

## Application of First Order Linear Differential Equation on the Spread of HIV/AIDS Using Logistic Model

(A Case study of Yobe State Specialist Hospital, Damaturu)

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### Abstract

### Original Research Article

Despite the fact that HIV/AIDS can be contacted through casual contact, sexual intercourse, blood transmission, using razor blend, are still stigmatized in the society. In this research work, the application of first order linear differential equation in the spread of HIV/AIDS obtained using logistic model variable method, It has been discovered that the concept of each year a large number of people all over the world die from HIV/AIDS. Although there are many complicating factors behind the spread of HIV, we still believe that relevant mathematical models can provide a good insight of the dynamics of the spread of it. If we can provide a satisfactory profile of this dynamics, it will certainly help government officials to make timely remedial actions. In the present work, we have established mathematical and logistic models of epidemiology for the spread of HIV. We have made a search for equilibrium points for the system and discussed about their stabilities. Efforts have been made to find the solution of the proposed system. Based on extensive analysis relevant comments are made on mutual co- existence of the group infected by HIV and the group not infected by that. To enlighten the public on how to avoid discrimination and stigmatization of people who are living with HIV/AIDS using mathematical model.

**Keywords:** First Order, Linear Differential Equation, Logistic Model, Mathematical Modeling, HIV/AIDS.

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## INTRODUCTION

Logistics Model is the growth from three perspectives: its sensitivity to initial conditions; its relationship to analogous difference equation models; and the formulation of stochastic models with mean logistic growth. The results indicate that although the logistic model is appealing in terms of its simplicity, its appeal is questionable in terms of its realism.

The process of developing a mathematical model is termed mathematical modeling [1]. Logistics models are used in the natural sciences (such as physics, biology, earth sciences, meteorology) and engineering disciplines (such as computer sciences, artificial intelligence), as well in the social sciences (such as economics, psychology, sociology, political sciences). Physician engineers, statisticians, operations research analysis, and economists use mathematical models most extensively. A model may help to explain a system and to study the effects of different components, and to make predictions about certain events [2].

Logistics models can take many forms, including but not limited to dynamical systems, statistical models, differential equations, or game theoretic models. These and other types of models can overlap, with a given model involving a variety of abstract structures. In general, logistics models may include logical models. In many cases, the quantity of scientific field depends on how well Logistics models developed on the theoretical side agree with results of repeatable experiments.

Lack of agreement between theoretical logistics models and experimental measurements often leads to important advance as better theories are developed. Logistics models are usually composed of relationships and variable relationships can be described by operators, such as algebraic operators, function differential operators, etc. variables abstractions of system parameters of increase that can be quantified. Operators can act with or without variables. Mathematical models can be classified in the following ways:

### Linear and Nonlinear

If all the operators in a Logistics model exhibit linearity, the resulting mathematical model is defined as linear. A model is considered nonlinear otherwise. The definition of linear and nonlinearity is dependent on context, and linear models may have nonlinear expression in them. For example, in statistical linear model, it is assumed that a relationship is linear in the parameters, but it may be nonlinear in the predictor variables. Similarly, a differential equation is said to be linear if it can be written with linear differential operators, but it can still have nonlinear expressions in it.

In a logistics-programming model, if the objective functions and constraints are represented entirely by linear equations, then the model is regarded as a linear model. If one or more of the objective functions or constraints are represented with nonlinear equations, then the model is known as a nonlinear model. Nonlinearity, even in simple systems, is often associated with phenomena such as chaos and irreversibility. Although there are exceptions, nonlinear systems and models tend to be more difficult to study than linear ones. A common approach to nonlinear problems is linearization, but this can be problematic if one is trying to study aspects such as irreversibility, which are strongly tied to nonlinearity.

### Static vs. Dynamic

A dynamic model accounts for time-dependent changes in the state of system, while a static (or steady state) model calculates the system in equilibrium, and thus is time-invariant. Dynamic models typically are represented by differential equations.

### Explicit vs. Implicit

If all of the input parameters of the overall model are known, and the output parameters can be calculated by a finite series of computations (known as linear programming, not to be explicit). But sometimes it is the output parameters which are known, and the corresponding inputs must be solved for by an iterative procedure, such as Newton's method (if the model is linear) or Boyden's method (if non-linear). For example, a jet engine's physical properties such as turbine and nozzle throat areas can be explicitly calculated given a design thermodynamic cycle (air and fuel flow rates, pressures, and temperatures) at a specific flight condition and power setting, but the engine's operating cycles at other flight conditions and power settings cannot explicitly calculated from the constant physical properties.

### Discrete vs. Continuous

A discrete model treats objects as discrete, such as the particles in a molecular model or the states in a statistical model; while a continuous model represents the objects in a continuous manner, such as the velocity field of fluid in pipe flows, temperatures

and stresses in a solid, and electric field that applies continuously over the entire model due to a point charge.

### Deterministic vs. Probabilistic (Stochastic)

A deterministic model is one in which every set of previous states of these variables; therefore, a deterministic model always performs the same way for a given set of initial conditions. Conversely, in a stochastic model- usually called a "statistical model"- randomness is present, and variable states are not described by unique values, but rather by probability distributions.

