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Bianchi Type-V Cosmological Model in *f (Q, T)* Theory of Gravitation

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Abstract **Review Article**

In this research, the LRS Bianchi type-V cosmological model of the universe for matter and radiation field, the energy momentum-tensor is specified by [Coley and Dunn 1990] with *f (Q, T)* theory of gravity. We have taken the linear form $f(Q,T)=\alpha Q+\beta T$ where α and β are constants. To solve the field equations, we have used the relation between shear scalar and expansion scalar. And also used the matter distribution obeys the γ -law of equation of state. Various physical and geometrical properties of the derived cosmological model are investigated. Also, state finder parameter of the obtained cosmological model is discussed.
Keywords: LRS Bianchi type-V, *f (Q, T)* theory of gravity, matter and radiation field.

1. INTRODUCTION

Based on a geometrical variable called non-metricity Q, which characterizes the characteristics of gravitational interaction, a novel theory of gravity known as symmetric teleparallel theory has recently been established [1]. The length interpretation of a vector under parallel transit is characterized geometrically by the non-metricity Q. Additionally, the $f(Q)$ theory was created from the symmetric teleparallel theory [2]. The literature has examined a number of geometrical and physical facets of $f(Q)$ gravity theory [3–10]. Additionally, observational data were used to study the $f(Q)$ theory of gravity in [11,12]. The BBN formalism and data in [13] to extract restrictions on $f(Q)$ gravity. Solanke et al [14] Bianchi type V cosmological model with cold dark matter and holographic dark energy with *f(Q,T)* theory of gravity.

The Bianchi type V cosmological model marks out homogeneous and anisotropic universes with various scale factors along each spatial direction, which is a natural generalization of the FRW model of the universe. Chandel et al. [15] studied Bianchi type-V dark energy cosmological model in modified scale covariant theory of gravitation. Solanke et al. [16] obtained the two fluid Bianchi Type-V cosmological models consisting of matter and radiating source in the *f(R, T)* theory of gravity.

Mete et al. [17] investigated dynamics of anisotropic universes in LRS Bianchi type-V cosmological model with perfect fluid in generic viable non-minimally coupled *f(R, T)* Gravity. Ladke et al [18] studied the non-vacuum perfect fluid solution of LRS Bianchi type-V space-time in the *f(R)* theory of gravity to obtained energy density and pressure of the universe by using stiff matter. Pawar et al [19] investigated anisotropic Bianchi type-V cosmological model filled with perfect fluid in the framework of *f(R. T)* gravity.

Our paper is organized as follows. In section 1, Introduction, In section 2, mathematical formulation of *f (Q,T)* theory of gravity. Section 3, metric and field equations. Section 4, solutions of field equations. Section 5, the physical and geometrical properties of the model, Section 6. State finder diagnostic and the last section contain some conclusion.

2. Mathematical formulation of *f (Q,T)* Theory of gravity

For $f(Q,T)$ gravity the expressions for Einstein's Hilbert action is defined as

$$S = \int \left[\frac{1}{16\pi} f(Q,T) + L_M \right] \sqrt{-g}\, d^4 x \tag{1}$$

Where $f(Q,T)$ is the function of *Q* i.e. non-metricity and *T* i.e. energy momentum tensor. Here $g \equiv \det(g_{\mu\nu})$. Also, the non metricity tensor is given by

$$Q \equiv g^{ij} \left(L^{\alpha}{}_{\beta i} L^{\beta}{}_{j\alpha} - L^{\alpha}{}_{\beta\alpha} L^{\beta}{}_{ij} \right) \tag{2}$$

Here $L^{\alpha}{}_{\beta\gamma}$ is denoted for deformation tensor and is given by

$$L^{\alpha}{}_{\beta\gamma} \equiv -\frac{1}{2} g^{\alpha\mu} \left(\nabla_{\gamma g \beta\mu} + \nabla_{\beta g \mu\gamma i} - \nabla_{\mu g \beta\gamma} \right) \tag{3}$$

Also, energy momentum tensor is given by

$$T_{\mu\nu} \equiv -\frac{2}{\sqrt{-g}} \frac{\delta\left(\sqrt{-g}\, L_m\right)}{\delta g^{\mu\nu}} \tag{4}$$

$$\theta_{\mu\nu} \equiv g^{\alpha\beta} \left(\frac{\delta T_{\alpha\beta}}{\delta g^{\mu\nu j}} \right) \tag{5}$$

which implies that

$$\delta T = \delta\left(T_{\mu\nu} g^{\mu\nu}\right) = \left(T_{\mu\nu} + \theta_{\mu\nu}\right) \delta g^{\mu\nu} \tag{6}$$

