Steady Generalized Plane Couette Flow of Viscous Incompressible Fluid between Two Porous Parallel Plates through Porous Medium with Magnetic Field

Dr. Anand Swrup Sharma (D.Sc. Scholar) *

Professor, Dept. of Applied Sciences (Mathematics), Future University, Bareilly (U P), India

DOI: 10.36347/sjpms.2020.v07i08.004 | Received: 10.08.2020 | Accepted: 18.08.2020 | Published: 20.08.2020

*Corresponding author: Dr. Anand Swrup Sharma
Email: sharma.as09@gmail.com

Abstract

In this paper, we have investigated the steady generalized plane Couette flow of viscous incompressible fluid between two porous parallel plates through porous medium with magnetic field. We have studied the velocity, average velocity, shear stress, skin frictions, the volumetric flow, drag coefficients & stream lines.

Keywords: Steady Couette flow, viscous parallel plates, incompressible fluid, porous medium, & magnetic field.

Copyright © 2020: This is an open-access article distributed under the terms of the Creative Commons Attribution license which permits unrestricted use, distribution, and reproduction in any medium for non-commercial use (Non Commercial, or CC-BY-NC) provided the original author and source are credited.

NOMENCLATURE

\[ u = \text{Velocity component along } x\text{-axis} \]
\[ v = \text{Velocity component along } y\text{-axis} \]
\[ t = \text{The time} \]
\[ \rho = \text{The density of fluid} \]
\[ P = \text{The fluid pressure} \]
\[ k = \text{The thermal conductivity} \]
\[ \mu = \text{Coefficient of viscosity} \]
\[ \nu = \text{Kinematic viscosity} \]
\[ Q = \text{The volumetric flow} \]

INTRODUCTION

**FORMULATION OF THE PROBLEM**

Let us consider two infinite porous plates AB & CD separated by a distance 2h. The fluid enters in y direction. The velocity component along x-axis is a function of y only. The motion of incompressible fluid is in two dimension and is steady then

\[ u = u(y), \quad w = 0 \quad \& \quad \frac{\partial}{\partial t} = 0 \]

The equation of continuity for incompressible fluid

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad \text{put} \quad w = 0 \quad \& \quad \frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} = 0
\]

\[ \nabla \sqrt{\frac{1}{k}} u \quad \& \quad \nabla \sqrt{\frac{1}{\mu}} u
\]

\[ \nabla \sqrt{\frac{1}{k}} u \quad \& \quad \nabla \sqrt{\frac{1}{\mu}} u
\]

\[ \frac{v}{v} \quad \text{is independent of} \quad y \quad \text{but motion is along} \quad y-\text{axis. So we can say that} \quad v \quad \text{is constant velocity} \quad i.e. \quad v = v_0 \quad \text{or the fluid enters in flow region through one plate at the same constant velocity} \quad v_0.
\]

Also Navier–Stokes equations for incompressible fluid in the absence of body force when flow is steady

\[ v_0 \frac{d u}{d y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \frac{d^2 u}{d y^2} + \left( \frac{1}{k} + \frac{\sigma B_0^2}{\mu} \right) u \quad \text{.......... (1)} \quad \& \quad - \frac{1}{\rho} \frac{\partial p}{\partial y} = 0 \quad \text{.......... (2)}
\]

**SOLUTION OF THE PROBLEM**

Equation (2) shows that the pressure does not depend on y hence \( p \) is a function of \( x \) only & so equation (1) reduces to

\[ \frac{d p}{d x} = \rho \left( \nu \frac{d^2 u}{d y^2} - v_0 \frac{d u}{d y} + \nu \frac{u}{k} + \frac{\sigma B_0^2}{\mu} u \right) \quad \Rightarrow \quad \frac{d^2 u}{d y^2} - \frac{v_0}{\nu} \frac{d u}{d y} + \left( \frac{1}{k} + \frac{\sigma B_0^2}{\mu} \right) u = -\frac{P}{\rho u} \quad \text{where} \quad \frac{d p}{d x} = -P
\]

\[ A.E. \quad m^2 - \frac{v_0}{\nu} m + \left( \frac{1}{k} + \frac{\sigma B_0^2}{\mu} \right) = 0 \quad \Rightarrow \quad m = \frac{v_0}{\nu} \pm \sqrt{\left( \frac{v_0}{\nu} \right)^2 - 4 \left( \frac{1}{k} + \frac{\sigma B_0^2}{\mu} \right)} = \frac{v_0}{2\nu} \pm \frac{\sqrt{\left( \frac{v_0}{\nu} \right)^2 - 4 \left( \frac{1}{k} + \frac{\sigma B_0^2}{\mu} \right)}}{2}
\]

