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An Exact and Simple Solution to the "Trisecting an Angle" Problem Using Straightedge and Compass

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Abstract **Original Research Article**

There are three classical problems in ancient Greek mathematics that were highly influential in the development of geometry: squaring the circle**,** trisecting an angle, and doubling the cube. The problem of angle trisection, in particular, involves constructing an angle that is exactly one-third of a given arbitrary angle using only two tools: an unmarked straightedge and a compass. This thesis focuses on the problem of trisecting an arbitrary angle. I present a classical straightedge-and-compass construction that achieves exact trisection, avoiding the explicit use of π and employing a ruler-based geometric analysis and synthesis approach. While it is relatively straightforward to trisect certain special angles (e.g., a right angle), trisecting a general angle has historically been considered impossible under classical constraints. The origins of the trisection problem are difficult to date precisely. The result of this research provides an exact construction-based solution to the long-standing challenge of trisecting an angle using only a straightedge and compass in Euclidean geometry. A solution to this classical problem of trisecting an angle—a challenge that has persisted since ancient Greece—was published in the journal *SJPMS* [9]. Although that solution employs only theorems and corollaries from high school geometry, it remains somewhat complex. This article, titled "An Exact and Simple Solution to the 'Trisecting an Angle' Problem Using Straightedge and Compass" presents a shorter and simpler solution compared to the 2024 publication. Using the methods of analysis and synthesis**,** along with straightedge and compass constructions in Euclidean geometry, this paper arrives at an exact—not approximate—solution. Additionally, the study introduces a newly invented tool called the "Trisection Ruler," which enables quick and precise trisection of any given angle.

Keywords: Angle trisection, divide angle into three equal parts, compass and straightedge construction, doubling the cube, squaring the circle.

1- INTRODUCTION

For over three millennia, three classical geometric problems from ancient Greece—Squaring the Circle, Doubling the Cube, and Trisecting an Angle—have challenged the ingenuity of mathematicians. Originally posed with the constraint of using only an unmarked straightedge and compass, these problems were eventually declared impossible to solve within the framework of Euclidean geometry. Foundational impossibility proofs by 19th-century mathematicians such as Pierre Wantzel and Ferdinand von Lindemann—employing algebraic field theory and transcendental number theory—seemed to confirm that exact solutions were unattainable [4,5,2]. Specifically, the transcendence of π implied that a square with an area exactly equal to that of a given circle could not be constructed through purely geometric means. Yet, while these algebraic impossibility theorems are rigorous within their own domains, they diverge in nature from the original geometric spirit of the ancient challenges. The historical emphasis on purely constructive geometry—devoid of algebra, arithmetic, or trigonometry—leaves room for a critical re-evaluation.

The problem of trisecting an arbitrary angle is one of the three classical challenges of ancient Greek mathematics, alongside the problems of squaring the circle and doubling the cube. These geometric problems have captivated the interest of mathematicians for over three millennia. Traditionally posed with the constraint of using only an unmarked straightedge and compass, they have long been considered unsolvable within the framework of Euclidean geometry.

In the 19th century, foundational impossibility proofs by Pierre Wantzel and Ferdinand von Lindemann—based on algebraic field theory and transcendental number theory—seemed to definitively confirm the infeasibility of exact solutions to these problems. In particular, the transcendence of π, as established by Lindemann in 1882, rendered squaring the circle impossible by classical construction *(Ferdinand von Lindemann, 1882, "Ueber die Zahl π", Mathematische Annalen 20 (1882), pp. 213-225.).* Similarly, Wantzel's 1837 result proved that arbitrary angle trisection and cube duplication were algebraically unattainable using only compass and straightedge [4,5].

Despite the mathematical rigor of these impossibility proofs within their respective algebraic and arithmetic frameworks, they diverge from the spirit of classical geometry, which is fundamentally constructive and geometric in nature ONLY. In this light, it is reasonable to re-evaluate these ancient problems from a purely geometric standpoint, devoid of algebraic abstractions [2,3,19].

This article asserts that the problem of angle trisection—long deemed impossible—has been effectively resolved within the realm of Euclidean geometry. The solution, developed using only a straightedge and compass, is rooted in the classical methods of analysis and synthesis, and is presented in the previous paper, "*Exact Angle Trisection with Straightedge and Compass by Secondary Geometry*" in 2023 [9]. This construction yields a complete and rigorous solution to the angle trisection problem, although the method remains relatively complex in its current form.

Building upon this foundation, the present research introduces a significantly simplified and exact solution to the trisection of arbitrary angles, remaining strictly within the constraints of Euclidean tools. Furthermore, a novel geometric instrument—the Trisection Ruler—was devised during the course of this work. This innovative tool allows for the rapid and precise trisection of any given angle and represents a practical advancement in constructive geometry.

This research also draws upon a philosophical principle, inspired by the Lao Tzu's aphorism in the *Tao Te Ching* book: "The Great Tao is simple, very simple" (大道至簡). Emphasizing the power of simplicity and concentric structure, the author develops exact geometric solutions not only to angle trisection but also to the long-standing problems of squaring the circle and doubling the cube. These solutions were derived exclusively through geometric reasoning [7,10].

The paper titled "Exact Solution to the Squaring the Circle Problem" (SJPMS, 2024) [7] presents a counterexample to the impossibility proof traditionally attributed to Lindemann. It provides a precise construction of a square with area equal to that of a given circle using only Euclidean tools—without invoking the transcendental nature of π. In addition, the inverse problem—"Circling the Square"—was solved and published in 2024 [8], and extended further to a newly posed problem: "Circling the Regular Hexagon."