### Deductive, inductive, or floating

A deductive model is a logical structure based on a theory. An inductive model arises from empirical finding and generalization from them. The floating model rests on neither theory nor observation, but is merely the invocation of expected structure. Application of mathematics in social sciences outside of economics has been criticized for unfounded models. Application of catastrophe theory in science has been characterized as a floating model.

### STATEMENT OF THE PROBLEM

AIDS is the leading cause of death in Sub-Saharan Africa, especially in the southern part of the continent where nine countries with the highest HIV prevalence worldwide are all located in this sub region, with each of these countries experiencing adult HIV prevalence greater than 10%. With an estimated adult HIV prevalence of 26% in 2017, Swaziland has the most severe level of infection in the world.

Acquired Immune Deficiency Syndrome (AIDS) is a viral disease caused by Human Immune Virus (HIV). According to the World Health Organization (WHO) the number of reported case of AIDs by the year 1988 was much less in Nigeria than in most countries of the world. This gave us great hope that there was time to warn people of the danger that appeared to be approaching but it was then discovered that the low reported incidence is not quite as hopeful as we thought. The HIV-2 of West Africa was not discovered until 1986, three years after the discovery of HIV-1. In addition, it was only later that it was realized that a different kind of blood test was needed to detect the HIV-2. Tests that could detect both virus were not readily available until the end of 1989 or early 1990.

The statement from the specialist hospital Damaturu (2017) reported that the estimated number of people living with HIV/AIDS as at the of September, 2001 aged 0-49 years is 35 million and 3.3 million of them are children from age 0-15 years and children who have lost one or both parent due to AIDS are 1 million (UNAIDS, report on the global AIDS epidemic, UNAIDS fact sheet, 2017) [3]. Therefore, the researcher is concerned that:

- Despite the campaign on the fact that HIV/AIDS cannot be contacted through casual contact, sharing of cloth, dishes, spoons, toilet, seat, hugging and handshake the victims are still stigmatized in the society.
- The attitude of the public is as result of improper information and ignorance about the disease.

## AIM AND OBJECTIVES OF THE STUDY

This study is aimed at addressing public perception toward HIV/AIDS victims increasing public awareness about the disease pattern of spread. It is also designed to achieve the following objectives:

To determine alternative method of improving the health standard of HIV/AIDS victims and promote interaction between the victims and non-victims of the HIV/AIDS using logistic model.

## SIGNIFICANCE OF THE STUDY

The research will create more awareness on the need to avoid stigmatization of the victims of HIV/AIDS and provide information about HIV infection.

## SCOPE OF THE STUDY

This study is restricted to the public perception towards HIV/AIDS victims and it will look into public discrimination and stigma attachment to HIV/AIDS victims.

## LIMITATION OF THE STUDY

This research works is limited to state Specialist Hospital Damaturu, Yobe -Nigeria.

## RESEARCH QUESTION

1. Does the public discriminate against and stigmatized the HIV/AIDS victims?
2. Does ignorance about the disease pattern contributes to such attitudes.
3. Are there means of preventing the disease?
4. How effective are the preventive measures?

## SOME REVIEWS

Jefferies [4], viewed AIDS as new sexually transmitted disease (STD) which develop in people who are infected with the HIV. Through not all who are infected with HIV go on to develop AIDS, but they may be unknowingly pass the virus onto other. A disease kills people once it has developed.

A. I. Enagi [5], considers a deterministic compartmental model of tuberculosis control strategy adopted by national tuberculosis and leprosy control program. He established the disease free and the endemic equilibrium state and carried out the stability analysis of the disease free and the endemic equilibrium state. He also carried out numerical simulations of the model to have an insight into the dynamics of the

model. He found out that the disease free equilibrium state is stable. The numerical simulation showed that it will be very difficult to complete eradicate tuberculosis from Nigeria using this method adopted by national tuberculosis and leprosy control program.

Mugisha *et al.*, [6], formulated mathematical models for the dynamics of tuberculosis in density population required to minimize and therefore eradicate tuberculosis. Both numerical and qualitative analyses were done and the effect of various in the area size and recruitment rates was investigated. Analysis showed that there existence disease free – equilibrium point provided the characteristics area is greater than the probability of survival from latent stage to infectious stage and the number of latent infectious produced but a typical characteristics area per individual should be at least 0.25 square kilometer in order to minimize tuberculosis incidence. His work suggested that characteristic area can as well be looked at as environmental stressor that can lead to tuberculosis.

Jama [7], the tuberculosis (TB) mortality rate has declined by 37% worldwide since 2000, but the disease still remains 1 of the top 10 causes of death, according to the most recent Global TB Report 2017, released by the World Health Organization. Greater political commitment is needed to address the burden of the disease and meet the goal of ending the TB epidemic by 2030. Tuberculosis was the leading cause of death from a single infectious agent in 2016, ranking above HIV/AIDS. Worldwide, 10.4 million new TB infections were estimated in 2016—10% of which occurred in people with HIV—and about 1.7 million people died of the disease. More than 600 000 new cases of TB with resistance to the most effective first-line drug, rifampicin, were reported, including 490 000 multidrug-resistant TB infections.

Lerner [8], in his paper, revisiting the tuberculosis research of Thomas holnee (2009–2015), examined the association between stress and tuberculosis. He urged that disease may lead to disease immune function and this thus to clinical disease. His study suggested that persons who had experienced stressful situation, such as divorce, death of spouse, or loss of a job were more likely to develop tuberculosis and less likely to recover from it. The scholars in the study also devised a numerical scale (social readjustment rating scale) that qualified stress events with control groups. They also emphasized the need to understand each patient history and view his/her tuberculosis infection as the culmination of a life of emotional hardship. However their study has been criticized by Theorell [9], for its inability to consider the fact that everybody respond differently to stressful situations.

