

Regular Polygonising a Circle with Straightedge and Compass in Euclidean Geometry

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Abstract

Original Research Article

The topic of this article means “one can construct a regular polygon with n sides, $n \in \mathbb{N}$, of which area is equal exactly to a given circle (O, r) , using a straightedge and a compass only”. No scientific theory lasts forever, but specific research and discoveries continuously build upon each other. The three classic ancient Greek mathematical challenges likely referring to are “Doubling the Cube”, “Trisecting an Angle” and “Squaring A Circle”, all famously proven Impossible under strict compass-and-straightedge constraints, by Pierre Wantzel (1837) using field theory and algebraic methods [4], then also by Ferdinand von Lindemann (1882) after proving π is transcendental. These original Greek challenges remain impossible under classical rules since their proofs rely on deep algebraic/transcendental properties settled in the 19th century. Recent claims may involve reinterpretations or unrelated advances but do overturn the conclusions above. Among these, the “Squaring A Circle” problem and related problems involving π have captivated both professional and amateur mathematicians for millennia. The title of this paper refers to the concept of “constructing a regular polygon with n sides has the exact area of a given circle,” or “Regular Polygonising A Circle” for short. This research idea arose after the “Squaring A Circle” problem was studied and solved and published in “SJPMS” in 2024 [6]. This paper presents an exact solution to constructing a regular polygon with n sides, that is concentric with and has the same area as a given circle. The solution does not rely on the number π and adheres strictly to the constraints of Euclidean Geometry, using only a straightedge and compass. The technique of “ANALYSIS” is employed to solve the “Regular Polygonising A Circle” problem precisely and exactly with only a straightedge and compass, without altering any premise of the problem. This independent research demonstrates the solution to the challenge using only these tools. All mathematical tools and propositions in this solution are derived from Euclidean geometry. The methodology involves geometric methods to arrange the given circle and its equal-area regular polygon with n sides (assumed existing) into a concentric position.

Keywords: Polygonising the Circle; regular polygonising circle; circle polygonise; polygon area of a circle; constructing a regular polygon with the same area as a circle; Euclidean geometry; straightedge and compass.

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INTRODUCTION

For over three millennia, three classical geometric well-known problems from ancient Greece Squaring the Circle, Doubling the Cube, and Trisecting an Angle emerged and challenged the ingenuity of mathematicians. These problems, first posed by the constraint of using only an unmarked straightedge and compass, have long been declared impossible to solve within the framework of Euclidean geometry. Foundational impossibility proofs by 19th-century mathematicians such as Pierre Wantzel and Ferdinand Von Lindemann [2], [3] and [4], - using algebraic field theory and transcendental number theory, - seem to

confirm that exact solutions were unattainable. Specifically, the transcendence of π implies that a square with an area exactly equal to that of a given circle cannot be constructed using purely geometric means [2] and [3]. However, while these algebraic impossibility theorems are rigorous within their own domains, they diverge in nature from the original geometric spirit of ancient challenges. The historical emphasis on purely constructive geometry, devoid of algebra, arithmetic or trigonometry, leaves space for critical re-evaluation [6].

Among them, Hippocrates meticulously studied the problem of squaring the circle. The problem is stated as follows: Using only a straightedge and a compass, is

it possible to construct a square with an area equal to a given circle? The problems were proven unsolvable using only straightedge and compass in the 19th century. In 1882, mathematician Ferdinand von Lindemann proved that π is an irrational number, which means that it is impossible to construct a square with the same area as a given circle using only a straightedge and a compass, as posed by Hippocrates. One of the most fascinating aspects of this problem is that it has captured the interest of mathematicians throughout the history of mathematics. From the earliest mathematical documents to today's mathematics, the problem and its relation to π have intrigued both professional and amateur mathematicians for millennia.

A significant step forward in proving the impossibility of squaring the circle occurred in 1761 when Lambert proved that π is irrational [2] & [3]. This, however, was not sufficient to demonstrate the impossibility of squaring the circle using a straightedge and compass, as certain algebraic numbers can be constructed with these tools. Despite the proof of the impossibility of squaring the circle, the problem has continued to captivate mathematicians and the general public alike, remaining an important topic in the history and philosophy of mathematics.