Let \[ \sqrt{\left( \frac{v_0}{2\nu} \right)^2 - \left( \frac{1}{k} + \frac{\sigma B_0^2}{\mu} \right)} = A \quad \text{&} \quad \frac{1}{k} + \frac{\sigma B_0^2}{\mu} = B \quad \& \quad \frac{\nu_0}{2\nu} > \left( \frac{1}{k} + \frac{\sigma B_0^2}{\mu} \right)
\]

\[ \therefore C.F. = e^{\frac{v_0}{2\nu} y} \left[ C_1 \cosh Ay + C_2 \sinh Ay \right] \quad \text{&} \quad P.I. = -\frac{P}{\mu B} \Rightarrow u(y) = e^{\frac{v_0}{2\nu} y} \left[ C_1 \cosh Ay + C_2 \sinh Ay \right] - \frac{P}{\mu B}
\]

Using boundary conditions \[ u = 0 \quad \text{at} \quad y = -h \quad \& \quad u = U \quad \text{at} \quad y = h
\]

\[ e^{-\frac{v_0}{2\nu} h} \left[ C_1 \cosh Ah - C_2 \sinh Ah \right] - \frac{P}{\mu B} = 0 \quad \text{.......... (3)} \quad \& \quad U = e^{\frac{v_0}{2\nu} h} \left[ C_1 \cosh Ah + C_2 \sinh Ah \right] - \frac{P}{\mu B} \quad \text{.......... (4)}
\]

\[ \Rightarrow \quad \frac{P}{\mu B} e^{\frac{v_0}{2\nu} h} = C_1 \cosh Ah - C_2 \sinh Ah \quad \& \quad \left( U + \frac{P}{\mu B} \right) e^{\frac{v_0}{2\nu} h} = C_1 \cosh Ah + C_2 \sinh Ah
\]

\[ C_1 = \frac{1}{2} \cosh Ah \left( U + \frac{P}{\mu B} \right) e^{\frac{v_0}{2\nu} h} + \frac{P}{\mu B} e^{\frac{v_0}{2\nu} h} \quad \& \quad C_2 = \frac{1}{2} \sinh Ah \left( U + \frac{P}{\mu B} \right) e^{\frac{v_0}{2\nu} h} - \frac{P}{\mu B} e^{\frac{v_0}{2\nu} h}
\]
THE AVERAGE VELOCITY DISTRIBUTION IN GENERALIZED PLANE COUETTE FLOW

\[
\begin{align*}
\bar{u}(y) &= \frac{v_0}{2} \frac{\cosh Ay}{\cosh Ah} 
\left( U + \frac{P}{\mu B} \right) e^{-\frac{y}{2\mu B}} 
+ \frac{v_0}{2} \frac{\sinh Ay}{\sinh Ah} 
\left( U + \frac{P}{\mu B} \right) e^{-\frac{y}{2\mu B}} 
- \frac{v_0}{\mu B} 
\right) 
- \frac{P}{\mu B} 
\end{align*}
\]

\[
\begin{align*}
u(y) &= \left( U + \frac{P}{\mu B} \right) \frac{v_0}{2} \frac{\sinh A(y + h)}{\cosh Ah} 
- \frac{P}{\mu B} \frac{v_0}{2} \frac{\sinh A(h)}{\cosh Ah} 
\left( U + \frac{P}{\mu B} \right) \frac{v_0}{2} \frac{\sinh A(y - h)}{\cosh Ah} 
- \frac{P}{\mu B}
\end{align*}
\]

\[
\sigma_{xy} = \frac{\mu}{\sinh 2Ah} \left[ A \left( U + \frac{P}{\mu B} \right) e^{-\frac{y}{2\mu B}} + \frac{P}{\mu B} \frac{v_0}{2} \sinh 2Ah - \frac{v_0}{2} \sinh A(y + h) \right] 
\]

\[
\left( \sigma_{xy} \right)_{y = -h} = \frac{\mu}{\sinh 2Ah} \left[ A \left( U + \frac{P}{\mu B} \right) e^{-\frac{y}{2\mu B}} + \frac{P}{\mu B} \frac{v_0}{2} \sinh 2Ah - \frac{v_0}{2} \sinh A(y + h) \right] 
\]

\[
\left( \sigma_{xy} \right)_{y = h} = \frac{\mu}{\sinh 2Ah} \left[ U + \frac{P}{\mu B} \right] \frac{v_0}{2} \sinh 2Ah + \frac{PA}{\mu B} \frac{v_0}{2} \sinh 2Ah 
\]