The latter challenge entails constructing a circle that is concentric with, and has the same area as, a given regular hexagon. This problem—never before addressed in the mathematical literature—arose naturally from the insights developed in solving the square-circling problem. The method involves identifying a regular dodecagon inscribed in the circle, which shares its twelve vertices with the extended sides of the hexagon. The research rigorously proves that this dodecagon, inscribed via compass and straightedge, leads to a circle with an area exactly equal to that of the hexagon. A new construction tool, the Regular Dodecagon Ruler, is introduced to facilitate this process [16,17].

The methodology of this article "*AN EXACT AND SIMPLE SOLUTION TO THE TRISECTING AN ANGLE PROBLEM USING STRAIGHTEDGE AND COMPASS"* is grounded in the ancient geometric techniques of analysis and synthesis. In the analytical phase, one assumes the problem has already been solved and works backward to reduce it to a known, solvable configuration. In the synthetic phase, the solution is reconstructed step-by-step from the initial conditions. These dual methods are essential to geometric reasoning and have been instrumental in the development of the new constructions presented in this paper.

Finally, this research contributes to the ongoing philosophical discussion concerning the nature of mathematical truth. While impossibility theorems hold within specific logical frameworks, the results here suggest that alternative constructive approaches—consistent with the original spirit of Euclidean geometry—may yield exact solutions previously deemed unattainable. This aligns with Karl Popper's philosophy of science: that knowledge remains provisional and subject to falsification by new evidence [1]. Thus, just as earlier "truths" in mathematics and science have been revised in light of new discoveries, the classical problems of antiquity may now be open to renewed understanding through purely geometric means.

2. PROPOSITIONS

For a given angle, one can use a straightedge and compass to easily construct its bisector. By standard definition, the two rays that divide a given angle into three equal smaller angles are called *trisectors*.

2.1 Angle Trisection Theorem:

Given an angle $\widehat{UOV}$less than 180°, its TRISECTORS can be constructed exactly and accurately using only a straightedge and compass.

NOTE: 'Aim to prove:

- With Geometric Analysis & Synthesis, one can use straightedge and compass to construct line segment AB which is perpendicular to the bisector of a given angle and the centre P of AB.
- The Circle (P, r = AP) has an upside-down regular semi-hexagon ABCD which intersects the two trisectors of the angle at points C & D.'

PROOF:

Let $\widehat{\mathbf{UOV}}$ be a given angle to be trisected and an arbitrary point A on the angle side, then draw AB which is perpendicular to the angle bisector at P (Figure 1 below). And then a circle (O, r = ½AB), having an arbitrary radius r, cuts sides OU & OV at points A & B. It is easy to use straightedge & compass to identify the centre P of AB (Figure 1 below).

If there exists trisectors OS & OT of the given angle, then OS is the bisector of the angle $\widehat{\mathbf{UOT}}$, and then with straightedge & compass, we can construct circle (P, AP) which intersects OS & OT at the points C & D, (Figure 1 below). Because OS is the bisector of the angle UÔT, circle (D, DP = AP = PB) creates 2 equal isosceles triangles EOD and DOC. Due to the fact that triangle DOC is an isosceles triangle, EOD is also an isosceles triangle (Figure 1 below).

Consider the intersect point Q of the circle (D, DP) and the bisector of the isosceles triangle EOD, where the triangle EQD is an equilateral triangle, due to DQ = DE are radiuses of the circle (D, DP) and Triangle EOD is isosceles with perpendicular bisector OQ showing EQ = QD (Figure 1 below),

triangle EQD is an equilateral triangle. Then with the symmetry property via the axe OD of the two isosceles triangles EOD & DOC, triangle DPC (red colour in Figure 1 below) is also equilateral with angle $\widehat{\mathbf{DPC}}$ = 60°. This fact shows that
$\widehat{APD}$ = $\widehat{CPB}$ = 60° too (Figure 1 below).

Thus, ABCD is an upside down regular semi-hexagon, which inscribed in the circle (P, AP) and has two vertices C & D lying in the trisectors OT & OS, respectively. And finally, the two TRISECCTORS OS & OT (blue colour in Figure 1) is constructive by a straightedge, as required.