Tracy *et al.*, [10], present a Meta – analysis of the literature on stress and immunity in humans. The result showed substantial evidence for a relation between stress and decrease in functional immune measures (proliferative response tomatoes and natural killer cell activity). The way neuroendocrine mechanism sand health practice might alternate stress has been discussed, evidence for the relations between stress and both functional and enumerative immune measures have been presented. The authors further stated that, stressor duration is important for immune outcomes, and interpersonal events are related to alterations incident immune parameters than social events. However, their work lacked a qualitative and numerical analysis which this study has covered.

According to Davidson [11], the World Health Organization (WHO) estimated that by the year 2000, there would be 20 million persons infected with HIV, the cumulative total since beginning of the epidemic being 40 million. About 90% of cases are in developing countries unable to afford the expensive medical care required to control progress of the disease. Recent serological test demonstrated HIV seropositive in an African male. In a blood sample taken in 1947 so far Sub-Sahara African has been hardest hit with the HIV epidemic with the prevalence between 15-50 years of age.

Adamu [12], stated that in 1992, 192 countries reported over 0.5 million AIDS cases. In the same year, about 10-12 million people were globally estimated to be HIV infected (including 3 million women and 1 million children) with AIDS developing in about 2-3 million of them. In Nigeria, 3.5 million people are HIV infected including 0.5 million children. In 1999, the sero-prevalence shows that Benue State has the highest rate 16.8%, Jigawa State was the lowest rate 1.7% while Yobe State has the rate 1.9%, and Damaturu Local Government has the highest rate 1.0% in Yobe state.

According to Walsh [13], HIV can infect cells, which carry a protein called CD4 on their surface. Their primary target is the T4 or T helper cells of the Immune system, but it also affects macrophages, cells in the mucus membrane and the both. One reverse transcriptase converts its RNA to DNA that is then integrated into the DNA of the host cell. HIV particles one then assembled & in the cytoplasm of the cell and escape by budding through the cell membrane, killing the cells are invaded and destroyed the body Immune system as weakened making the individual prone to the variety of opportunistic infection, malignant disease and neuro-psychiatric complications. There is strong association between the number of CD4, lymphocyte and the development of life threatening illness.

Coventry *et al.*, [14], defined discrimination as making unfair difference in ones treatment of people

while stigmatization was defined as branding someone as disgraceful.

According to Chapman [15], individuals who outwards signs of their physical condition are stigmatized by the society that they are treated differently. The averted glance, the imminent away from physical contact and the widespread remark all serve to demonstrate that the individual is different and does not fit into the society perception of an acceptable person.

Jatau [16], noted that the discrimination and stigma caused the patient to construct a new of life around a new set of attitude, a new set of social relationship and activities. He further states that the public perception response to victims situation as a direct expression of his defect. He suggested ways to handle such stigma and the patients. Treat the conditions symptomatically so as to remove or minimize its impacts help the patients to his/her disability and the condition they find themselves. Provide the needs of a patients so as to give him/her a feeling of belonging. The society should be made to perceive the victims as being sick instead of making them feel isolated and unwanted.

## METHODOLOGY

A variable separable method of solving Regression equation is employed completely so that the result obtained could be use as a tool for solving the problem.

## FRAME WORK

This research is mainly the application of statistical Regression equation with much emphasis on the linear regression equation.

## DEFINITION OF TERMS

- AIDS: acquired Immune Deficiency Syndrome [17]
- HIV: Human Immune Deficiency Virus [18]
- Disease: is anything that imposes some form of limitation on our physical, mental, emotional, social and psychological functioning [19].
- Stigmatization: is defined as branding someone as disgraceful.
- Discrimination: is making unfair differences in one's treatment of people.

## METHODOLOGY

Here the method and the process of computing the values in data obtained are done here in this chapter. Therefore, below is the data obtained from the Specialist Hospital Damaturu Yobe State. An epidemic of HIV and AIDs in Yobe state in the months of May 2018.

**DIFFERENTIAL EQUATION**

A differential equation is a mathematical equation that relates some function with its derivative. In application, the function usually represents physical quantities, derivatives represent their rate of change, and the differential equation defines the relationship between them.

Differential equations first came into existence with the intervention of calculus by Newton and Leibniz.

**SUSCEPTIBLE**

This refers to the potential people in the community of those that are likely to be infected in an outbreak.

**INFECTED**

The group of people being infected by a disease in an outbreak. It refers to infected persons.

**EPIDEMICS**

A large outbreak of a disease over a short period of time is called epidemics.

**INITIAL CONDITIONS**

Initial conditions are a condition or set of conditions on the solution that will allow us to determine which solution that we are after. Initial conditions are of the form  $Y(t_0)=y_0$  and  $Y'(t_0)=y'_0$ . In other words, initial conditions are the value of the solution and its derivative at a specific point. Differential equations are used hence only one solution will meet the given condition.

The number of initial conditions that are required for a given differential equation will depend upon the order of the differential equations.