THE SKIN FRICTIONS AT LOWER AND UPPER PLATE

\[
\left( \sigma_{xy} \right)_{y = -h} = \frac{\mu}{\sinh 2Ah} \left[ A \left( U + \frac{P}{\mu B} \right) e^{-\frac{y}{2\mu B}} + \frac{P}{\mu B} \frac{v_0}{2} \sinh 2Ah - \frac{v_0}{2} \sinh A(y + h) \right] 
\]

\[
\left( \sigma_{xy} \right)_{y = h} = \frac{\mu}{\sinh 2Ah} \left[ U + \frac{P}{\mu B} \right] \frac{v_0}{2} \sinh 2Ah + \frac{PA}{\mu B} \frac{v_0}{2} \sinh 2Ah 
\]

THE AVERAGE VELOCITY DISTRIBUTION IN GENERALIZED PLANE COUETTE FLOW

\[
\begin{align*}
\bar{u}(y) &= \frac{v_0}{2} \frac{\cosh Ay}{\cosh Ah} 
\left( U + \frac{P}{\mu B} \right) e^{-\frac{y}{2\mu B}} 
+ \frac{v_0}{2} \frac{\sinh Ay}{\sinh Ah} 
\left( U + \frac{P}{\mu B} \right) e^{-\frac{y}{2\mu B}} 
- \frac{v_0}{\mu B} 
\right) 
- \frac{P}{\mu B} 
\end{align*}
\]

\[
\begin{align*}
u(y) &= \left( U + \frac{P}{\mu B} \right) \frac{v_0}{2} \frac{\sinh A(y + h)}{\cosh Ah} 
- \frac{P}{\mu B} \frac{v_0}{2} \frac{\sinh A(h)}{\cosh Ah} 
\left( U + \frac{P}{\mu B} \right) \frac{v_0}{2} \frac{\sinh A(y - h)}{\cosh Ah} 
- \frac{P}{\mu B}
\end{align*}
\]

\[
\sigma_{xy} = \frac{\mu}{\sinh 2Ah} \left[ A \left( U + \frac{P}{\mu B} \right) e^{-\frac{y}{2\mu B}} + \frac{P}{\mu B} \frac{v_0}{2} \sinh 2Ah - \frac{v_0}{2} \sinh A(y + h) \right] 
\]

\[
\left( \sigma_{xy} \right)_{y = -h} = \frac{\mu}{\sinh 2Ah} \left[ A \left( U + \frac{P}{\mu B} \right) e^{-\frac{y}{2\mu B}} + \frac{P}{\mu B} \frac{v_0}{2} \sinh 2Ah - \frac{v_0}{2} \sinh A(y + h) \right] 
\]

\[
\left( \sigma_{xy} \right)_{y = h} = \frac{\mu}{\sinh 2Ah} \left[ U + \frac{P}{\mu B} \right] \frac{v_0}{2} \sinh 2Ah + \frac{PA}{\mu B} \frac{v_0}{2} \sinh 2Ah 
\]

\[
\begin{align*}
\bar{u}(y) &= \frac{v_0}{2} \frac{\cosh Ay}{\cosh Ah} 
\left( U + \frac{P}{\mu B} \right) e^{-\frac{y}{2\mu B}} 
+ \frac{v_0}{2} \frac{\sinh Ay}{\sinh Ah} 
\left( U + \frac{P}{\mu B} \right) e^{-\frac{y}{2\mu B}} 
- \frac{v_0}{\mu B} 
\right) 
- \frac{P}{\mu B} 
\end{align*}
\]

\[
\begin{align*}
u(y) &= \left( U + \frac{P}{\mu B} \right) \frac{v_0}{2} \frac{\sinh A(y + h)}{\cosh Ah} 
- \frac{P}{\mu B} \frac{v_0}{2} \frac{\sinh A(h)}{\cosh Ah} 
\left( U + \frac{P}{\mu B} \right) \frac{v_0}{2} \frac{\sinh A(y - h)}{\cosh Ah} 
- \frac{P}{\mu B}
\end{align*}
\]

\[
\sigma_{xy} = \frac{\mu}{\sinh 2Ah} \left[ A \left( U + \frac{P}{\mu B} \right) e^{-\frac{y}{2\mu B}} + \frac{P}{\mu B} \frac{v_0}{2} \sinh 2Ah - \frac{v_0}{2} \sinh A(y + h) \right] 
\]

\[
\left( \sigma_{xy} \right)_{y = -h} = \frac{\mu}{\sinh 2Ah} \left[ A \left( U + \frac{P}{\mu B} \right) e^{-\frac{y}{2\mu B}} + \frac{P}{\mu B} \frac{v_0}{2} \sinh 2Ah - \frac{v_0}{2} \sinh A(y + h) \right] 
\]

\[
\left( \sigma_{xy} \right)_{y = h} = \frac{\mu}{\sinh 2Ah} \left[ U + \frac{P}{\mu B} \right] \frac{v_0}{2} \sinh 2Ah + \frac{PA}{\mu B} \frac{v_0}{2} \sinh 2Ah 
\]