By varying the gravitational action (1), the field equation for extended symmetrical teleparallel gravity, $f(Q,T)$ gravity is:

$$-\frac{2}{\sqrt{-g}} \nabla_{\alpha} \left(f_Q \sqrt{-g}\, P^{\alpha}{}_{\mu\nu} \right) - \frac{1}{2} f g_{\mu\nu} + f_T \left(T_{\mu\nu} + \theta_{\mu\nu j} \right) - f_Q \left(P_{\mu\alpha\beta} Q_{\nu}{}^{\alpha\beta} - 2 Q^{\alpha\beta}{}_{\mu} P_{\alpha\beta\nu} \right) = 8\pi T_{\mu\nu} \tag{7}$$

$P^{\alpha}{}_{\mu\nu}$ is denoted for model's super potential and is given by

$$P^{\alpha}{}_{\mu\nu} = -\frac{1}{2} L^{\alpha}{}_{\mu\nu} + \frac{1}{4} \left(Q^{\alpha} - \tilde{Q}^{\alpha} \right) g_{\mu\nu} - \frac{1}{2} \delta^{\alpha} \left({}_{\mu} Q_{\nu} \right) \tag{8}$$

where $Q_{\alpha} \equiv Q_{\alpha\varpi}{}^{\mu}$, $\tilde{Q}^{\alpha} \equiv Q^{\mu}{}_{\alpha\mu}$ is trace of non metricity tensor and $T = T_{\alpha\beta} g^{\alpha\beta}$ is trace of energy momentum tensor.

3. Metric and field Equations

LRS Bianchi type-V cosmological model of the universe

$$ds^2 = -dt^2 + A^2 dx^2 + e^{2x} \left(B^2 dy^2 + C^2 dz^2 \right) \tag{9}$$

where A, B and C are cosmic scale factors and functions of cosmological time *t* only.

The large-scale structure of the universe is significantly influenced by the anisotropic cosmological model (Kantowski and Sachs, 1966). Several authors who have studied cosmology using relativistic cosmological models have not provided adequate clarity to take consideration in a regular expansion of the early phases of the universe.

For matter and radiation field, the energy momentum-tensor is specified by [20]

$$T_{ij} = T_{ij}{}^{(m)} + T_{ij}{}^{(r)} \tag{10}$$

where $T_{ij}{}^{(m)}$ is the energy momentum-tensor for the matter described by a perfect fluid with density ρ_m, pressure p_m and the four velocity $u_i{}^{(m)} = (1,0,0,0)$ where $g^{ij} u_i{}^{(m)} u_j{}^{(m)} = 1$

$T_{ij}^{(r)}$ is the energy momentum-tensor for the radiation field with density ρ_r, pressure $p_r = \frac{1}{3}\rho_r$ and the four velocities $u_i^{(r)} = (1,0,0,0)$ were $g^{ij} u_i^{(r)} u_j^{(r)} = 1$

Thus,

$$T_{ij}^{(m)} = (\rho_m + p_m)\, u_i^{(m)} u_j^{(m)} + p_m\, g_{ij} \tag{11}$$

From Eq. (11), we have

$$T_1^1 = T_2^2 = T_3^3 = p_m + \frac{1}{3}\rho_r \text{ and } T_4^4 = -(\rho_m + \rho_r) \tag{12}$$

The trace of energy-momentum tensor is given by

$$T = T_1^1 + T_2^2 + T_3^3 + T_4^4 = 3p_m - \rho_m \tag{13}$$

Thus, the field equation (7) for the given cosmological model (9) yields

$$\frac{1}{2} f - FQ = -8\pi \widetilde{G} p_m + 8\pi (1 + \widetilde{G})(\rho_m + \rho_r) \tag{14}$$

$$-\frac{1}{2} f + F\left(\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{2\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}\dot{C}}{AC}\right) = 8\pi\, p_m + \frac{8\pi}{3}(1 + \widetilde{G})\,\rho_r \tag{15}$$

$$-\frac{1}{2} f + F\left(\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{2\dot{A}\dot{C}}{AC} + \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC}\right) = 8\pi\, p_m + \frac{8\pi}{3}(1 + \widetilde{G})\,\rho_r \tag{16}$$

$$-\frac{1}{2} f + F\left(\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{2\dot{A}\dot{B}}{AB} + \frac{\dot{A}\dot{C}}{AC} + \frac{\dot{B}\dot{C}}{BC}\right) = 8\pi\, p_m + \frac{8\pi}{3}(1 + \widetilde{G})\,\rho_r \tag{17}$$

where $F \equiv f_Q$, $f_T \equiv 8\pi \widetilde{G}$ and dot represents the derivative with respect to cosmic time t.