**INITIAL VALUE PROBLEM**

An initial value problem is a differential equation along with the appropriate number of initial conditions, e.g.  $6x^2y'' + 18xy' + 3y = 0$

$$Y(6) = y_0 - y \quad Y'(6) = \frac{1}{216}$$

**VARIABLE SEPARABLE METHOD**

The differential equation  $\frac{dN}{dt} = ky(N - y)$  is called a first-order differential equation and can be solved by the separable method, as follows:

$$\frac{dN}{dt} = ky(N - y) \dots \dots \dots (2.0)$$

$$\frac{dN}{y(N - y)} = k dt$$

$$\therefore \int \frac{dN}{y(N - y)} = \int k dt$$

$$\int \frac{1}{y(N - y)} dN = k dt + c$$

$$\text{but } \frac{1}{y(N - y)} = \frac{A}{y} + \frac{B}{N - y}$$

$$\frac{A(N - y) + By}{y(N - y)} = \frac{1}{y(N - y)}$$

$$\therefore A(N - y) + By = 1 \dots \dots \dots (2.1)$$

Let  $y=N$

$$A(N - N) + BN = 1$$

$$A(0) + BN = 1$$

$$BN = 1 \quad B = \frac{1}{N}$$

Also

$$A(N - N) + BN = 1$$

$$BN = 1, B = \frac{1}{N}$$

Also

Let y=0

$$A(N - 0) + B(0) = 1$$

$$AN + 0 = 1$$

$$AN = 1, A = \frac{1}{N}$$

Hence

$$\frac{A}{y} + \frac{B}{N - y} dN = \frac{1}{y} + \frac{1}{N - y} \dots\dots\dots(2,2)$$

Therefore

$$\int \frac{1}{y(N - y)} dt = \int \left( \frac{A}{y} + \frac{B}{N - y} dN \right)$$

$$\int \left[ \frac{1}{y} + \frac{1}{N - y} \right] dt = \frac{1}{N} \int \frac{1}{y} + \frac{1}{N - y} = kt + c$$

$$\frac{1}{N} (\ln |N| - \ln |N - y|) = kt + c$$

$$1(\ln |N| - \ln |N - y|) = Nkt + c$$

$$\ln \left| \frac{N}{N - y} \right| = Nkt + c$$

$$\frac{N}{N - y} = Ne^{kt+c} \dots\dots\dots(2,3)$$

For some constant N and K substituting the value of initial condition in (4) above lead us to logistic equation.

**RESULT AND DISCUSSION**

In this section, we are used the model formulated in the methodology aspect to compute the result at various time and the result obtained can be discuss to enable used depict the epidemic or otherwise from equation.

- S= Susceptible people
- T= Time period
- I= Infected persons

Since the rate of speed of HIV/AIDS disease is directly proportional to the infected and non-infected person the Yobe state has a total five hundred (500) sample and a population of thirsting thousand (13,000) people. If the infected

people is represented as N then the non – infected people is 500 – N” hence  $\frac{dN}{dt} \propto N(500 - N)$

$$\frac{dN}{dt} KN(500 - N)$$

Variable separable

$$\frac{dN}{N(500 - N)} = kdt$$

Integrating both side

$$\int \frac{dN}{N(500 - N)} = k \int dt$$

$$\therefore \frac{1}{N(500 - N)} = \frac{A}{N} + \frac{B}{500 - N}$$

$$\frac{A}{N} + \frac{B}{500 - N} = \frac{1}{N(500 - N)}$$

$$\frac{A(500 - N) + BN}{N(500 - N)} + \frac{1B}{N(500 - N)}$$

$$A(500 - N) + BN = 1$$

Let N=500

$$A(500 - 500) + B(500) = 1$$

$$0 + 500B = 1$$

Again Let N=0

$$0 + 500B = 1$$

$$B = \frac{1}{500}$$

Again let N=0

$$A(500 - N) + BN = 1$$

$$A(500 - 0) + B(0) = 1$$

$$500A + 0 = 1$$

$$500A = 1$$

$$A = \frac{1}{500}$$

$$\therefore \frac{A}{N} + \frac{B}{500 - N} = \frac{1}{500N} + \frac{1}{500(500 - N)}$$

$$\int \frac{1}{500N} + \int \frac{1}{500(500 - N)} dN = K \int dt$$

$$\frac{1}{500} = \left[ \int \frac{1}{N} dN + \int \frac{1}{500 - N} dN \right] = Kt + C$$

$$\frac{1}{500} (\ln|N| - \ln|500 - N|) + D = Kt + C$$

$$\frac{1}{500} (\ln|N| - \ln|500 - N|) = Kt + C$$

$$\ln|N| - \ln|500 - N| - (Kt + C)500$$

$$\ln \left| \frac{N}{500 - N} \right| = R_1 t + C$$

Where K and are constant and N is always between 1 and 13,000 we can drop the absolute value.

$$\ln \left| \frac{N}{500 - N} \right| = R_1 t + C$$

At the initial condition

$$\begin{aligned} N(t) &= N & N(0) &= 1 \\ \ln \left| \frac{N}{500 - N} \right| &= R_1(0) + C_1 \\ \ln \left| \frac{N}{499} \right| &= C \end{aligned}$$

Again  $N(2) = 84$

Implies that

$$\ln \left| \frac{84}{500 - 84} \right| = 2k + \ln \left| \frac{1}{499} \right|$$

From here we can plug in the values of N. by substituting their initial condition it lead is to logistic equation as prove in equation (3,0)chapter three. This means that it will first cause on exponential increase, which will show down and eventually level it.