© 2020 Scholars Journal of Physics, Mathematics and Statistics | Published by SAS Publishers, India
\[
\frac{P}{4\mu h B^2 \sinh 2Ah} \left\{ \frac{v_0}{2\nu} e^{v_0 e^{2Ah}} - e^{2Ah} - e^{v_0 e^{2Ah}} + e^{2Ah} \right\} - A \left\{ e^{v_0 e^{2Ah}} - e^{2Ah} + e^{v_0 e^{2Ah}} - e^{2Ah} \right\} - \frac{P}{\mu B}
\]

Since
\[
\left( \frac{v_0}{2\nu} \right)^2 - \left( \frac{1}{k} + \frac{\sigma B^2_{0}}{\mu} \right) = A \quad \left( \frac{1}{k} + \frac{\sigma B^2_{0}}{\mu} \right) = B \Rightarrow \left( \frac{v_0}{2\nu} \right)^2 - A^2 = B
\]

\[
= \left( \frac{U + \frac{P}{\mu B}}{2h \sinh 2Ah} \right) \left( \frac{v_0}{2\nu} \sinh 2Ah - \frac{v_0}{2\nu} \cosh 2Ah + A e^{-v_0 / \nu} \right) - \frac{P}{2h \mu B^2 \sinh 2Ah} \left( \frac{v_0}{2\nu} \sinh 2Ah + \frac{v_0}{2\nu} \cosh 2Ah - A e^{-v_0 / \nu} \right) - \frac{P}{\mu B}
\]

\[
u_{av} = \frac{U}{2h \sinh 2Ah} \left( \frac{v_0}{2\nu} \sinh 2Ah - \frac{v_0}{2\nu} \cosh 2Ah + A e^{-v_0 / \nu} \right) - \frac{PA}{\mu B^2 \sinh 2Ah} \left( \cosh 2Ah - \cosh \left( \frac{v_0}{\nu} \right) \right) - \frac{2hP}{\mu B} \quad \ldots \ldots \ldots (9)
\]

**THE VOLUMETRIC FLOW:** \( Q = 2h(u)_{av} \)

\[
Q = \frac{U}{B \sinh 2Ah} \left( \frac{v_0}{2\nu} \sinh 2Ah - \frac{v_0}{2\nu} \cosh 2Ah + A e^{-v_0 / \nu} \right) - \frac{2PA}{\mu B^2 \sinh 2Ah} \left( \cosh 2Ah - \cosh \left( \frac{v_0}{\nu} \right) \right) - \frac{2hP}{\mu B} \quad \ldots \ldots \ldots (10)
\]

**THE DRAG COEFFICIENTS:**

\[
(C_f)_{y = h} = \left( \frac{\sigma_{xy}}{\rho \nu_{av}^2} \right)_{y = h} \quad \& \quad (C_f')_{y = -h} = \left( \frac{\sigma_{xy}}{\rho \nu_{av}^2} \right)_{y = -h}
\]

\[
(C_f)_{y = h} = \left[ \frac{8\mu h^2 B^4 \sinh 2Ah \left( U + \frac{P}{\mu B} \right) \left( \frac{v_0}{2\nu} \sinh 2Ah + A e^{-v_0 / \nu} \right) - \frac{PA}{\mu B} \left( \frac{v_0}{\nu} \right) e^{-v_0 / \nu} \right]}{\rho \mu B} \left( \frac{v_0}{2\nu} \sinh 2Ah - \frac{v_0}{2\nu} \cosh 2Ah + A e^{-v_0 / \nu} \right) - 2AP \left( \cosh 2Ah - \cosh \left( \frac{v_0}{\nu} \right) \right) - 2PBh \sinh 2Ah \right] \quad \ldots \ldots \ldots (11)
\]

\[
(C_f')_{y = -h} = \left[ \frac{8\mu h^2 B^4 \sinh 2Ah \left( A \left( U + \frac{P}{\mu B} \right) e^{-v_0 / \nu} + \frac{P}{\mu B} \left( \frac{v_0}{2\nu} \sinh 2Ah - \frac{v_0}{2\nu} \cosh 2Ah \right) \right)}{\rho \mu B} \left( \frac{v_0}{2\nu} \sinh 2Ah - \frac{v_0}{2\nu} \cosh 2Ah \right) - 2AP \left( \cosh 2Ah - \cosh \left( \frac{v_0}{\nu} \right) \right) - 2PBh \sinh 2Ah \right] \quad \ldots \ldots \ldots (12)
\]