In 1837, French mathematician L. Wantzel proved that the three classical problems of ancient Greece are impossible to solve using only a straightedge and compass. Although based on algebra, Pierre Laurent Wantzel (1837) declared the verdict, "**Arbitrary angle trisection is impossible**", I experienced that my achieved procedure was possibly not arrested by this verdict [4]. After applying my method to trisect an arbitrary angle, I hope readers can trust algebra cannot affect the successful geometrical constructions of angle trisection.

Therefore, the problems of squaring the circle, doubling the cube, and trisecting an angle have been studied for centuries and remain unsolvable with these tools to this day. As of December 2022, no mathematician has found exact solutions to classical problems such as "Doubling the Cube," "Squaring the Circle," and "Trisecting an Angle" using only a compass and straightedge.

However, in 2023, my research article, published on the SJPMS journal, [6] as an exact solution to the "Squaring the Circle" problem, provides a counter-proof to the impossibility stated by Wantzel in 1837, [4].

This article arises from such a re-evaluation and documents a new geometric approach remaining strictly within the constraints of compass and straightedge that aims to construct precise solutions to several of these age-old problems. Beginning with a geometric interpretation of Lao Tzu's aphorism "*The Great Way is*

simple," the author explores the power of simplicity and concentric arrangement. Through detailed analysis, an exact method for Squaring the Circle was developed, followed by solutions to Doubling the Cube and Trisecting an Angle, each derived using geometric reasoning alone in 2023, 2024 and 2025 [6], [8] and [9].

In the past, knowledge was often considered scientific if it could be confirmed through specific evidence or experimentations. However, Karl Popper, in his book *Logik der Forschung* (The Logic of Scientific Discovery), - published in 1934, demonstrated that a fundamental characteristic of scientific hypotheses is their ability to be proven wrong (falsifiability). Anything that cannot be refuted by evidence is temporarily regarded as true until new evidence emerges. For instance, in astronomy, the Big Bang theory is widely accepted; however, in the future, anyone who discovers a flaw in this theory will be acknowledged by the entire physics community. Furthermore, no scientific theory lasts forever; rather, it is specific research and discoveries that continually build upon one another [1].

Starting from accepted premises, without proof, one uses deductive reasoning to arrive at theorems and corollaries. With different premises, we develop different mathematical systems. For example, the premise "from a point outside a line, only one parallel line can be drawn to the given line" leads to Euclidean geometry. If we assume that no parallel lines can be drawn from that point, we enter the realm of Riemannian geometry. Alternatively, Lobachevskian geometry assumes that an infinite number of parallel lines can be drawn through that point.

It is important to clarify that this does not imply that a square with an area equal to a circle does not exist. If the circle has area A , a square with a side length equal to the square root of A would have the same area. This does not imply that it is impossible to solve the problem using only a straightedge and a compass. Thus, I provided the exact solution to the "Squaring the Circle" problem without altering the premises of Euclidean geometry or the constraints of straightedge and compass [6]. In 2024, I also solved the inverse problem, titled "Circling The Square with Straightedge & Compass in Euclidean Geometry," which was published in the SJPMS [7].

"Regular Polygonising A Circle" refers to the process of "constructing a regular polygon with n sides, $n \in \mathbb{N}$, that has the same area as a given circle and is concentric with that circle." This concept came to me spontaneously after solving the problem of "squaring the circle." I thought, "If one can square the circle, can one also regularly polygonise the circle using only a straightedge and compass in Euclidean Geometry?" In other words, "Regular Polygonising A Circle" is a new challenge problem that arose from the exact solution presented in my "Squaring the Circle" paper, published

in *SJPMS* [6]. Therefore, the “*Regular Polygonising A Circle*” challenge and its solution did not exist in the field of mathematics until it was solved and published in this paper.