$\approx 19$  People infected the fourth day.

**Table-1: To show the susceptible, infected people and the time period**

T	0	1	2	3	4
S	500	346	416	429	215
I	1	154	84	71	286

**Table-2: To show the susceptible, infected people and the time period**

T	0	1	2	3	4
S	13,000	12,992	12,910	12,458	77,90
I	1	8	90	542	5210

S=susceptible people  
T=time period  
I=infected persons

Since the rate of speed of HIV/AIDS disease is directly proportional to the infected and non-infected person the Yobe state has a total population of thirsting thousand (13,000) people. If the infected people is represented as N then the non – infected people is  $13,000 - N$ ” hence

$$\begin{aligned} \frac{dN}{dt} &\propto N(13,000 - N) \\ \frac{dN}{dt} &= KN(13,000 - N) \end{aligned}$$

Variable separable

$$\begin{aligned} \frac{dN}{N(13,000 - N)} &= k dt \\ \text{Integrating both side} \\ \int \frac{dN}{N(13,000 - N)} &= k \int dt \\ \therefore \frac{1}{N(13,000 - N)} &= \frac{A}{N} + \frac{B}{13,000 - N} \\ \frac{A}{N} + \frac{B}{13,000 - N} &= \frac{1}{N(13,000 - N)} \\ \frac{A(13000 - N) + BN}{N(13,000 - N)} &+ \frac{1B}{N(13,000 - N)} \end{aligned}$$



$$A(13,000 - N) + BN = 1$$

Let  $N=13,000$

$$A(13,000 - 13,000) + B(13,000) = 1$$

$$0 + 13,000B = 1$$

Again Let  $N=0$

$$0 + 13,000B = 1$$

$$B = \frac{1}{13,000}$$

Again let  $N=0$

$$A(13,000 - N) + BN = 1$$

$$A(13,000 - 0) + B(0) = 1$$

$$13000A + 0 = 1$$

$$13000A = 1$$

$$A = \frac{1}{13,000}$$

$$\therefore \frac{A}{N} + \frac{B}{13,000 - N} = \frac{1}{13,000} + \frac{1}{13,000 - N}$$

$$\int \frac{1}{13,000} + \int \frac{1}{13,000 - N} dN = K \int dt$$

$$\frac{1}{13,000} = \left[ \int \frac{1}{N} dN + \int \frac{1}{13,000 - N} dN \right] = Kt + C$$

$$\frac{1}{13,000} (\ln|N| - \ln|13,000 - N|) + D = Kt + C$$

$$\frac{1}{13,000} (\ln|N| - \ln|13,000 - N|) = Kt + C$$

$$\ln|N| - \ln|13,000 - N| - (Kt + C) \cdot 13,000$$

$$\ln \left| \frac{N}{13,000 - N} \right| = R_1 t + C$$

Where  $K$  and  $D$  are constant and  $N$  is always between 1 and 13,000 we can drop the absolute value.

$$\ln \left| \frac{N}{13,000 - N} \right| = R_1 t + C$$

At the initial condition  $N(t) = N(0) = 1$

$$\ln \left| \frac{N}{13,000 - N} \right| = R_1(0) + C_1$$

$$\ln \left| \frac{N}{12,999} \right| = C$$

Again  $N(2) = 90$

Implies that

$$\ln \left| \frac{90}{13,000 - 90} \right| = 2k + \ln \left| \frac{1}{12,999} \right|$$

From here we can plug in the values of N. by substituting their initial condition it lead is to logistic equation as prove in equation (3,0). This means that it will first cause on exponential increase, which will show down and eventually level it.

$N(2) = 90$  from table(2)

$$P(t) = \frac{kp_o e^{rt}}{k + p_o(e^{rt} - 1)}$$

$$P(2) = \frac{13,000 \times 1 \times e^{2r}}{13,000 + 1(e^{2r} - 1)}$$

$$\therefore 90 = \frac{13,000 \times 1 \times e^{2r}}{13,000 + 1(e^{2r} - 1)}$$

$$90 = \frac{13,000e^{2r}}{13,000 - 1 + (e^{2r})} \Rightarrow 90 = \frac{13000e^{2r}}{12,999 + e^{2r}}$$

$$90(12,999 + e^{2r}) = 13,000e^{2r}$$

$$11,69910 + 90e^{2r} = 13,000e^{2r}$$

$$11,69910 = 13000e^{2r} - 90e^{2r}$$

$$\frac{1169910}{12910} = e^{2r}$$

$$\ln 90.62 = 2r$$

$$r = \frac{1}{2} \ln 90.62$$

$$\therefore P(t) = \frac{kp_o e^{\frac{1}{2} \ln 90.62(t)}}{k + p_o \left( e^{\frac{1}{2} \ln 90.62(t)} - 1 \right)}$$

The above equation is the general model depicting the number of infected population in the host society at any point in time in the table (1) the letter “S” means the susceptible people (meaning the people that can be infected in the host society ), the rate at which (S) changes with time in the host society once epidemic expected is S and it is always negative, expressed as  $S = \text{total population} + \text{rate of change in susceptible} \times \text{the time (in days)}$ .

