\cdot\xi\right) + C_3, \quad 0<\xi<\xi_4,$$

where C_3 is an integral constant.

Case 2. When $(a^2\delta-\frac{4c}{a})^2+24\cdot\frac{e}{a}\le 0$, we need to consider three types of orbits, as orbits $\Pi_1(\Pi_3)$, $\Pi_2^+(\Pi_2^-)$ and Ω shown in Figure 1(b-c).

Consider the orbits Π_1 or Π_3 whose energy is not equal to the energy of cusp C. Similar to the calculation of the orbit Γ_1, it is not difficult to get the eighth type of traveling wave solution of (2.4)

$$v_8(\xi)=r_{13}+\sqrt{3}\cdot\left|r_{13}+\frac{1}{12}\left(a^2\delta-\frac{4c}{a}\right)\right|-\frac{2\sqrt{3}\cdot\left|r_{13}+\frac{1}{12}\left(a^2\delta-\frac{4c}{a}\right)\right|}{1-cn\left(\sqrt[4]{3\left(r_{13}+\frac{1}{12}\left(a^2\delta-\frac{4c}{a}\right)\right)^2}\cdot|\xi|\right)},\quad 0<|\xi|<\xi_5,$$

where the real parameter $r_{13}\neq\frac{c}{3a}-\frac{a^2\delta}{12}$ and $\xi_5=\frac{2}{\sqrt[4]{3\left(r_{13}+\frac{1}{12}\left(a^2\delta-\frac{4c}{a}\right)\right)^2}}\int_0^{\frac{\pi}{2}}\frac{d\vartheta}{\sqrt{1-\frac{\sqrt{3}\cdot\left|r_{13}+\frac{1}{12}\left(a^2\delta-\frac{4c}{a}\right)\right|+\frac{1}{8}\left(a^2\delta-\frac{4c}{a}\right)+\frac{3}{2}r_{13}}{2\sqrt{3}\cdot\left|r_{13}+\frac{1}{12}\left(a^2\delta-\frac{4c}{a}\right)\right|}\cdot\sin^2\vartheta}}$.

We integrate $v_8(\xi)$ once to obtain the eighth type of traveling wave solution of (1.1) as follows

$$u_8(\xi)=\left(r_{13}-\sqrt{3}\cdot\left|r_{13}+\frac{1}{12}\left(a^2\delta-\frac{4c}{a}\right)\right|\right)\cdot\xi+\sqrt[4]{3\left(r_{13}+\frac{1}{12}\left(a^2\delta-\frac{4c}{a}\right)\right)^2}\cdot\left[E\left(\sqrt[4]{3\left(r_{13}+\frac{1}{12}\left(a^2\delta-\frac{4c}{a}\right)\right)^2}\cdot\xi\right)+\frac{sn\left(\sqrt[4]{3\left(r_{13}+\frac{1}{12}\left(a^2\delta-\frac{4c}{a}\right)\right)^2}\cdot\xi\right)\cdot dn\left(\sqrt[4]{3\left(r_{13}+\frac{1}{12}\left(a^2\delta-\frac{4c}{a}\right)\right)^2}\cdot\xi\right)}{1-cn\left(\sqrt[4]{3\left(r_{13}+\frac{1}{12}\left(a^2\delta-\frac{4c}{a}\right)\right)^2}\cdot\xi\right)}\right],$$

where $0<|\xi|<\xi_5$

Then, we consider another unbounded orbits Π_2^+ or Π_2^- whose energy is equal to the energy of cusp C. Its expression can be determined by the following integrals

$$\int_{-\infty}^{v}\frac{dv}{(r_{14}-v)\sqrt{r_{14}-v}}=\int_0^{\xi}2d\xi,\quad \xi>0,$$
$$-\int_{v}^{-\infty}\frac{dv}{(r_{14}-v)\sqrt{r_{14}-v}}=\int_{\xi}^{0}2d\xi,\quad \xi<0,$$

where the real parameter $r_{14}=\frac{c}{3a}-\frac{a^2\delta}{12}$ satisfies the relation $-\infty<v<r_{14}$. By a direct calculation, the ninth type of traveling wave solution of (2.4) has the following form

$$v_9(\xi)=r_{14}-\frac{1}{\xi^2},\quad |\xi|>0.$$

Therefore, the ninth type of traveling wave solution of (1.1) can be given by

$$u_9(\xi)=\int v_9(\xi)\,d\xi=r_{14}\cdot\xi+\frac{1}{\xi}+C_4,\quad |\xi|>0.$$

where C_4 is an integral constant.

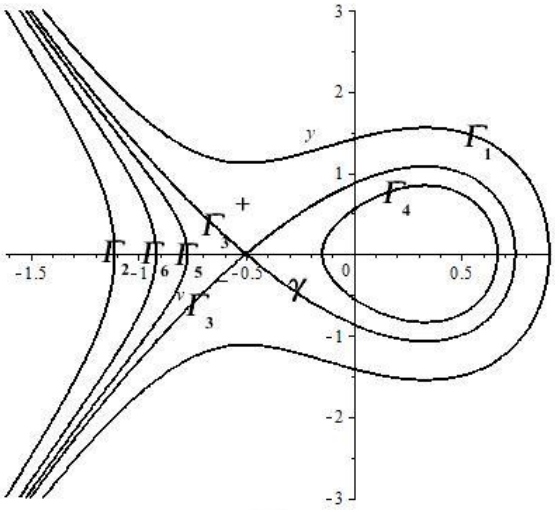
Finally, similar calculation process can be applied to compute the corresponding traveling wave solution of the orbit Ω, so wo directly give its expression as follows