For LRS Bianchi type-V cosmological model, the average scale factor *a(t)* and spatial volume *V* are given by

$$a = (ABC)^{\frac{1}{3}}, \tag{18}$$

$$V = a^3 = (ABC). \tag{19}$$

The mean generalized Hubble parameter *H* for this model is,

$$H = \frac{1}{3}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) = \frac{\dot{a}}{a} \tag{20}$$

where H_1, H_2, H_3 are the directional Hubble parameters defined by

$$H_1 = \frac{\dot{A}}{A},\ H_2 = \frac{\dot{B}}{B},\ H_3 = \frac{\dot{C}}{C}$$

The average anisotropy parameter is

$$A_m = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{\Delta H_i}{H}\right)^2, \text{ where } \Delta H_i = H_i - H. \tag{21}$$

The expansion scalar θ and the shear scalar σ can be expressed as follows:

$$\theta = 3H = \frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}, \tag{22}$$

$$\sigma^2 = \frac{1}{2}\left[\sum_i H_i^{\,2} - \frac{\theta^2}{3}\right]. \tag{23}$$

4. Solutions of field Equations

There are different forms of $f(Q,T)$ gravity, or sake of simplicity, we have taken the linear form $f(Q,T) = \alpha Q + \beta T$ where α and β are constants.

Here $F = f_Q = \alpha,$ and $f_T = \beta = 8\pi \widetilde{G}$, $\widetilde{G} = \dfrac{\beta}{8\pi}$

According to Thorne (1967), observations of the velocity-red-shift relation for extragalactic sources indicate that the universe's Hubble expansion is currently isotropic within 30% (Kantowski and Sachs, 1966, Kristian and Sachs, 1966).

We have system of the equations (14) to (17) are four equations and six unknowns A, B, C, p_m, ρ_m & ρ_r. Hence we use the relation between shear scalar and expansion scalar.

$$A^2 = lBC$$

without loss of generality, we take $l = 1$

$$A^2 = BC \tag{24}$$

Anisotropic relations gives

$$B = C^k \tag{25}$$

and the matter distribution obeys the γ-law of equation of state,

$$p_m = (\gamma - 1)\rho_m, \qquad 1 \le \gamma \le 2 \tag{26}$$

Subtracting equations (16) from (17) and using relations (24) and (25) we get,

$$A = \left[\frac{3(k+1)}{2}(c_1 t + c_2)\right]^{\frac{1}{3}} \tag{27}$$

$$B = \left[\frac{3(k+1)}{2}(c_1 t + c_2)\right]^{\frac{2k}{3(k+1)}} \tag{28}$$

and $$C = \left[\frac{3(k+1)}{2}(c_1 t + c_2)\right]^{\frac{2}{3(k+1)}} \tag{29}$$

Where c_1 and c_2 is constants of integration.

LRS Bianchi type-V cosmological model in equation (9) takes the form

$$ds^2 = -dt^2 + \left[\frac{3(k+1)}{2}(c_1 t + c_2)\right]^{\frac{2}{3}} dx^2 + e^{2x}\left\{\left[\frac{3(k+1)}{2}(c_1 t + c_2)\right]^{\frac{4k}{3(k+1)}} dy^2 + \left[\frac{3(k+1)}{2}(c_1 t + c_2)\right]^{\frac{4k}{3(k+1)}} dz^2\right\} \tag{30}$$

5. The Physical and Geometrical Properties of the model

For the LRS Bianchi type-V cosmological model in $f(Q,T)$ gravity, the average scale factor is found to be