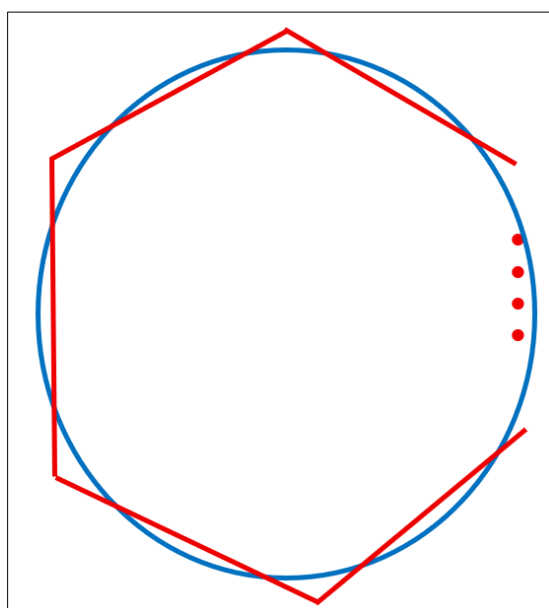
In seeking solutions to such problems, geometers developed a special technique called “ANALYSIS.” They assumed that the problem had been solved, and by investigating the properties of the solution, they worked backward to identify an equivalent problem that could be solved based on the given conditions. To obtain a formally correct solution to the original problem, geometers reverse the process, starting with the data to solve the equivalent problem derived through analysis, and then use that solution to solve the original problem. This reverse procedure is known as “SYNTHESIS.” Analysis and synthesis are fundamental methods for mathematical reasoning and problem-solving. Analysis involves breaking down complex mathematical problems or structures into simpler and, - more manageable components. It enables mathematicians to understand the underlying principles, identify patterns, and isolate essential variables. Through analytical methods, one can derive conclusions, test hypotheses, and explore the logical consequences of these assumptions. Conversely, synthesis entails the combination of previously understood elements to construct more complex systems or develop general theories. It often follows analysis, - using the insights gained to build new models, solve broader problems, or formulate proofs. The interplay between analysis and synthesis fosters deeper comprehension of mathematical concepts and drives the progression of mathematical theory. Together, these complementary approaches are vital to the advancement of mathematics, underpinning both theoretical inquiry and practical applications.

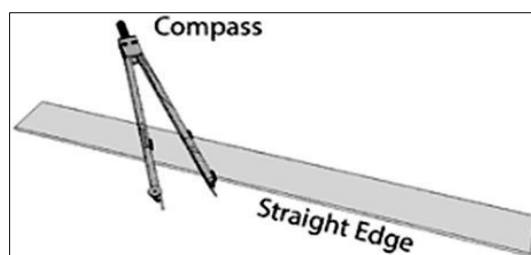
I have adopted both the “ANALYSIS” and “SYNTHESIS” techniques to solve the “Regular Triangling A Circle” problem exactly, using only a straightedge, compass, in Euclidean Geometry, without involving the irrational number π .

The inspiration for this research arose after the solution to the “Squaring the Circle” problem was published in *SJPMS* in 2024 [6]. If it is possible to square a circle using Euclidean geometry, the question arises: *why is it difficult to regularly polygonise a circle?* In this context, the given circle contains an inscribed regular polygon with $2n$ sides, $n \in \mathbb{N}$ which any two parallel sides extend, leading to a regular polygon with n sides. This *regular polygon with n sides* serves as the solution to the new problem of the “regular polygon circle A.” The remaining task is to prove that the area of this regular polygon is equal to the area of the given circle. This paper also introduces a new mathematical tool, the “Regular Polygon Ruler,” and provides a proof for the following:

1. A regular polygon with n sides intersects the given circle at the $2n$ vertices of a special polygon;
2. The construction process forms a regular polygon with $2n$ sides inscribed in the given circle.

This paper’s solution to the “Regular Polygonising A Circle” problem is naturally leads to the end of the process from solutions to “Regular Triangling A Circle”, “Squaring A Circle”, “Regular Pentagoning A Circle”, ... , and “Regular Polygonising A Circle” problems.





I. PROOFS OF NEW PROPOSITIONS

I.1 Theorem 1:

If there exists a regular polygon with n sides, $n \in \mathbb{N}$, that has the same area as a given circle (O, r) , then

n sides of the hexagon cut the circle at $2n$ points to form an inscribed polygon with $2n$ sides in the circle.

