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The Consistent Criteria for Testing Hypotheses

Z. Zerakidze*, M. Chania
Gori University, Gori, Georgia; Sukhumi University Tbilisi, Georgia

***Corresponding author**
Z. Zerakidze

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Abstract: The present theory a consistent criteria for testing hypotheses can be used, for example, in the reliably predication of different engineering designs. In the paper there are discussed statistical structures $\{E, S, \mu_i, i \in I\}$. We prove sufficient conditions for extence of such criteria and we prove conditions for extence exstremal points.
Keywords: Consistent, Testing, reliably, engineering designs.

INTRODUCTION

In the general theory of testing hypotheses there often arises a problem of transition from weakly separated family of probability measure to the corresponding strongly separated family.In the ZF theory Z. Zerakidze (see[3] –[4]) proved that the countable family of probability, ortogonaly and strongly separability are aquivalent.

The consistent criteria for testing hypotheses

Let (E, S) be a measurable space with a geven family of probability measures: $\{\mu_i, i \in I\}$. The following definitions are taken from the works ([1]-[8]).

Definition 2.1. An object $\{E, S, \mu_i, i \in I\}$ is called a statistical structure.

Definition 2.2. A statistical structure $\{E, S, \mu_i, i \in I\}$ is called orthogonal if the family of probability measures $\{\mu_i, i \in I\}$ are pairwise singular measures.

Definition 2.3. A statistical structure $\{E, S, \mu_i, i \in I\}$ is called weakly separable if there exists a family S - measurable sets $\{X_i, i \in I\}$ such that the relations are fulfilled:

$$\mu_i(X_j) = \begin{cases} 1, & if\, i = j \\ 0, & if \quad i \neq j \end{cases}$$

Definition 2.4. A statistical structure $\{E, S, \mu_i, i \in I\}$ is called strongly separable if there exists a family S - measurable sets $\{X_i, i \in I\}$ such that the relations are fulfilled:

$$1) \quad \mu_i(X_i) = 1, \ \forall i \in I;$$

$$2) \quad X_i \cap X_j = \varnothing, \ \forall i, j, \ i \neq j, i, j \in I;$$

$$3) \bigcup_{i \in I} X_i = E.$$

Let H be the set hypotheses and $\{\mu_h, h \in H\}$ be probability measures definded on the measurable space (E, S). For each $h \in H$ denote $\overline{\mu}_h$ the complection of the measure μ_h, and denote by dom($\overline{\mu}_h$) the σ -algebra of all $\overline{\mu}_h$ measurable subsets of E. Let

$$S_1 = \bigcap_{h \in H} dom(\overline{\mu}_h).$$

Let H be the set of hypotheses and $B(H)$ be σ -algebra of subsets of which contains all finite subsets of H .

Definition 2.5. We Will say that the singular statistical structure $\{E, S_1, \overline{\mu_h}, h \in H\}$ admits a counsistent criteria for testing hypotheses if there exists at last one measurable mapping

$$\delta : (E, S_1) \to (H, B(H)),$$

such that

$$\overline{\mu_h}(\{x : \delta(x) = h\}) = 1, \ \ \forall h \in H.$$

Remark 2.1. The definition and construction of the consistent criteria is studied z. zerakidze (see[2]).

Definition 2.6. Let G some σ -subalgebra of σ -algebra S_1 . Algebra G is called free (relatively hypotheses $h \in H$), if all restriction of probability measures $\{\mu_h, h \in H\}$ on the algebra G much up.

Definition 2.7. A statistical structure $\{E, S_1, \overline{\mu_h}, h \in H\}$ is called isolated, if minimal σ -algebra D relatively which measurable all function with from $h \to \mu_h(A), \ A \in S_1$ devides points on H .

Definition 2.8. A statistical structure $\{E, S_1, \overline{\mu_h}, h \in H\}$ is called strongly isolated if σ -algebra D contains all finite subsets of H .

Definition 2.9. A statistical structure $\{E, S_1, \overline{\mu_h}, h \in H\}$ is called decomposable, if there exist two such sub algebra $S_2, S_3 \subset S_1$ whose union generates σ -algebra $S_1 \cdot S_2$ is sufficient and S_3 is free.The such couple (S_2, S_3) is called decomposition of statistical structure $\{E, S_1, \overline{\mu_h}, h \in H\}$. For any set $G \subset 2^H$ by symbol $< G >$ we will denote the algebra generated by set G and $\sigma < G >$ the σ -algebra generated by set G .

Let $I^* = \bigcap_{h \in H} \{A \in S_1 : \mu_h(A) = 0\}$

Definition 2.10. Algebra $B_1 \subset S_1$ is called minimal sufficient, if B_1 is sufficient and for any sufficient algebra $B_1^{'}$ fulfilled condition $B_1 \subset \sigma < B_1^{'} \cup I^* >$.