$$a = \left[\frac{3(k+1)}{2}(c_1 t + c_2)\right]^{\frac{1}{3}} \tag{31}$$

The spatial volume is found to be

$$V = \left[\frac{3(k+1)}{2}(c_1 t + c_2)\right] \tag{32}$$

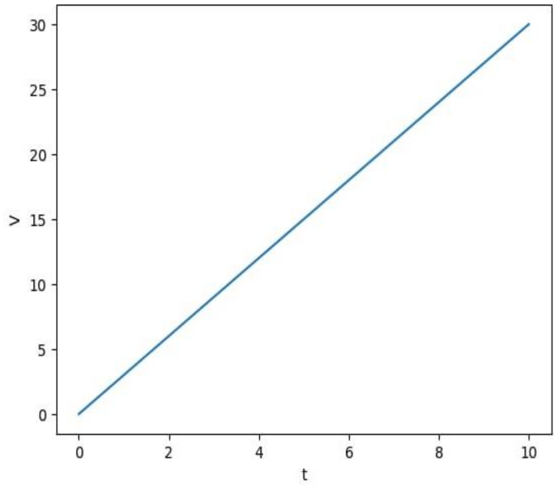

Fig. 1 Plot of Spatial volume versus time

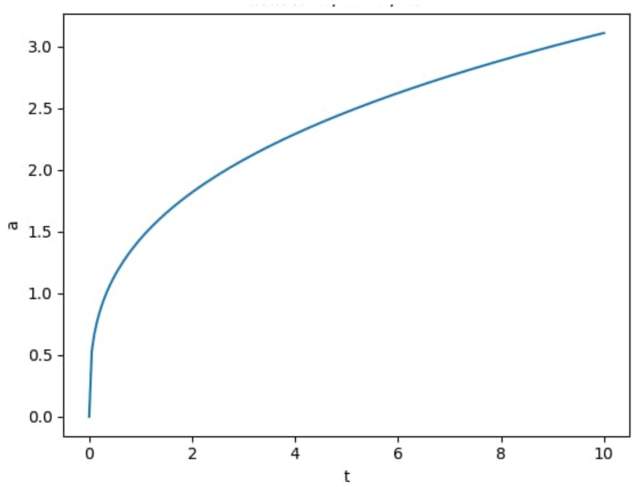

Fig. 2 Plot of Average scale factor versus time

The Hubble parameter is found to be

$$H = \frac{c_1}{3}(c_1 t + c_2)^{-1} \tag{33}$$

The expansion scalar (θ), Shear scalar (σ^2), deceleration parameter (q) and mean anisotropy parameter (Δ) are found to be

$$\theta = c_1 (c_1 t + c_2)^{-1} \tag{34}$$

$$\sigma^2 = \frac{1}{4}(k-1)^2 c_1{}^2 \left[\frac{3(k+1)}{2}(c_1 t + c_2)\right]^{-2} \tag{35}$$