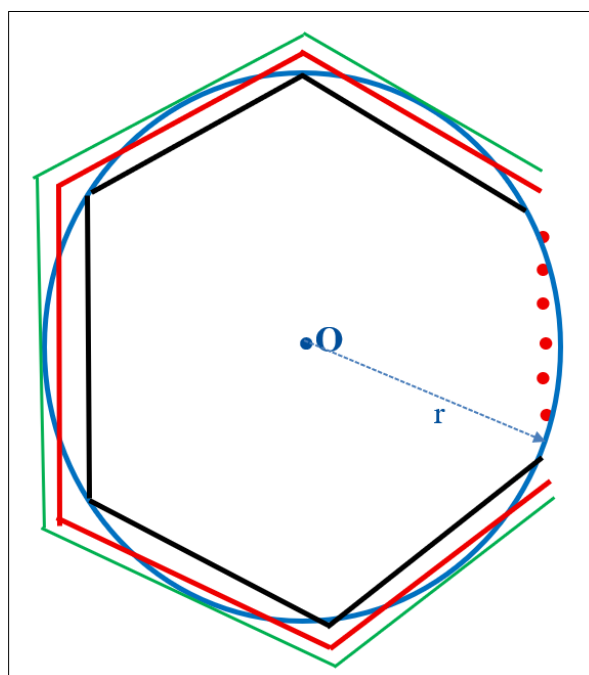


Figure 1: Red polygon is assumed to has the same area as the given circle (blue color), black polygon is inscribed in the circle & the green polygon is the circumscribed polygon. They are all concentric to the circle center and parallel sides

PROOF:

Assuming that the red regular polygon with n sides in **Figure 1** above is the required shape that has the same area as the area of a given circle (O, r) and is concentrated within the circle (blue color), then

- The area of this hexagon is less than the area of the circumscribed regular polygon (green) of the circle.,
- The area of this polygon (red) is larger than the

area of the inscribed regular polygon (black) in the circle.

Thus, this regular polygon (red color) with n sides has to cut the circle at $2n$ points $a, b, c, d, e, f, g, h, i, \dots, x, y, z$ (figure below). These $2n$ points are vertices of an inscribed regular polygon with $2n$ sides in the given circle, as required and described in Figure 2 below.

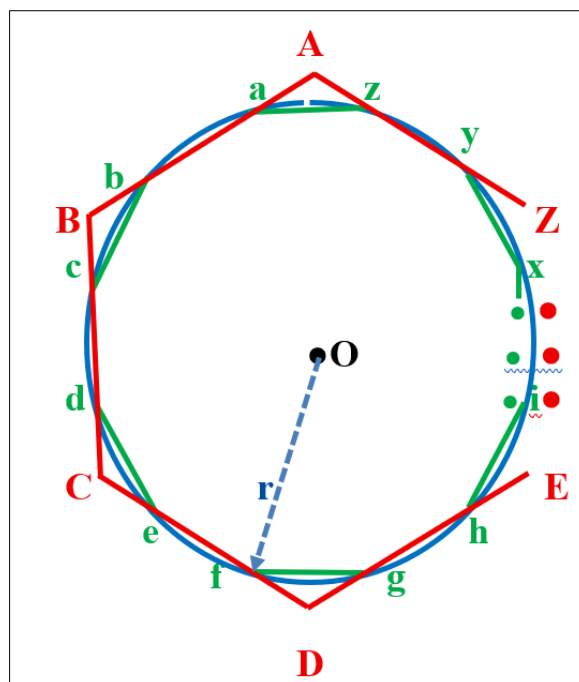


Figure 2: The given blue circle (O, r) , the regular polygon $ABCDE...Z$ with n sides (red color) and the inscribed regular polygon $abcdefghi...xyz$ with $2n$ sides in the given circle (O, r)

I.2 Theorem 2:

If there exists a regular polygon $ABCDE...Z$, with n sides, that has the same area as a given circle (O, r) , then the polygon cuts the circumference of the circle (O, r) at $2n$ points $a, b, c, d, e, f, g, h, i, \dots, x, y$ & z which are $2n$ vertices of an inscribed regular polygon of the given circle.

PROOF:

Assume that there exists a regular polygon $ABCD...YZ$ (red color in Figure 2 below), that has the same area $\square r^2$ as the area of a given circle (O, r) and is concentric to the circle.