Let $\Im$ some σ -subalgebra of algebra S_1 and μ -probability measure defined on $\Im$, we will denote by $S_\mu(S_1, \Im)$ the set of finite and finity additive continuations of measure μ on the σ -algebra S_1 and let $exS_\mu(S_1, \Im)$ the set its exstremal points. $S_\mu^\sigma(S_1, \Im)$ the set of all countable additive continuations of measure μ on the σ -algebra S_1 and $exS_\mu^\sigma(S_1, \Im)$ the set its exstremal points.

Is known, that $exS_\mu(S_1, \Im) \neq \varnothing$, but the set $ex\, S_\mu^\sigma(S_1, \Im)$ my be empty ([7]).

Example 2.1.

In the terminology of [8] let $ba(\Sigma, \nu, \Sigma')$ denote the set of all $\mu \in ba(S, \Sigma')$ with $\mu \geq 0$ and $\mu(S) = 1$, such that $\mu / \Sigma = \nu$, where Σ' is a field of subset of a set S , Σ is subfield of Σ' and $\nu \in ba(S, \Sigma)$ with $\nu \geq 0$ and $\nu(S) = 1$. The set $ca(\Sigma, \nu, \Sigma')$, where Σ and Σ', $\Sigma \subset \Sigma'$, denote σ -fields and ν is a probability measure on Σ , is defined ih the same way. Whereas in the case $ca(\Sigma, \nu, \Sigma')$ the set of exstremal points may be empty. Take , for example, for S the set of real numbers, $\Sigma = \{B \subset S / B\}$ resp. B^c is countable, Σ' is defined to be set of Borel subsets of S, and ν is defined by $\nu(B) = 0$, resp.1 if B, resp. B^c is countable.

1. The counsistent criteria for testing hypotheses ih Hilbert space of measures and extremal points.

Let M^σ be a real linear space of all alternating finite measures on S .

Definition 3.1. A linear subset $M_H \subset M^\sigma$ is called a Hilbert space of measures if:

1) One can introduce on M_H a scale product (μ,ν), $\mu,\nu \in M_H$ is the Hilbert space and every mutually singular measures μ and ν, $\mu,\nu \in M_H$, the scale product $(\mu,\nu) = 0$;
2) If $\nu \in M_H$ and $| f(x) | \le 1$, then $\mu_f(A) = \int_A f(x)\nu(dx) \in M_H$, where $f(x)$ is a S_1-measurable real function and $(\nu_f,\nu_f) \le (\nu,\nu)$;
3) If $\nu \in M_H$, $\nu_n > 0$, $\nu_n(E) < +\infty m$ $n = 1,2,\ldots$ and $\nu_n \downarrow 0$, then for any $\nu \in M_H$ $\lim_{n\to\infty}(\nu_n,\mu) = 0$.

Remark 3.1. The definition and construction of the Hilbert space of measures is studied Z. Zerakidze (see[4]) The following theorem has also been proved in this paper (see[4])

Theorem 3.1. Let M_H is Hilberts space of measures then M_H is the straight sum Hilbert spaces $H_2(\overline{\mu_h})$ so $M_H = \bigoplus_{h\in H} H_2(\overline{\mu_h})$, where $H_2(\overline{\mu_h})$ is the family of measures $\nu(A) = \int_A f(x)\overline{\mu_h}(dx)$, $\forall A \in S_1$, that $\int_E | f(x) |^2 \overline{\mu_h}(dx) < +\infty$ and $\| \nu \|_{H_2(\overline{\mu}_h)} = \left(\int_A | f(x) |^2 \overline{\mu}_h(dx) \right)^{\frac{1}{2}}$.

Theorem3.2. Let $M_H = \bigoplus_{h\in H} H_2(\overline{\mu_h})$ be a Hilbert space of measure, E be the complete separable metric space, S_1 be Borel σ-algebra in E and $card H \le 2^{\chi_0}$, then if the correspondence

$$f \leftrightarrow \psi_f,$$

given by the equality

$$\int f(x)\nu(dx) = (\psi_f,\nu), \quad \forall \nu \in M_H$$

be one-to-one. Denote by $F = F(M_H)$ the set of real functions f for which $\int f(x)\overline{\mu_h}(dx)$ is defined $\forall \overline{\mu}_h \in M_H$.

Then the statistical structure $\{E, S_1, \overline{\mu_h}, h \in H\}$ admits a consistent criteria for testing hypotheses, and if the statistical structures $\{E, S_1, \overline{\mu_h}, h \in H\}$ is decomposition, (S_2, S_3) then $\overline{\mu}_h \in exS^{\sigma}_{\overline{\mu}_h}(S_1, S_3)$, $\forall h \in H$.