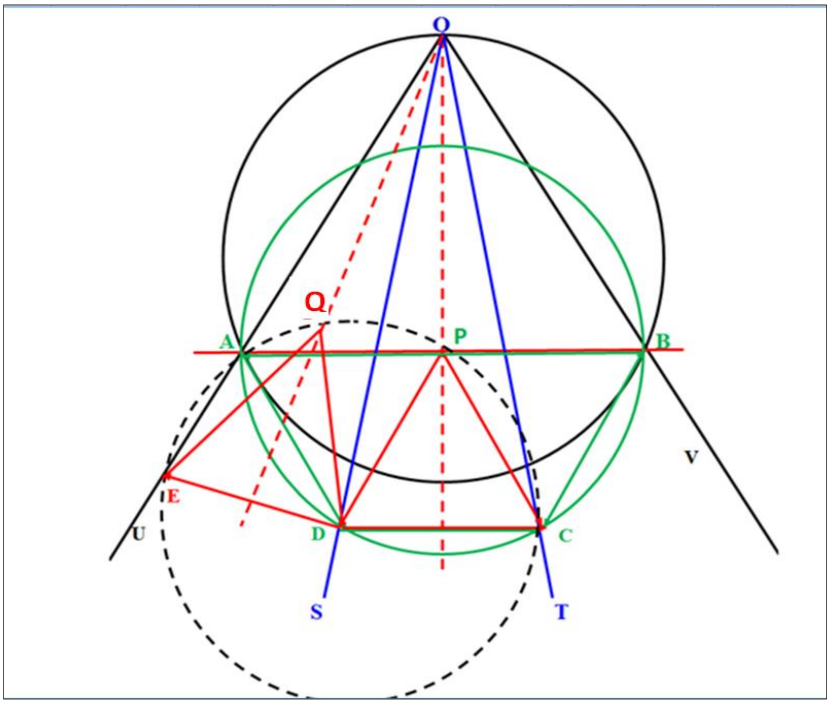


Figure 1: The upside down regular semi-hexagon ABCD and two trisectors OS and OT of the given angle $\widehat{\mathbf{UOV}}$

2.2 Definition of Trisection Ruler:

Let $\widehat{\mathbf{UOV}}$ be a given angle with internal trisectors OS and OT. Let AB be a line segment perpendicular to the angle bisector, and let P be the midpoint of AB. Define C(P, AP) as the circle centred at point P with radius AP.

If the geometric figure formed—denoted as the inverted semi-hexagon ABCD—satisfies the construction criteria specified in Figure 2, then this figure is called a *TRISECTION RULER* for the given angle (Figure 2, below).

"Note that there exist infinitely many points A on the given angle ray OU; therefore, one can have infinitely many Trisection Rulers for a given angle."

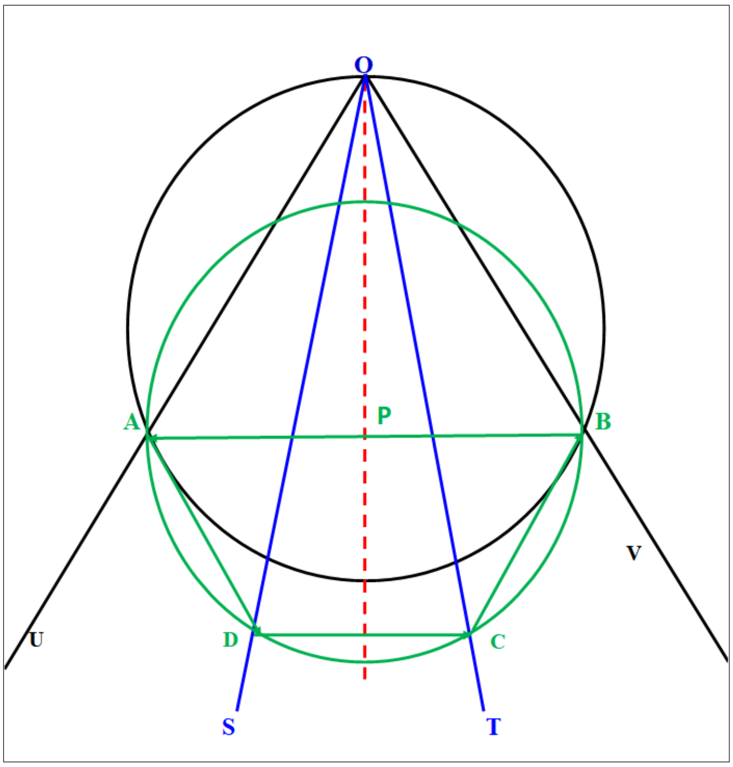


Figure 2: Trisection Ruler ABCD and the two trisectors OS & OT of the given angle $\widehat{UOV}$.

2.3 Angle Trisection Corollary

Let R be a Trisection Ruler ABCD of an angle $\widehat{UOV}$. Then, OD and OC are the two trisectors of the given angle $\widehat{UOV}$.

Proof:

Consider the angle $\widehat{UOV}$ with a Trisection Ruler ABCD as defined in sections 2.1 and 2.2. Next, draw the circle centred at D with radius AD such that it intersects line OU at point E and passes through point C. This gives us the relationship:

DE = DC (1)

From point E, draw a line parallel to OD. From point O, draw line OF parallel to DE to form parallelogram ODEF with the following properties:

- OE is the shorter diagonal,
- EF = OD and OF = DE.

Next, from point C, draw a line parallel to OD, and from point O, draw line OG parallel to CD to create parallelogram ODCG (as shown in green in Figure 3 below), with the following properties:

- CD = OG and OG = OF = DE,
- OD = CG and CG = EF = OD,
- OC is the shorter diagonal.

By the properties of the parallelograms ODEF and ODCG, we conclude that they are equal, leading to the equality of their shorter diagonals:

OE = OC (2)

From equations (1) and (2), and considering the shared side OD, we find that triangles ODE and OCD are congruent and isosceles. Thus, we have:

Angle $\widehat{UOS}$ = angle $\widehat{SOT}$ (3)

A similar proof can be applied to show that the shorter diagonals OD and OE' of the equal parallelograms OCDF' and OCE'G' are also equal (*Figure 4, below*). Consequently, this implies that triangles OCD and OCE' are congruent, leading to:

OC = OE' and angle $\widehat{SOT}$ = angle $\widehat{TOV}$ (4)

Thus, from equations (3) and (4), we conclude that the three angles $\widehat{UOS}$, $\widehat{SOT}$ and $\widehat{TOV}$ are equal as required.