According to Theorem 1 in Section I.1, this regular polygon's sides cut the circumference of the circle at $2n$ points $a, b, c, d, e, f, g, \dots, x, y$ & z (Figure 3, below). Let circle (O, R) be the circumscribed circle of the regular polygon $ABCD...YZ$. Extend chord ab (Figure 3 below) of the circle (O, R) at points A' & B' , and then connect the center points O to A' . From point A' ,

draw a symmetric straight line segment $A'Z'$ (the black arrow line in Figure 3), which cuts the circle (O, R) at Z' , OA' is the bisector of angle $B'A'Z'$. Because of the symmetrical property of lines $A'B'$ and $A'Z'$ and circle chords ab & yz (Figure 3 below), line $A'Z'$ overlaps the chord yz of the circle (O, r) . Similar extensions of chords $cd, ef, g, \dots$ & $\dots x$ and connections $B'O, C'O, D'O, \dots$ & $Z'O$ and the similar symmetric proofs above show that $A'B'C'D'...Z'$ is also an inscribed regular polygon with n sides in the circle (O, R) . Therefore, these two regular polygons are equal, and their intersection area is polygon $abcdefgh...xyz$, - with $2n$ sides - (Figure 3 below), which is a regular inscribed polygon with $2n$ sides in the given circle (O, r) . This shows that the side extensions of this $2n$ polygon intersect exactly at n vertices of the required regular polygon $ABCD...YZ$, and n vertices of polygon $A'B'C'D'...Z'$, equally (Figure 3). Based on the symmetric and concentric properties of the two polygons,

$$\text{Polygon } ABCD...YZ = \text{Polygon } A'B'C'D'...Z' \quad (1)$$

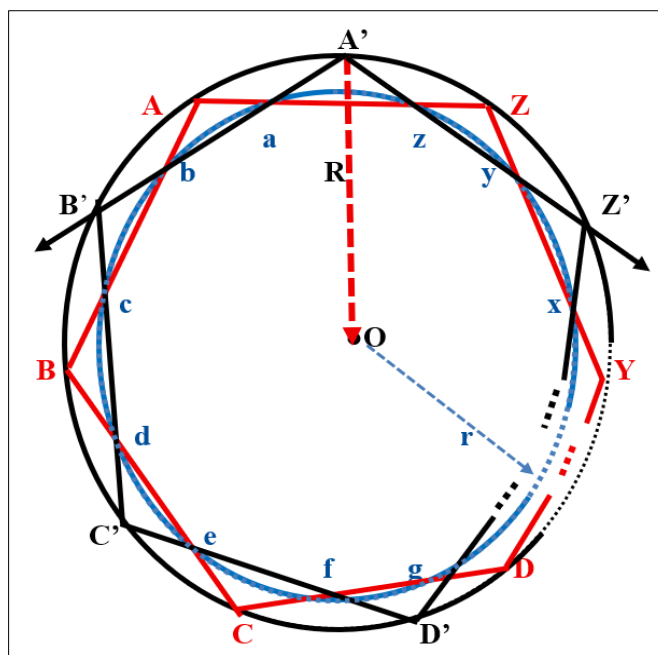


Figure 3: The given circle (O, r) – blue color –, the regular polygon $ABCD...YZ$ with n sides and the regular $A'B'C'D'...Z'$ with n sides

Because (O, r) and (O, R) are the two concentric circles, symmetric properties, and equal distances from the center O to $2n$ chords $ab, bc, cd, de, ef, fg, g..., ...x, xy, yz$ & $za, abcdefg...xyz$ is a regular polygon with $2n$ sides as follows:

$$ab = bc = cd = de = ef = fg = g... = ...x = xy = yz = za \quad (2).$$

From (2), the inscribed polygon $abcdefg...xyz$ of the given circle (O, r) is an identification of the resulting regular polygon $ABCD...YZ$, as required.

I.3 DEFINITION:

Let $n \in \mathbb{N}$, and given a circle (O, r) , area $= \pi r^2$, then any regular polygon with $2n$ sides inscribed in the circle is defined as “REGULAR POLYGON RULER” of the circle from Theorem 2 in section I.2 above (Figure 4 below).