$$S = 13,000 + S(t) \Rightarrow S = 13,000 - S(\theta)t$$

Also “I” means the infected people, the change in the number of infected people in the society of host with time is I,

Since,

$$\therefore P(t) = \frac{kp_o e^{\frac{1}{2} \ln 90.62(t)}}{k + p_o \left( e^{\frac{1}{2} \ln 90.62(t)} - 1 \right)}$$

When the time t=0 is zero

$$P(t) = \frac{k p_o e^{\frac{1}{2} \ln 90.62(t)}}{k + p_o \left( e^{\frac{1}{2} \ln 90.62(t)} - 1 \right)}$$

$$P(0) = \frac{13,000 \times 1 \times e^{\frac{1}{2} \ln 90.62(0)}}{13,000 + 1 \left( e^{\frac{1}{2} \ln 90.62(0)} - 1 \right)} = \frac{13,000 e^0}{13,000 - 1 e^0}$$

$$= \frac{13,000 \times 1}{13,000 + 1 - 1} = \frac{13,000}{13,000} = 1$$

Initially when the time (t) is one t=1

$$P(2) = \frac{13,000 \times 1 \times e^{\frac{1}{2} \ln 90.62(2)}}{13,000 + 1 \left( e^{\frac{1}{2} \ln 90.62(2)} - 1 \right)} = \frac{13,000 e^{4.5067}}{13,000 - 1 + e^{4.5067}}$$

$$= \frac{13,000 \times 90.6223}{12,999 + 90.6223} = \frac{1178089.9}{13089.6223} = 90.0018$$

People were infected the second day.

≈ 90      When t=3

$$P(3) = \frac{13,000 \times 1 \times e^{\frac{1}{2} \ln 90.62(3)}}{13,000 + 1 \left( e^{\frac{1}{2} \ln 90.62(3)} - 1 \right)} = \frac{13,000 e^{6.7600}}{12,999 + e^{6.7600}}$$

$$= \frac{13,000 \times 862.6422}{12,999 + 862.6422} = \frac{11214.3486}{13861.6422} = 8090.202$$

≈ 8090      People where infected the third day.

When t=4

$$P(4) = \frac{13,000 \times 1 \times e^{\frac{1}{2} \ln 90.62(4)}}{13,000 + 1 \left( e^{\frac{1}{2} \ln 90.62(4)} - 1 \right)} = \frac{13,000 e^{9.0133}}{12,999 + e^{9.0133}}$$

$$= \frac{13,000 \times 8211.5749}{12,999 + 8211.5749} = \frac{10675073.7}{21210.5749} = 5032.8892$$

≈ 5033      People infected the fourth day

Below is the showing the computed value of the infected and non – infected person in the society of host over a short period of time by the model.

Base on the table1 and Table-2 above the value of infected people can be plotted against the time period and also the values of non – infected can also be plotted against the period.

**INITIAL VALUE PROBLEM**

An initial value problem is a differential equation along in the appropriate number of initial conditions e.g  $6x^2 y'' + 18x y' + 3y = 0$

$$Y(6) = ys - y \quad Y(6) = \frac{1}{216}$$

**LOGISTIC EQUATION**

$$\frac{dp}{dt} = ap; \quad \frac{dp}{dt} = kp \quad \text{For some } k > 0$$

If we want the new born growth rate to approach zero as p approach the maximum carrying capacity we can introduce a limiting factor of H-P

$$\frac{dp}{dt} = kp(H - P) \dots \dots \dots (2,4)$$

This is the logistic differential equation  $\frac{dp}{dt} = p(a - bp)$  with a solution

$$P(t) \frac{ace^{at}}{1 + bce^{at}} = \frac{ace^{at}}{e^{at} + (e^{-at} + bc)} = \frac{ac}{bc + e^{-at}}$$

$$P(t) \frac{ac}{bc + e^{-at}} \dots \dots \dots (2,5)$$

If  $t=0$  then  $P(0) = P_0 \neq \frac{a}{b}$  we find then  $C = \frac{P_0}{a - bP_0}$  and so after substituting and simplifying the solution becomes

$$P(t) \frac{ap_0}{bp_0 + (a - bp_0)e^{-at}} \dots \dots \dots (2,5)$$

Were a and b are constant with  $a > b$  at  $t=0$

$$P(0) = p_0 \text{ then}$$

$$P(t) \frac{ac}{bc + e^{-at}} \dots \dots \dots (2,6)$$

at,  $t=0$

$$P_0 = \frac{ac}{bc + e^0} \cdot P_0 \frac{ac}{bc + 1}$$

$$P_0(bc + 1) = ac$$

$$P_0bc + P_0 = ac$$

$$P_0bc - ac = -P_0$$

$$ac - P_0bc = P_0$$

$$C = \frac{P_0}{a - P_0b}$$

$$C(a - P_0b) = P_0$$

Substituting the value of C in to equation (12)

$$P(t) \frac{ac}{bc + e^{-at}}$$

$$= a \left[ \frac{P_0}{a - bP_0} \right] \div \left[ \frac{P_0}{a - P_0b} \right] + \frac{e^{-at}}{1}$$

$$\frac{aP_0}{a - bP_0} \div \frac{bP_0(a - P_0b)e^{-at}}{a - P_0b}$$

$$P(t) = \frac{aP_0}{a - P_0b} \times \frac{a - bP_0}{bP_0 + (a - P_0b)e^{-at}}$$

$$P(t) = \frac{aP_0}{bP_0 + (a - P_0b)e^{-at}} \dots\dots\dots(2,7)$$

Multiply equation (1,2) by  $\frac{e^{at}}{e^{at}} = 1$

$$P(t) = \frac{aP_0e^{at}}{bP_0e^{at} + (a - P_0b)} \dots\dots\dots(2,8)$$

Where

- $P_0$  = population of the host society at the initial time
- $b$  = the final population after some long time also called the maximum carrying capacity with limit growth
- $a$  = the initial growth rate
- $t$  = time measure on days.