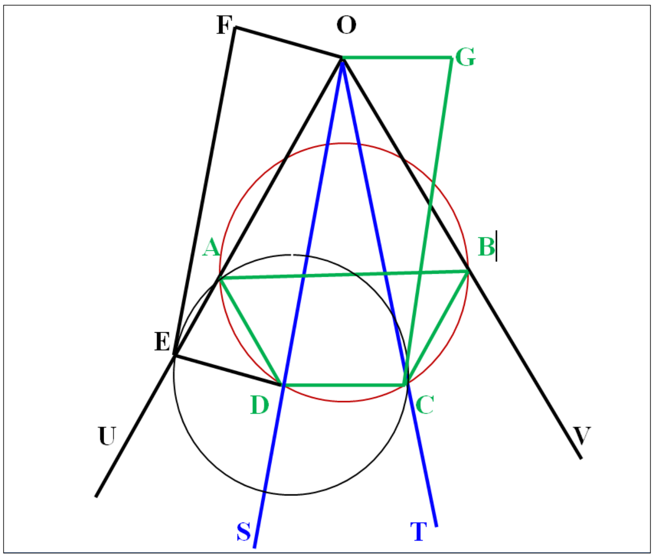


Figure 3: The circle (D, AD), the 2 equal parallelograms ODEF (black colour) & ODCG (green colour) and the two trisectors OS & OT of the given angle $\widehat{UOV}$

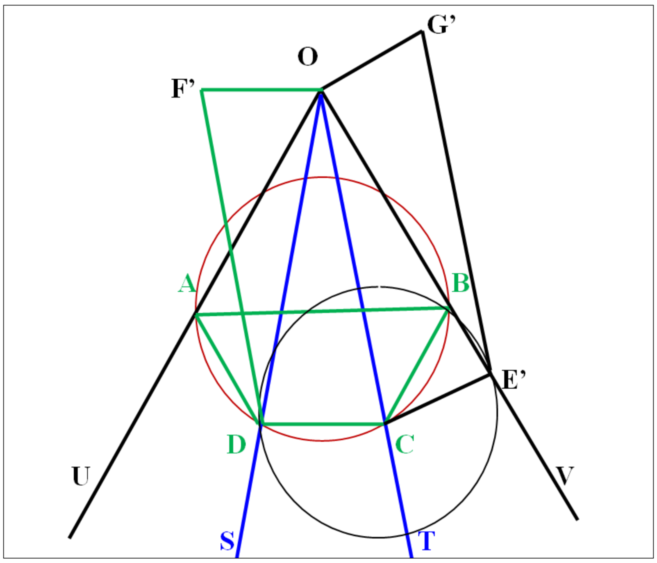


Figure 4: Circle (C, CD) two equal parallelograms OCDF' and OCE'G', and two equal diagonals OD and OE' for the similar proof above

Note that the above sections 2.1 & 2.3 can be combined into a short THEOREM, as follow:

"Exact trisection of a given angle with straightedge & compass is constructible if and only if the angle has a Trisection Ruler defined in Section 2.2 above"

2.4 ANGLE TRISECTION METHOD

By the Trisection Theorem, the definition of the Trisection Ruler and section 2.3 above, an angle trisection method can be described as follows (Figure 5 below):

Step 1: Using a straightedge and compass, choose an arbitrary point A on one side of the given angle. Then draw a line segment through A that is perpendicular to the bisector of the angle (see Figure 3 below). This line intersects the other side of the angle at point B.

Step 2: With the straightedge and compass, draw a semi-hexagon ABCD, where the large base AB is upside down.

Step 3: Connect OC and OD to create the two trisectors OS and OT of the given angle $\widehat{UOV}$ (see Figure 5 below).

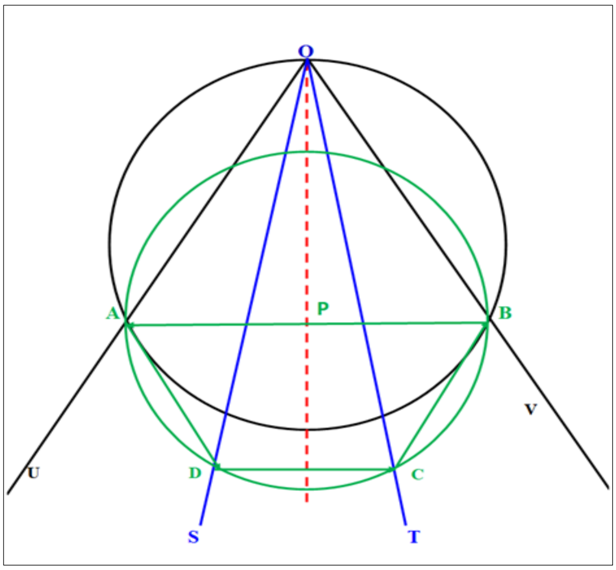


Figure 5: Shapes (Circles, Regular Semi-hexagon) for creating the Trisection Ruler in the Angle Trisection Method which is applied for a given angle $\widehat{\text{UOV}}$

3. SPECIAL CASE

3.1 Right Angle

It is very easy to divide a given right angle UÔV into 3 equal angles, 30° each. Choose an arbitrary length p, p ⊂ r, in the vertical side OU of UÔV to mark a point A, OA = p, p is an arbitrary length. Then take a compass & a straightedge to draw the circle (A, p) and an equilateral triangle AÔQ located in the right of the circles (A, p). And then, OQ is one trisectors of the angle $\widehat{\mathbf{UOV}}$, by the expression
{ $\widehat{\text{UOV}} - \widehat{\text{AOQ}} = 90° - 60° = 30° = \widehat{\text{QOV}}$ }.