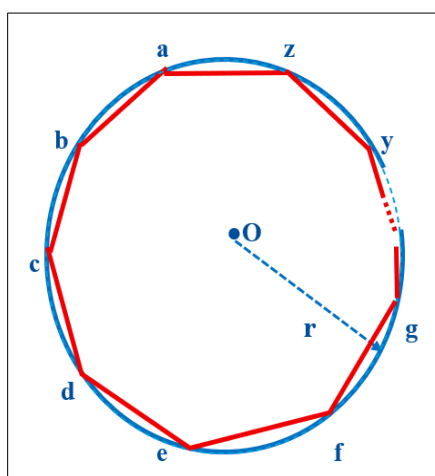


Figure 4: “REGULAR POLYGON RULER” $abcdefg...yz$ (blue colour) of a given circle (O, r) is the regular dodecagon inscribed in the given Circle (O, r)

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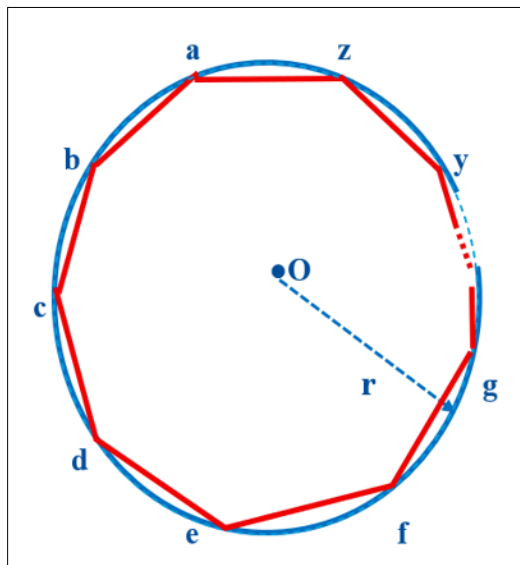
Given a circle (O, r) and a number $n \in \mathbb{N}$ then a REGULAR POLYGON RULER with $2n$ sides of the circle is a mathematics tool to construct a regular polygon with n sides, that has the same area as the area

of the given circle, using only a straightedge & compass in Euclidean Geometry.

PROOF:

Given a circle (O, r) , area $= \pi r^2$, and a number $n \in \mathbb{N}$, then by the Definition in section I.3 above, there

exists a regular polygon $abcdefg...xyz$ with $2n$ sides inscribed in the circle, which is the Regular Polygon Ruler, described in the following figure.



Extend n sides $bc, de, fg, \dots, xy$ & za of the regular polygon $abcdefg...xyz$ (Regular Polygon Ruler) to obtain a $ABCD...YZ$ (red color in Figure 5). According to Theorem 2 in Section I.2, this red polygon has n sides, and is a regular polygon having the same area πr^2 as the area of the given circle. Therefore, the regular polygon $ABCD...YZ$ is the solution to the “Polygonising A Circle” challenge problem (Figure 5 below).

Note: If one draws the concentric circle (O, R) that is the circumscribed circle of the above regular polygon $ABCD...YZ$, then the black regular polygon $A'B'C'D'...Z'$ (in Figure 5 below) is also equal to the resulting regular hexagon $ABCD...YZ$. Therefore, one REGULAR POLYGON RULER of this challenge problem produces 2 constructive equal solutions using only straightedge & compass in Euclidean Geometry, as required (Figure 5 below).

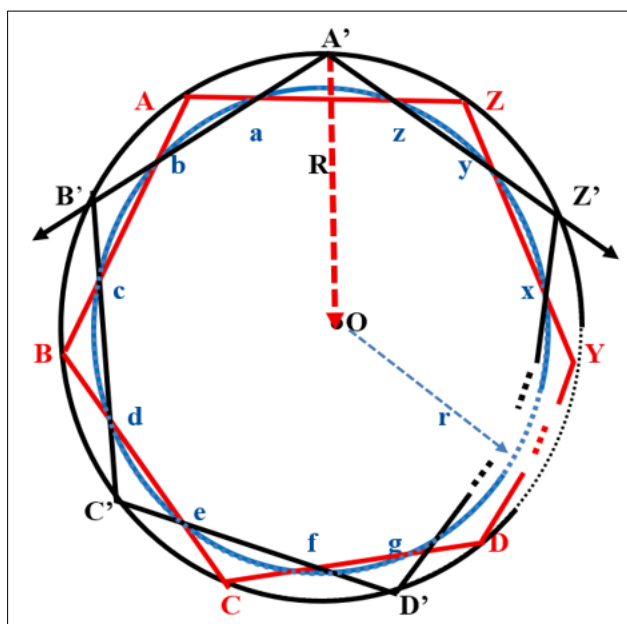


Figure 5: The regular polygon $abcdefg...yz$ (blue colour) inscribed in the given circle (O, r) and the resulting polygon $ABCD...YZ$ of the “Polygonising A Circle” challenge problem