**DETERMINATION MODELING TECHNIQUES**

The first step of determination modeling consists of having a complete and realistic picture of the biology of the diseases. Example duration of the period of infection, incubation periods, immune status after infection.

The second step is to collect the data on the demographic epidemiology and the biologic characteristic of the infection (transmission rate) and the population (birth rate and death rate). The third state a sir model is selected.

**A THRESHOLD THEOREM OF KERMACK AND MC KENDRICK OF SPREAD OF HIV AND AID DISEASE**

Before going on with sir model it is important to understand how epidemics set up in a population for disease conferring long lasting immunity infection. For example: HIV/AIDS. The number of susceptible (s) decrease with time. Before the outbreak of a first HIV/AIDS case the population of susceptible (s) is 100% in the population because everyone is susceptible, the proportion of expose (E), infected (I) and recovered (R) is zero (0), when epidemic start to spread susceptible

decrease while immune and infection increase until everyone get immunized.

The potential of infected person in a population depend on the basic in a population depends on the basic reproduction number (R) is define as the average number of person directly infected by an infection disease during his/her entire infection period when he/she enters a totally susceptible persons. The development and the size of infection are determined by (R) that relies on.

- The attack rate (risk of transmission per contact)
- The number of potentially infected contact i.e the average person in a population has per unit time.
- The duration has per the infecting period.

If at any time (R) gets smaller than one i.e the disease eventually disappears from the population because on average each infected person cannot ensure transmission of the infection agent to one susceptible. This result in a new waves of infected been lesser amplitude then the proceeding one and finally to disease elimination on other hand it ( $R_0=1$ ) the disease remain endemic as one infection agent to one infections agent to one susceptible on the average. Lastly if ( $R_0=1$ ) en epidemic build up. This effort has been established by kermack and Mc Kendrick and expands body the introduction of infections individual into a community of susceptible does not automatically give rise to an epidemic outbreak.

**Table-1: To show the susceptible, infected people and the time period**

T	0	1	2	3	4
S	500	346	416	429	215
I	1	154	84	71	286

S=susceptible people  
 T=time period  
 I=infected persons

Since the rate of speed of HIV/AIDS disease is directly proportional to the infected and non infected person the Yobe state has a total population of thirsting

thousand (13,000) people. If the infected people is represented as N then the non – infected people is 13,000 – N.

$N(2) = 84$  from Table-1

$$P(t) = \frac{kp_o e^{rt}}{k + p_o(e^{rt} - 1)}$$

$$P(2) = \frac{500 \times 1 \times e^{2r}}{500 + 1(e^{2r} - 1)}$$

$$\therefore 90 = \frac{500 \times 1 \times e^{2r}}{500 + 1(e^{2r} - 1)}$$

$$90 = \frac{500e^{2r}}{500 - 1 + (e^{2r})} \Rightarrow 90 = \frac{500e^{2r}}{499 + e^{2r}}$$

$$84(499 + e^{2r}) = 500e^{2r}$$

$$41916 + 84e^{2r} = 500e^{2r}$$

$$41916 = 416e^{2r} - 84e^{2r}$$

$$\frac{41916}{416} = e^{2r}$$

$$\ln 100.76 = 2r$$

$$r = \frac{1}{2} \ln 100.76$$

$$\therefore P(t) = \frac{kp_o e^{\frac{1}{2} \ln 100.76(t)}}{k + p_o \left( e^{\frac{1}{2} \ln 100.76(t)} - 1 \right)}$$

The above equation is the general model depicting the number of infected population in the host society at any point in time in the Table-1 the letter ‘‘S’’ means the susceptible people (meaning the people that can be infected in the host society), the rate at which (S) changes with time in the host society once epidemic expected is S and it is always negative, expressed as  $S = \text{total population} + \text{rate of change in susceptible} \times \text{time (in days)}$ .

$$S = 500 + S(t) \Rightarrow S = 500 - S(\theta)t$$

Also ‘‘I’’ means the infected people, the change in the number of infected people in the society of host with time is I,

Since,

$$\therefore P(t) = \frac{kp_o e^{\frac{1}{2} \ln 100.76(t)}}{k + p_o \left( e^{\frac{1}{2} \ln 100.76(t)} - 1 \right)}$$

When the time  $t=0$  is zero,

$$P(t) = \frac{kp_o e^{\frac{1}{2} \ln 100.76(t)}}{k + p_o \left( e^{\frac{1}{2} \ln 100.76(t)} - 1 \right)}$$

$$P(0) = \frac{500 \times 1 \times e^{\frac{1}{2} \ln 100.76(0)}}{500 + 1 \left( e^{\frac{1}{2} \ln 100.76(0)} - 1 \right)} = \frac{500e^0}{500 - 1e^0} = \frac{500 \times 1}{500 + 1 - 1} = \frac{500}{500} = 1 \text{ (person infected)}$$

initially.