At the end, draw the bisector of the 60-degree $\widehat{\mathbf{AOQ}}$ to get the other Trisectors (red arrow in Figure 6 below) of the given angle $\widehat{\mathbf{UOV}}$ = 90°. This method uses only a compass & a straightedge.

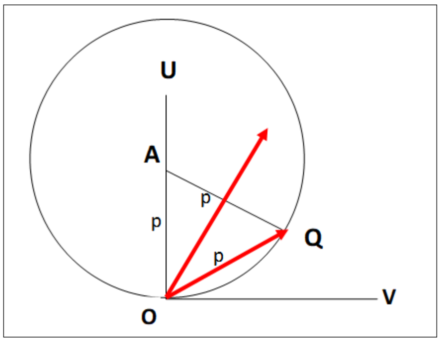


Figure 6: Trisectors (red colour) constructed for a given right angle $\widehat{UOV}$.

3.2 Flat angle (Straight-line angle/Angle = 180°)

For an angle $\widehat{\mathbf{UOV}}$ = 180°, with a compass & a straightedge, draw a equilateral triangle OAB, A is located in the left side OU of the given $\widehat{\mathbf{UOV}}$ and B is located under the horizon line OA, then OB is one trisector of the given flat angle $\widehat{\mathbf{UOV}}$. The other trisector of $\widehat{\mathbf{UOV}}$ is just the bisector of the angle $\widehat{\mathbf{BOV}}$, say OC, as the following Figure 7 below. This method uses only a compass & a straightedge.

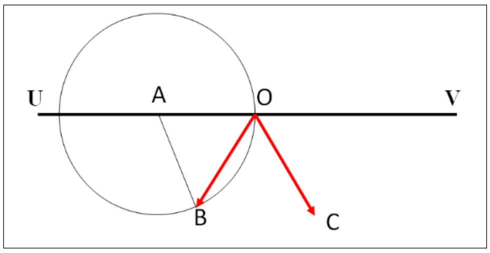


Figure 7: Two Trisectors OB & OC (red colour) of a given flat angle.

_3.3_ 180°<Angle < 270°

For a given angle $\widehat{UOV}$, 180° < $\widehat{UOV}$ < 270°, let OA be an extensive line of side OU of $\widehat{UOV}$ to the right-hand side (Figure 6 below). This straight line divides $\widehat{UOV}$ into 2 angles, which are a flat angle $\widehat{UOA}$ = 180° and an angle $\widehat{VOA}$ < 90°.

Then apply Section 2.4 above and use a compass & a straightedge to draw 2 trisectors (2 Trisecting Lines) OM & ON of the flat angle $\widehat{UOA}$ (these two trisectors divide this flat angle into three 60-degree angles). And then apply Section 2 above to draw two trisectors of the angle $\widehat{VOA}$, using a compass & a straightedge, where these two trisectors divide $\widehat{VOA}$ into 3 equal small angles, which are less than 30° (Figure 8 below).

The remain work is to place consecutively, one angle 60° and one small angle of a-third angle $\widehat{VOA}$) to the left side OU of the given angle $\widehat{UOV}$, to get one trisectors OP. Then use straightedge & compass to draw the bisector OQ of the angle $\widehat{POV}$, which is also the other trisectors of the given angle $\widehat{UOV}$, as required (Figure 8 below).

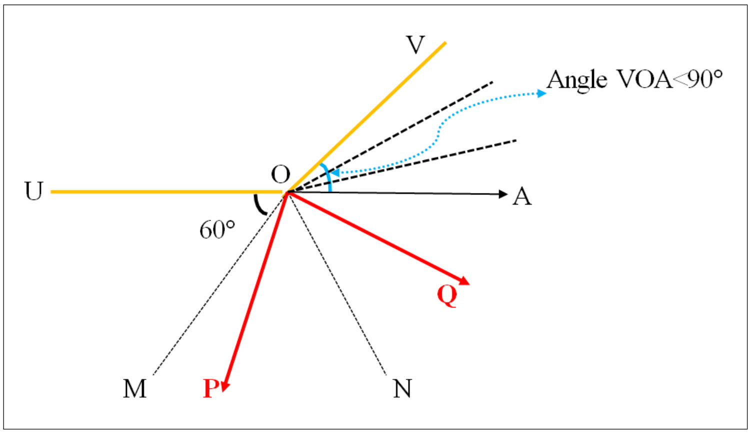


Figure 8: **Two trisectors OP & OQ of the angle $\widehat{UOV}$, 180° < $\widehat{UOV}$ < 270°**

_3.4_ 270° Angle

For any angle $\widehat{UOV}$ = 270°, which is equal to 3x90° (3 right angles), we only need to lengthen the angle sides from the angle vertex to get its 2 trisectors OA & OB, as the described Figure 9 below:

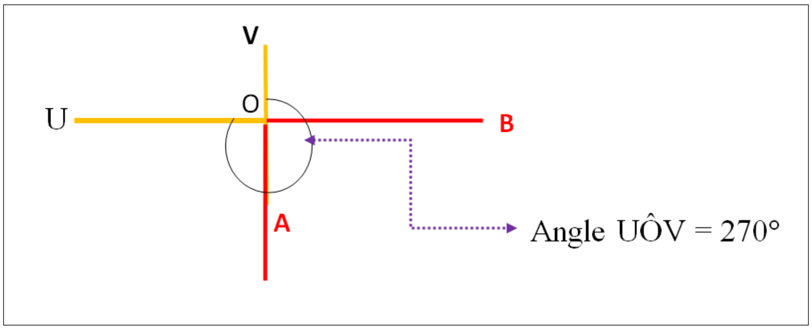


Figure 9: The two Trisectors OA & OB of a given angle $\widehat{UOV}$ = 270°

_3.5_ 270°<Angle < 360°

For a given angle $\widehat{UOV}$, 270° < $\widehat{UOV}$ < 360°, lengthen the side OU of $\widehat{UOV}$ to the right, then draw a perpendicular straight line to OU which is MN to get angle $\widehat{VOM}$ < 90° (Figure 10 below). And then apply section 2.4 above, to create two trisectors of the angle $\widehat{VOM}$, using a compass & a straightedge, where these two trisectors divide $\widehat{VOM}$ into 3 equal angles named 1, 2 & 3 in Figure 10 below.