III. DISCUSSION AND CONCLUSION

Can mathematicians use a compass and a straightedge to construct a regular polygon (triangle,

square, pentagon, hexagon, ...) of equal area to a given circle, using only a straightedge and compass? The results of my independent research successfully get an

exact solution to the ancient Greek “Squaring A Circle” problem challenge showing the idea that “If one can solve “squaring a circle” then one can also solve “regular triangling a circle”, “regular pentagoning a circle”, “regular hexagoning a circle” problems too ... This study proves that it is certain to successfully construct a regular polygon that has the same area πr^2 as the area of a given circle (O, r) with only straightedge and compass in Euclidean Geometry. In additional, this regular polygon has the same area πr^2 as the area πr^2 of a resulting square from my solution to the “Squaring The Circle” problem, published on the SJPMs on 31/10/2024 [6]. It is interesting that if the same given circle (O, r) to “Squaring A Circle” and “Polygonising A Circle” problem, then we can see these resulting square & polygon have the same area πr^2 as the area of the given circle.

This construction method is quite different from the approximation and is based on using only a straightedge and compass within Euclidean Geometry.

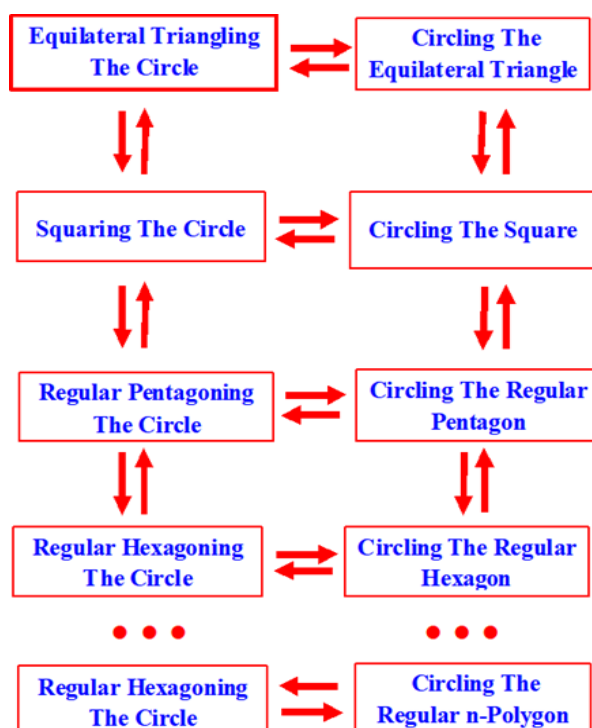
The inspiration for this research arose after the solution to the “Squaring The Circle” problem was published in SJPMs in 2024 [6]. If it is possible to square a circle using only a straightedge and compass in Euclidean geometry, the question arises: why is it difficult to regular polygonising a circle equidistantly, similar to how one squares a circle? In this context, the given circle contains an inscribed regular polygon having $2n$ sides, with n pairs of non-consecutive sides extending from it, leading to a regular polygon having n

sides. This regular polygon with n side serves as the solution to the new problem “regular polygonising a circle”. The remaining task is to prove that the area of this regular polygon is exactly equal to the area of the given circle. This paper also introduces a new mathematical tool, called the “Regular Polygon Ruler,” and provides a proof for the following statements: (1) A concentric regular polygon with n sides intersects the given circle at $2n$ vertices and forms a special polygon with $2n$ side, and with two non-consecutive sides symmetrically positioned; (2) This solution construction forms a Regular Polygon Ruler ($2n$ sides) inscribed in the given circle.

Upstream from this method of exact “Regular Polygonising A Circle” above, we can deduce, conversely, to get a new mathematical challenge “Circling A Regular Polygon” with a straightedge & a compass only, in Euclidean Geometry. This can be described in detail as follows.

Given a regular polygon with n sides a and area A , a & $A \in R$, then use a straightedge and a compass to construct an accurate circle, which has the exact area A . Then, how solving this new geometry problem is still an open research interest, to obtain a circular area without using the traditional constant π .

Finally, derivatives from the “Squaring The Circle” problem (ancient Greek challenge) for further researches, can be described in the following diagram:



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