When the time (t) is one t=1,

$$P(2) = \frac{500 \times 1 \times e^{\frac{1}{2} \ln 10076(2)}}{500 + 1 \left( e^{\frac{1}{2} \ln 10076(2)} - 1 \right)} = \frac{500e^{21.652}}{500 - 1 + e^{21.652}}$$

$$= \frac{500 \times 14.1824}{499 + 14.1824} = \frac{7091.2}{513.1824} = 13.8180$$

≈ 14 People were infected the second day.

When t=3,

$$P(3) = \frac{500 \times 1 \times e^{\frac{1}{2} \ln 10076(3)}}{500 + 1 \left( e^{\frac{1}{2} \ln 10076(3)} - 1 \right)} = \frac{500e^{2.8557}}{499 + e^{2.8557}}$$

$$= \frac{500 \times 17.3857}{499 + 17.3857} = \frac{8692.85}{516.3857} = 16.83$$

≈ 17 People where infected the third day.

When t=4

$$P(4) = \frac{500 \times 1 \times e^{\frac{1}{2} \ln 10076(4)}}{500 + 1 \left( e^{\frac{1}{2} \ln 10076(4)} - 1 \right)} = \frac{500e^{2.9995}}{499 + e^{2.9995}}$$

$$= \frac{500 \times 20.0755}{499 + 20.0755} = \frac{10037.75}{519.0755} = 19.33$$

≈ 19 People infected the fourth day.

Below is the showing the computed value of the infected and non – infected person in the society of host over a short period of time by the model.

T	0	1	2	3	4
S	500	349	416	429	215
I	1	10	14	17	19

Base on the above table the value of infected people can be plotted against the time period and also the values of non – infected can also be plotted against the period.

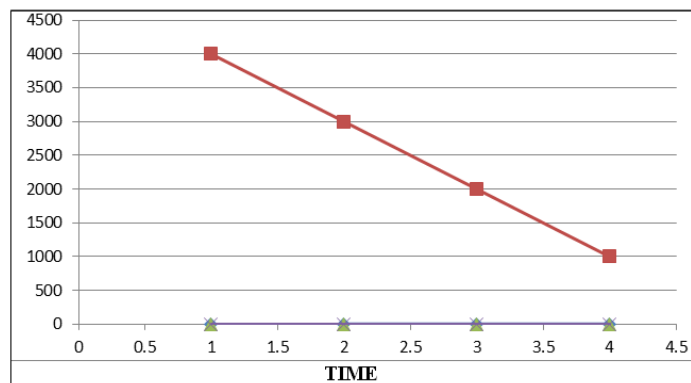


Fig-1: The Graph of Susceptible Persons against the Time

However, the Table-1 and the above graph depicts that an epidemics has set up and the level of the epidemic is totally of the final stayed affecting on average of 286. People in just four days, that is it infected positive is steady from the onset and letter it

exponentially increase causing every large damage to the society. Also the graph below depicted the non – infected people against the time in the same society of host at that period in time.

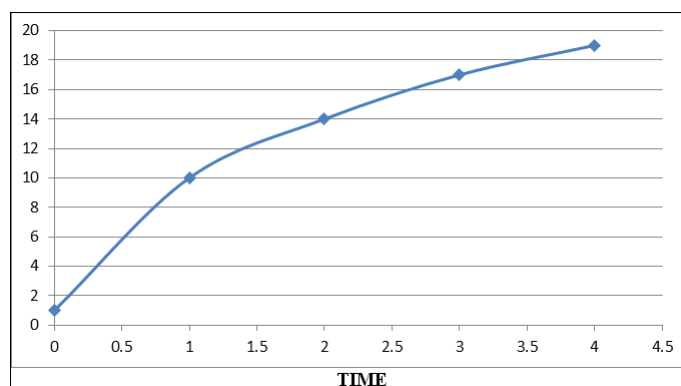


Fig-2: The Graph of Infected Person against the Time

According to the graph there is a decline in the number of non – infected person in the society of host, although, the number of non – infected (500-N) decline steady and latter reduces sharply showing a threat to the society.

## CONCLUSION

Based on the findings so far, the HIV and AIDS has set up affecting the positive fraction of the society of host with as average of infected person daily. The HIV and AIDS level was slow in the first place but later it increases exponentially indicating a sign of danger to the said society of host.

## RECOMMENDATION

Sequel to the finding obtained in this study it is obvious that the results obtained have depicted the number of people likely to be infected over a period of time and make a reasonable for best of how many people to be infected in a certain time to enable a proper decision and supply in case of an outbreak (HIV & AIDS) like this in a given society of host with known population number.

Although the study has not make it to our consumption the death that occurred since inception of the epidemic the number of recovered people and the inclining and declining, in the trend of their immune system during HIV and AIDs when on treatment. Further study could be conducted by any interested candidate to make clarification of these not known parameter enlisted above by the study.

A study of this nature need data from the host societies which were normally monitored and checked by health agencies difficulties is often encountered while trying to collect the data for the project.

In conclusion, we would like to say that since there is an outbreak in the said society of host a special care unit be build by the government in at least every unit of local government so that we can get proper health care.

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