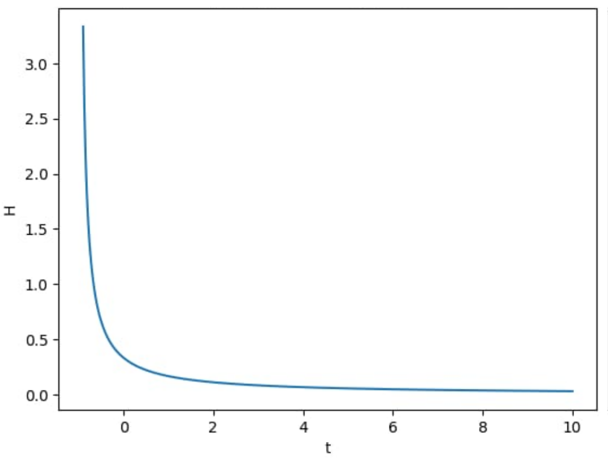

Fig. 3 Plot of Hubble parameter versus time

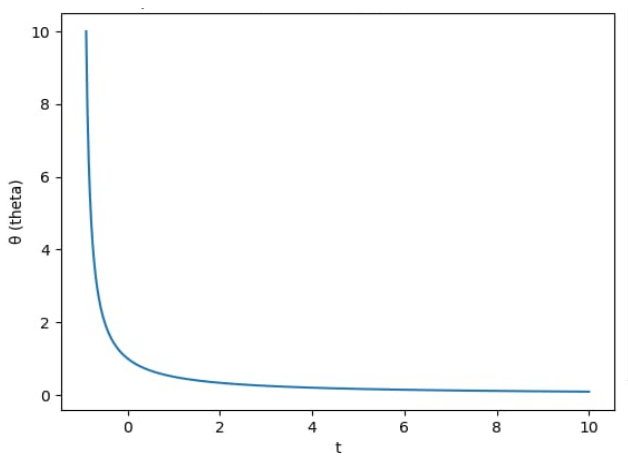

Fig. 4 Plot of Expansion scalar versus time

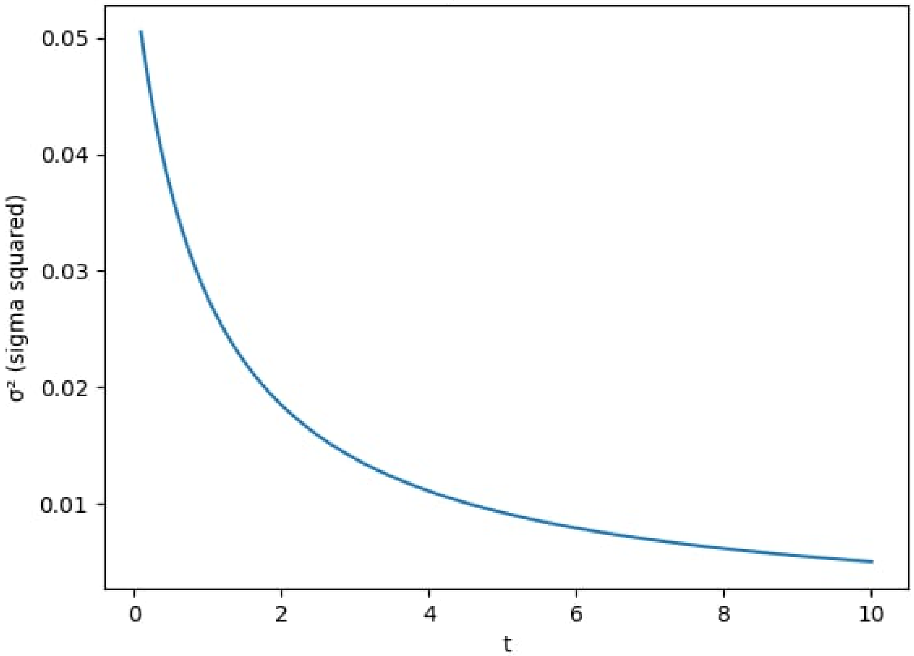

Fig. 5 Plot of Shear scalar versus time

$$q = 2 \tag{36}$$

$$\Delta = \frac{2}{3}(k-1)\left[\frac{(k-1)}{(k+1)}\right]^2 \tag{37}$$

Also, the pressure for matter and density for matter and radiation field of obtained cosmological model is

$$p_m = \frac{-\frac{1}{3}\alpha(\gamma-1)(k^2+4k+1)c_1{}^2}{\left(8\pi\gamma+\frac{3\gamma\beta}{2}-\frac{32\pi}{3}-\frac{8\beta}{3}\right)}\left[\frac{3(k+1)}{2}(c_1t+c_2)\right]^{-2} \tag{38}$$

$$\rho_m = \frac{-\frac{1}{3}\alpha(k^2+4k+1)c_1{}^2}{\left(8\pi\gamma+\frac{3\gamma\beta}{2}-\frac{32\pi}{3}-\frac{8\beta}{3}\right)}\left[\frac{3(k+1)}{2}(c_1t+c_2)\right]^{-2} \tag{39}$$

$$\rho_r = \left(\frac{-4\pi\gamma+8\pi-\frac{11\gamma\beta}{12}+2\beta}{8\pi\gamma+\frac{3\gamma\beta}{2}-\frac{32\pi}{3}-\frac{8\beta}{3}}\right)\left(\frac{\alpha}{8\pi+\beta}\right)(k^2+4k+1)c_1{}^2\left[\frac{3(k+1)}{2}(c_1t+c_2)\right]^{-2} \tag{40}$$

6. State finder diagnostic

The state finder parameters used to characterize the expansion of the Universe, including epochs dominated by matter and radiation, are the dimensionless pair $\{r, s\}$.

It is constructed with the help of scale factor of universe and its derivative up to order three.

$$r = \frac{\dddot{a}}{aH^3} \quad \text{and} \quad S = \frac{r-1}{3\left(q-\frac{1}{2}\right)}$$

The state finder is a dimensionless quantity beyond H and q the next step is r and s is the linear combination of r and deceleration parameter q. Parameters of state finder diagnostic are found to be

$$r = 10 \quad \text{and} \quad s = 2 \tag{41}$$

Particular values of the state finder diagnostic pair are connected foe standard model are as follows:

$\{r,\ s\} = \{1,\ 0\}$ represents the Λ CDM model.

$\{r,\ s\} = \{1,\ 1\}$ represents the SCDM model.

$\{r,\ s\} = (-\infty,\ \infty)$ represents the Einstein static universe.

CONCLUSION

In this paper, we have studied the LRS Bianchi Type V cosmological model in theory of gravity for matter and radiation field. We have taken the energy momentum tensor for the matter described by a perfect fluid. While solving the field equations we get four equations and six unknowns. So we have used the relation between shear scalar and expansion scalar. And also used the matter distribution obeys the -law of equation of state. We have graphically calculated the different physical and geometrical properties of the obtained cosmological model. Figure 1 and Figure 2, shows that behavior of spatial volume and average scale factor, both parameters are increases as time increases. Also, Figure 3, Figure 4 and Figure 5, show that as time decreases Hubble parameter, expansion scalar and shear scalar decreases. It concludes that, all the parameters are well behaved. Also, we have investigated the state finder parameter of the obtained model.

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