Proof. Let $f \in F(M_H)$ is corresponded with $\overline{\mu}_{h'} \in M_H$ for which $\int f(x)\overline{\mu}_h(dx) = (\overline{\mu}_{h'}, \overline{\mu}_h)$, then $\overline{\mu}_{h'}, \overline{\mu}_{h''} \in M_H$ we have $\int f_{h'}(x)\overline{\mu}_{h'}(dx) = (\mu_{h'}, \mu_{h''}) = \int f_1(x) f_2(x)\mu_{h'}(dx) = \int f_{h'}(x) f_2(x)\mu_{h'}(dx)$. So $f_{h'}(x) = f_1$ for almost with respect to measure $\overline{\mu}_{h'}$ and

$f_{h'}(x) > 0$, $\int f_{h'}^2(x)\overline{\mu}_{h'}(dx) < +\infty$, $\overline{\mu}_{h'}^* = \int f_{h'}(x)\mu_{h'}(dx)$, then $\int f_{h'}^*(x)\overline{\mu}_{h''}(dx) = (\overline{\mu}_{h'}, \overline{\mu}_{h''}) = 0$, $\forall h' \neq h''$,

On other $\overline{\mu}_{h'}(E - X_{h'}) = 0$, $X_{h'} = \{x : f_{h'}^*(x) > 0\}$. Hence it follows that $\overline{\mu}_{h'}(X_{h''}) = \begin{cases} 1, & if \quad h' = h'' \\ 0, & if \quad h' \neq h'' \end{cases}$, the statistical structure $\{E, S_1, \overline{\mu_h}, h \in H\}$ is weakly separable. Represent $\{\overline{\mu}_h, h \in H\}$, card $H \leq 2^{\chi_0}$ as an inductive sequence $\overline{\mu}_h < \omega_1$, where ω_1 denotes the first ordinal number of the power of the set H.

Sense the statistical structure $\{E, S_1, \overline{\mu_h}, h \in H\}$ is weakly separable, there exists a family S -measurable sets $\{X_h, h \in H\}$ such that the following are fulfilled

$$\overline{\mu}_{h'}(X_{h''}) = \begin{cases} 1, & if \quad h' = h'' \\ 0, & if \quad h' \neq h'' \end{cases}, \quad \forall h', h'' \in [0, \omega_1).$$

We define ω_1 sequence of parts Z_h of the space E, so that the following relations are fulfilled:

1) Z_h is Borel subset in E for all $h < \omega_1$;
2) $Z_h \subset X_h$ for all $h < \omega_1$;
3) $Z_{h'} \cap X_{h''} = \varnothing$ for all $h' < \omega_1$, $h'' < \omega_1$, $h' \neq h''$;

Assume that $Z_{h_0} = X_{h_0}$, Let further the particular sequence $\{Z_{h_j}\}_{j<i}$ be already defined for $i < \omega_1$. It is clear, that $\mu^*(\bigcup_{j<i} Z_{h_j}) = 0$, (see[3]). Thus there exists a Borel subset Y_{h_i} of space E such that the following relations are valid: $\bigcup_{j<i} Z_{h_j} \subset Y_{h_i}$ and $\overline{\mu}(Y_{h_i}) = 0$. Assume $Z_{h_i} = X_{h_i} - Y_{h_i}$, there by the ω_1 sequence of $\{Z_{h_j}\}_{j<\omega_1}$ disjuntive measurable subsets of space E is ciunstructed, Therefore $\overline{\mu}_h(X_h) = 1$, $\forall h < \omega_1$. A statistical structure $\{E, S_1, \overline{\mu_h}, h \in H\}$, card $H \leq 2^{\chi_0}$ is strongly separated there there exists a family $\{Z_h\}_{h\in H}$ of elements of σ -algebra $S_1 = \bigcap_{h\in H} dom(\overline{\mu}_h)$ such that:

1. $\overline{\mu}_h(X_h) = 1$, $\forall h \in H$;
2. $Z_{h'} \cap Z_{h''} = \varnothing$, $\forall h', h'' \in H$, $h' \neq h''$;
3. $\bigcup_{h\in H} X_h = E$.

For $x \in E$,we put $\delta(x) = h$, where h is unique hypotheses from the set H for which $x \in Z_{h'}$. The extence of such a unique hypotheses H can be proved using conditions 2), 3). Now let $Y \in B(H)$.

Then $\{x : \delta(x) \in Y\} = \bigcup_{h\in Y} Z_h$. We must show that $\{x : \delta(x) \in Y\} \in dom(\overline{\mu}_{h_0})$ for each $h_0 \in H$. If $h_0 \in Y$, then $\{x : \delta(x) \in Y\} = \bigcup_{h\in Y} Z_h = Z_{h_0} \bigcup (\bigcup_{h\in Y-\{h_0\}} Z_h)$.