The remain work is to attach one right angle 90° and one small angle among the three equal angles 1, 2, 3 to the horizontal side OU in the given angle $\widehat{UOV}$ to get one trisectors OP (Figure 10). And then draw the bisector of the angle $\widehat{POV}$ to get the other trisectors OQ of the given angle UÔV, 270° < $\widehat{UOV}$ < 360°, as required.

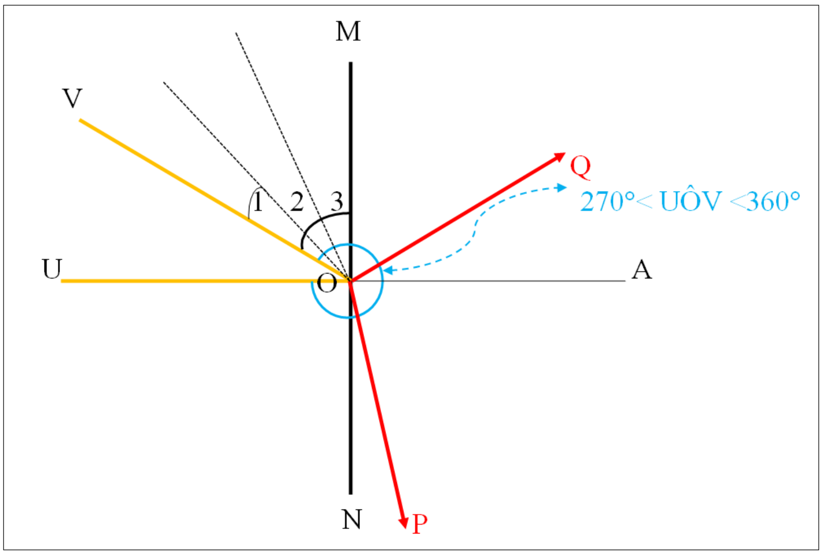


Figure 10: **Two Trisectors OP & OQ of a given angle $\widehat{UOV}$, 270°< $\widehat{UOV}$ < 360°**

***3.6* Angle 360° / 360°-Angle (Full corner / Perigon)**

For a given 360-degree angle $\widehat{UOV}$, from vertex O we can use a straightedge & a compass to construct a regular hexagon inscribed in a circle (O, r), r is an arbitrary length, r ⊂ R (Figure 11 below). Then connect the 3 vertices of the hexagon, which are not consecutive, to obtain an equilateral triangle inscribed in the circle (O, r). And then connect O to two vertices of the triangle to obtain the 2 trisectors OP & OQ of the given 360-degree angle $\widehat{UOV}$ (the other vertex of the triangle is connected to O to make a duplicated side of the given 360-degree angle $\widehat{UOV}$).

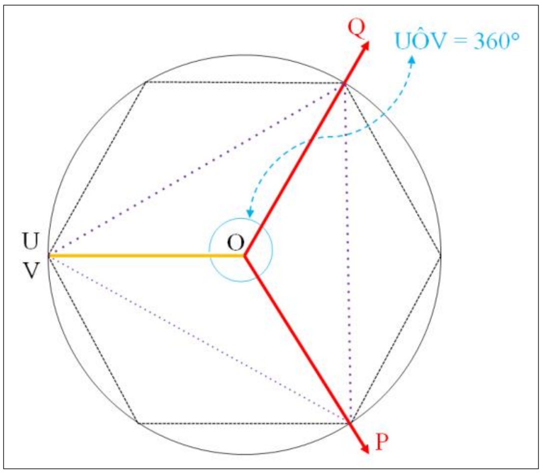


Figure 11: Two Trisectors OQ and OP of a given 360-degree angle

Summary of Research Findings

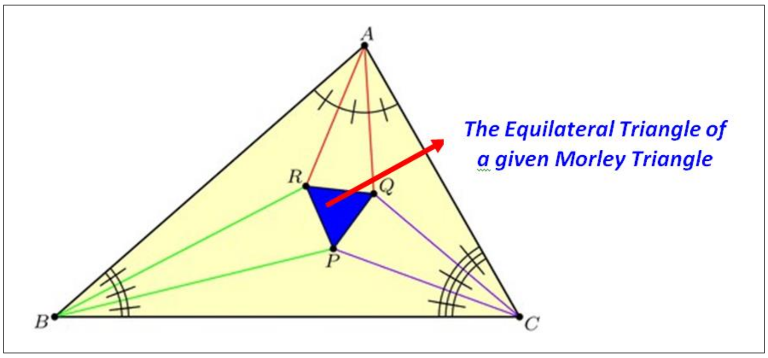
This research establishes several significant findings in the classical problem of angle trisection using only a compass and straightedge within the framework of Euclidean Geometry:

1. **Exact Construction of Angle Trisectors**: It is possible to construct two defined trisectors of any given angle exactly, using only a compass and a straightedge, within the principles of classical Euclidean geometry.
2. **Innovation of the Trisection Ruler**: The core advancement enabling this construction is the introduction of a conceptual tool termed the Trisection Ruler. This innovation allows the application of classical geometric techniques to address the historically unsolved problem of angle trisection.
3. **Geometric Nature of the Trisection Ruler**: The Trisection Ruler may be interpreted as a geometric parameter, dependent on the size of the given angle and an arbitrary parameter p, defining two lengths on either side of the angle's vertex.
4. **Refutation of Wantzel's Impossibility Proof**: This method serves as a counterexample to Pierre Wantzel's 1837 algebraic proof of the impossibility of angle trisection using classical tools. By providing a constructive solution, this result challenges the scope and assumptions of Wantzel's argument.
5. **Application to Morley's Triangle**: The construction of angle trisectors further leads to a

simple method for drawing Morley's Triangle. The procedure is as follows:

- Construct the trisectors of the three internal angles, $\hat{A}$, $\hat{B}$, and $\hat{C}$, of a given triangle.
- Determine the intersection points of each pair of adjacent trisectors: {A, B}, {B, C}, and {C, A}.
- Connect these three points to form an equilateral triangle, known as Morley's Triangle.

6. **Main Theorem (Angle Trisection Theorem):** Any given angle can be divided into three equal sub-angles, exactly and accurately, using only a compass and a straightedge, within the axioms of Euclidean geometry.



4. DISCUSSION & CONCLUSION

4.1 DISCUSSION

Innovation Cannot Thrive When Constrained by Traditional Theories

In the past, those who entered the construction industry typically gained some understanding of its history. One notable figure is Joseph Monier (1823–1906), the inventor of reinforced concrete. He first presented his invention at the Paris Exhibition in 1867 and was granted the world's first patent for reinforced concrete. Subsequently, he received additional patents for reinforced concrete pipes, tanks, beams, and other applications. The first reinforced concrete bridge was also built according to his design. However, Monier's invention was not initially recognized by construction engineers and leading experts in France and around the world. Bound by conventional theories that treated steel and concrete as separate materials, and lacking knowledge of their combined potential, they doubted the durability of the composite material. Furthermore, due to Monier's status as a common individual rather than an academic or professional insider, his work was largely disregarded. As a result, meaningful application of his invention was delayed until the late 19th and early 20th centuries. Nevertheless, reinforced concrete eventually became one of the greatest innovations in human history, revolutionizing the construction industry. This breakthrough is attributed to Monier, a self-taught inventor whose ideas ultimately prevailed despite initial resistance. His success was made possible by a few individuals in the engineering community who recognized the invention's value and either acquired the rights or continued to develop it. Today, it is widely acknowledged that without reinforced concrete, it would be impossible to construct skyscrapers, strong bridges, modern highways with overpasses and underpasses, and the vast infrastructure required for large contemporary cities. A similar example can be seen in the invention of Blockchain technology by Satoshi Nakamoto—an individual whose identity remains unknown due to a deliberate choice to remain anonymous. Like Monier, Nakamoto's work has had a profound impact on the world, despite coming from outside traditional academic or institutional frameworks. These examples demonstrate that strict adherence to established theories can hinder creativity and innovation. True progress often originates from those willing to think beyond conventional boundaries and from those who dare to explore uncharted territory. Another example of great invention is the Blockchain technology of Satoshi Nakamoto, a person who has a name but no one knows who he is.

In the past, knowledge was often considered scientific if it could be confirmed through specific evidence or experiments. However, Karl Popper, in his book Logik der Forschung (The Logic of Scientific Discovery), published in 1934, demonstrated that a fundamental characteristic of scientific hypotheses is their ability to be proven wrong (falsifiability). Anything that cannot be refuted by evidence is temporarily regarded as true until new evidence emerges. For instance, in astronomy, the Big Bang theory is widely accepted, but in the future, anyone who discovers a flaw in this theory will be acknowledged by the entire physics community. Furthermore, no scientific theory lasts forever; rather, it is specific research and discoveries that continually build upon one another [1]. Starting from accepted premises, without proof, one uses deductive reasoning to arrive at theorems and corollaries. With different premises, we develop different mathematical systems. For example, the premise "from a point outside a line, only one parallel line can be drawn to the given line" leads to Euclidean geometry. If we assume that no parallel lines can be drawn from that point, we enter the realm of Riemannian geometry. Alternatively, Lobachevskian geometry assumes that an infinite number of parallel lines can be drawn through that point. No scientific theory lasts forever, but specific research and discoveries continuously build upon each other. The three classic ancient Greek mathematical challenges likely referring to are “Doubling The Circle”, “Trisecting An Angle” and “Squaring The Circle”, all famously proven Impossible under strict compass-and-straightedge constraints, by Pierre Wantzel (1837) using field theory and algebraic methods, then also by Ferdinand von Lindemann (1882) after proving π is transcendental. These original Greek challenges remain impossible under classical rules since their proofs rely on deep algebraic/transcendental properties settled in the 19th century. Recent claims may involve reinterpretations or unrelated advances but do overturn these conclusions above. Among these, the "Squaring The Circle" problem and related problems involving π have captivated both professional and amateur mathematicians for millennia.