$$v_{10}(\xi)=r_{15}+\sqrt{3r_{15}^2+\frac{1}{2}\left(a^2\delta-\frac{4c}{a}\right)\cdot r_{15}-\frac{e}{2a}}-\frac{2\sqrt{3r_{15}^2+\frac{1}{2}\left(a^2\delta-\frac{4c}{a}\right)\cdot r_{15}-\frac{e}{2a}}}{1-cn\left(2\sqrt[4]{3r_{15}^2+\frac{1}{2}\left(a^2\delta-\frac{4c}{a}\right)\cdot r_{15}-\frac{e}{2a}}\cdot|\xi|\right)},$$

where the parameter r_{15} is a real number, $0<|\xi|<\xi_6$ and $\xi_6=\frac{2}{\sqrt[4]{3r_{15}^2+\frac{1}{2}\left(a^2\delta-\frac{4c}{a}\right)\cdot r_{15}-\frac{e}{2a}}}\int_0^{\frac{\pi}{2}}\frac{d\vartheta}{\sqrt{1-\frac{\sqrt{3r_{15}^2+\frac{1}{2}\left(a^2\delta-\frac{4c}{a}\right)\cdot r_{15}-\frac{e}{2a}}+\frac{1}{8}\left(a^2\delta-\frac{4c}{a}\right)+\frac{3}{2}r_{15}}{2\sqrt{3r_{15}^2+\frac{1}{2}\left(a^2\delta-\frac{4c}{a}\right)\cdot r_{15}-\frac{e}{2a}}}\cdot\sin^2\vartheta}}$. At last, the tenth type of traveling wave solution of (1.1) is

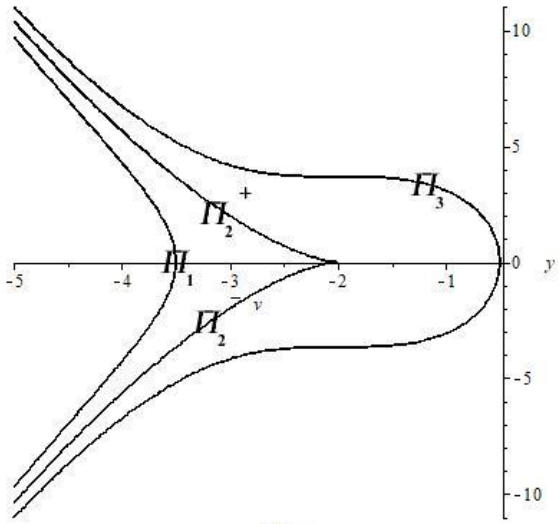
$$u_{10}(\xi)=\left(r_{15}-\sqrt{3r_{15}^2+\frac{1}{2}\left(a^2\delta-\frac{4c}{a}\right)\cdot r_{15}-\frac{e}{2a}}\right)\cdot\xi+\sqrt[4]{3r_{15}^2+\frac{1}{2}\left(a^2\delta-\frac{4c}{a}\right)\cdot r_{15}-\frac{e}{2a}}$$
$$\cdot\left[E\left(2\sqrt[4]{3r_{15}^2+\frac{1}{2}\left(a^2\delta-\frac{4c}{a}\right)\cdot r_{15}-\frac{e}{2a}}\cdot\xi\right)\right.$$
$$\left.+\frac{sn\left(2\sqrt[4]{3r_{15}^2+\frac{1}{2}\left(a^2\delta-\frac{4c}{a}\right)\cdot r_{15}-\frac{e}{2a}}\cdot\xi\right)\cdot dn\left(2\sqrt[4]{3r_{15}^2+\frac{1}{2}\left(a^2\delta-\frac{4c}{a}\right)\cdot r_{15}-\frac{e}{2a}}\cdot\xi\right)}{1-cn\left(2\sqrt[4]{3r_{15}^2+\frac{1}{2}\left(a^2\delta-\frac{4c}{a}\right)\cdot r_{15}-\frac{e}{2a}}\cdot\xi\right)}\right],$$

where $0<|\xi|<\xi_6$.



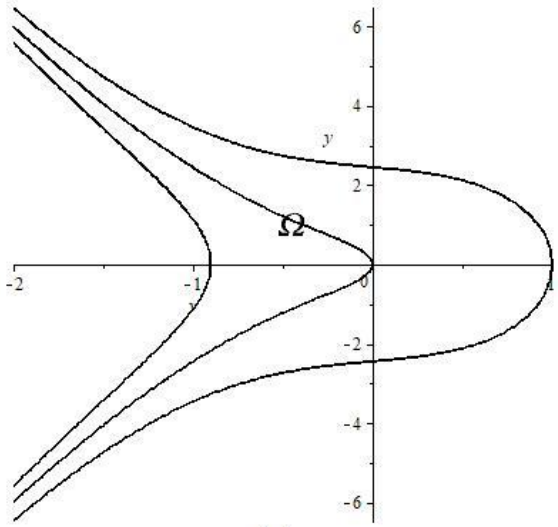
(a)

Fig-1



(b)

Fig-2



(c)

Fig-3: "The phase portraits of (2.4) in different parameter bifurcation sets"

CONCLUSIONS

In this paper, we apply the bifurcation method of dynamical system to study all types of traveling waves of the BKP equation. This method allows detailed analysis on phase space geometry of the traveling wave system to clearly identify all possible traveling waves and corresponding existence conditions of the BKP equation under different parameter ranges. Our results show that all traveling wave solutions of the BKP equation can be divided into ten types and their exact expressions can be completely given by direct calculations or elliptic integrals. These obtained solutions not only facilitate the verification of numerical solvers and the stability analysis of solutions, but also help to understand the dynamic behavior of the nonlinear wave field.

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