On the one hand, from the validity of the condition 1), 2), 3)it follows that

$$Z_{h_0} \in S_1 = \bigcap_{h\in H} dom(\overline{\mu}_h) \subseteq dom(\overline{\mu}_{h_0})$$

On the other hard, the validity of the condition $\bigcup_{h\in Y-\{h_0\}} Z_h \subseteq (E - Z_{h_0})$ implies that $\overline{\mu}_{h_0}(\bigcup_{h\in Y-\{h_0\}} Z_h) = 0$.

The last equality yields that $\bigcup_{h\in Y-\{h_0\}} Z_h \in dom(\overline{\mu}_{h_0})$. Since $dom(\overline{\mu}_{h_0})$ is σ -algebra, we deduce that

$\{x:\delta(x)\in Y\}=Z_{h_0}\cup(\bigcup_{h\in Y-\{h_0\}}Z_h)\in dom(\overline{\mu}_{h_0})$. If $h_0\notin Y$, then $\{x:\delta(x)\in Y\}=\bigcup_{h\in H}Z_h\subseteq(E-Z_{h_0})$ and we conclude that $\overline{\mu}_{h_0}\{x:\delta(x)\in Y\}=0$ the last relation implies that $\{x:\delta(x)\in Y\}\in dom(\overline{\mu}_{h_0})$ for an arbitrary $h_0\in H$. Hance $\{x:\delta(x)\in Y\}\in\bigcap_{h\in H}dom(\overline{\mu}_{h_0})=S_1$.

We have shown that the $map\,\delta:(E,S_1)\to(H,B(H))$ is measurable map. and $\overline{\mu}_h\{x:\delta(x)=h\}=1,\ \forall h\in H$. Thus the statistical structure $\{E,S_1,\overline{\mu_h},h\in H\}$ admits a consistent criteria for testing hypotheses. The extence of a consistent criterium for testing hypothases $\delta:(E,S_1)\to(H,B(H))$.
Let the linear operator u is denoted by

$$u(f)=\int_E f(x)\overline{\mu}_h(dx),\quad f\in B(E,S_1).$$

This operator u is appositive isometric operator with norm $\|u\|=1$ and $u:B(E_1,S_1)\to(H,B(H))\cdot(uB(E_1,S_1))=(H,B(H))$. σ -algebra $\delta^{-1}(H)$ is minimai sufficient.In what follows $B(E,S_1)$ will always measurable functions on (E,S_1) having the natural order and with norm $\|f\|=\sup_{x\in E}|f|$, as $S_1=\bigcap_{h\in H}dom(\overline{\mu}_h)$, then $S_1=\sigma<\delta^{-1}(B(H))\cup\Im^*>$,
Where $\Im^*=\bigcap_{h\in H}\{A\in S:\mu_h(A)=0\}$.

If statistical structure $\{E,S_1,\overline{\mu_h},h\in H\}$ is decomposable $(\delta^{-1}B(H),G_2)$, where $\delta^{-1}B(H)$ is sufficient algebra and G_2 is free algebra then algebras S_1 and G_2 also is decomposable and $\forall A\in S_1,\ A=C\Delta I,\ C\in\delta^{-1}(B(H))$, $I\in\Im^*$ and $\mu_h(A\Delta C)=0$. This denote that $\mu_h\in\exp S^{\sigma}_{\mu}(S_1,\delta^{-1}(B(H)),\ \forall h\in H$ (see [7], Theorem 1).

REFERENCES

1. Ibramhalilov IA. Skorokhod. Consistent criteria of parameters of random processes. Kiev. 1980.
2. Zerakidze Z. Generalization of Neumann-Pearson criterion. Collected scientific of works (in Georgian). The ministry of Edication and Science of Georgia, Gori state University, pp 63-69. Tbilisi 2005.
3. Zerakidze Z. On weekly divisible and divisible families of probability measures. Bull. Acad. Sci. Georgia SSR 113, 1984.
4. Zerakidze Hilbert space of measure Z. Ukr. Math. Journ. 1986; 38(2): p.148-154. Kiev.
5. Eliauri L, Mumladze M, Zerakidze Z. Consistent Criteria for checking Hypotheses. Journal of Mathematics and System Science. 2013 Oct 1;3(10):514.
6. Akaike H. A new look at the statistical model identification. IEEE transactions on automatic control. 1974 Dec;19(6):716-23.
7. Plachky D. Extremal and monogenic additive set functions. Proceedings of the American Mathematical Society. 1976;54(1):193-6.
8. Dunford N, Schwartz JT. Linear operators part I: general theory. New York: Interscience publishers; 1958.