4.2 CONCLUSION

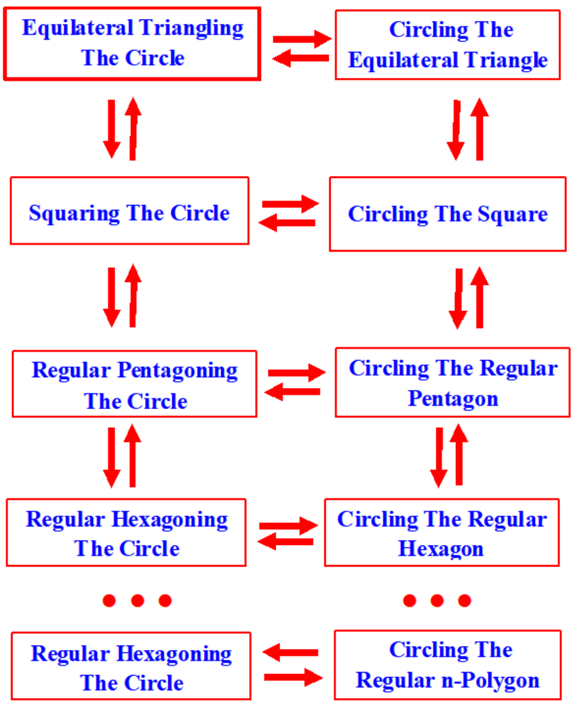
Most mathematicians and mathematics enthusiasts accept that the three classical Greek problems—Squaring the Circle**,** Doubling the Cube, and Trisecting an Angle - are impossible to solve using only a straightedge and compass. This consensus largely stems from the work of Pierre Wantzel (1837), who employed field theory and algebraic methods to prove the impossibility of certain geometric constructions. However, it is important to recognize that Squaring the Circle is fundamentally a geometric construction problem, and the algebraic approach may not fully address the nuances of Euclidean geometry. Further support for the impossibility of squaring the circle came from Ferdinand von Lindemann’s proof in 1882 that π is transcendental. From this, it is commonly concluded that since π cannot be constructed using a finite sequence of straightedge and compass steps, it is impossible to square the circle in the classical sense.

However, it is worth considering a different perspective. If one can construct a square of area A equal to that of a given circle (with centre O and radius r), then, using the formula $A = \pi r^2$, one could derive a numerical precision of π by measuring the square’s edge precisely with modern laser technology. From this measurement, π could be computed using the fact that π is equal accurately to A divided by r^2.

I hold a different point: *I believe I have constructed a valid solution to the “Squaring the Circle” problem using only a straightedge and compass, in accordance with the classical constraints and published* (see References). This belief strengthens my resolve as I pursue a solution to the Angle Trisection problem. The techniques of geometrical analysis and synthesis are instrumental in this effort. Suppose we are given an angle UÔV, that we wish to divide exactly into three equal small angles UÔS, SÔT, & TÔV. By analyzing the relationships among the angle’s parts and constructing a circle with diameter AB, where the line AB perpendicular to the bisector of the angle UÔV and intersects the angle sides at points A & B, we can demonstrate that segment AB functions as a Trisection Ruler’s key. This aids in achieving an accurate and geometrically valid trisection (Geometry Analysis). Continuing with the synthetic approach: from angle UÔV, select any point A on one side and draw line AB perpendicular to the bisector of UÔV. Then, construct a circle centred at point P with radius ½AB. Through this logical progression, we develop the necessary Trisection Ruler and establish a foundation for a rigorous proof, which will be presented in detail in the Proof of Angle Trisection Theorem, Definition of the Trisection Ruler and Angle Trisection Method in Section 2 above (Geometry Synthesis). Additionally, this study opens the door to new investigations into the "*Regular Heptagoning A Circle*" problem within Euclidean geometry, once again using only a straightedge and compass.

It is difficult to realise why the “Angle Trisection” challenge lasted thousands year meanwhile its solution within the classic Euclidean Geometry is proved simply as above.

The results of this research may also contribute to further explorations stemming from the ancient "Squaring the Circle" problem, as depicted in the following diagram:



Last but not least, I am confident to believe this research article is more simple, accurate & true than the articles published in the Mathematical-Gazette *(Cambridge University Press)*, as follows:

199. An Angle Trisection
2828. On the trisection of an angle
1729. Trisection of an angle
3022. The trisection of an angle
71.40 An apparatus for trisecting an angle
3,113. On note 3022—Trisection of the angle
92.52 Trisecting angles with ruler and compasses
1795. Trisection of an angle (Note 1729)
2666. An interative construction for the trisection of a given angle
100.28 Mixing angle trisection with Pythagorean triples
607. [K1. 21. b.] A Method of Trisecting any Angle
393. [K. 21. b.] Mechanical Construction for the Trisection of an Angle
70. A geometrical method of trisecting any angle with the aid of a rectangular hyperbola
and so on ….

Conflicts of Interest
The author declares that there is no conflict of interest regarding the publication of this paper.

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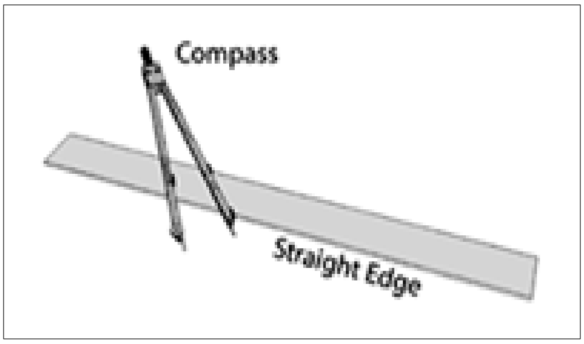

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