**THE STREAM LINE IN THE PLANE GENERALIZED COUETTE FLOW:**

\[
\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w} \quad \text{where} \quad \vec{q} = u \hat{i} + v \hat{j} + w \hat{k}
\]

\[
\Rightarrow \left[ \frac{1}{\sinh 2Ah} \left( U + \frac{P}{\mu B} \right) e^{v_0 / \nu} (y-h) \sinh A(y+h) - \frac{P}{\mu B} e^{v_0 / \nu} (y+h) \sinh A(y-h) \right] - \frac{P}{\mu B} = \frac{dy}{v} = \frac{dz}{0}
\]

Taking first two equations

\[
v_0 \int dx = \left[ \frac{1}{\sinh 2Ah} \left( U + \frac{P}{\mu B} \right) e^{v_0 / \nu} (y-h) \sinh A(y+h) - \frac{P}{\mu B} e^{v_0 / \nu} (y+h) \sinh A(y-h) \right] - \frac{P}{\mu B} \]

\[
dy + C_1
\]

© 2020 Scholars Journal of Physics, Mathematics and Statistics | Published by SAS Publishers, India
\[ v_0 \times = \frac{U + P}{\mu B} e^{2v_0(y-h)} \left( e^{A(y+h)} - e^{-A(y-h)} \right) dy + \frac{P}{\mu B} e^{2v_0(y+h)} \left( e^{A(y-h)} - e^{-A(y-h)} \right) dy + \frac{P}{\mu B} y = C_1 \]

\[ v_0 \times = \frac{U + P}{2 \sinh 2Ah} \left( \frac{V_0(y-h) + A(y+h)}{2v} - \frac{V_0(y-h) - A(y+h)}{2v} \right) dy + \frac{P}{2 \mu B \sinh 2Ah} \left( \frac{V_0(y+h) + A(y-h)}{2v} - \frac{V_0(y+h) - A(y-h)}{2v} \right) dy + \frac{P}{\mu B} y = C_1 \]

\[ v_0 \times = \frac{U + P + \mu B}{2 \sinh 2Ah} \left( \frac{V_0(y-h) + A(y+h)}{2v} - \frac{V_0(y-h) - A(y+h)}{2v} \right) + \frac{P}{2 \mu B \sinh 2Ah} \left( \frac{V_0(y+h) + A(y-h)}{2v} - \frac{V_0(y+h) - A(y-h)}{2v} \right) \]

\[ v_0 \times = \frac{U + P}{B \sinh 2Ah} \left( \frac{V_0(y-h) + A(y+h)}{2v} - \frac{V_0(y-h) - A(y+h)}{2v} \right) + \frac{P}{\mu B} V_0(y+h) - A(y+h) + \frac{P}{\mu B} y = C_1 \]

\[ v_0 \times = \frac{U + P}{B \sinh 2Ah} \left( \frac{V_0(y-h) + A(y+h)}{2v} - \frac{V_0(y-h) - A(y+h)}{2v} \right) + \frac{P}{\mu B} V_0(y+h) - A(y+h) + \frac{P}{\mu B} y = C_1 \]

\[ \text{curl } \vec{q} = \begin{pmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u & v & w \end{pmatrix} = \begin{pmatrix} i & \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 \end{pmatrix} \frac{1}{\sinh 2Ah} \left( \frac{V_0(y-h)}{2v} + A(y+h) - \frac{V_0(y-h)}{2v} - A(y+h) \right) \]

\[ \text{curl } \vec{q} = -\frac{1}{\sinh 2Ah} \left( \frac{V_0(y-h)}{2v} + A(y+h) - \frac{V_0(y-h)}{2v} - A(y+h) \right) \]

\[ \Rightarrow \text{motion of the fluid is rotational.} \]

\[ \text{Table for velocity: } P = 9, U = 6, \mu = 0.5, h = 0.5, v_0/y = 6 \text{ & } \frac{1}{k} = \frac{\sigma B_0^2}{\mu} = 16, \left( \frac{v_0}{2v} \right)^2 - \left( \frac{1}{k} + \frac{\sigma B_0^2}{\mu} \right) = 2 \]

<table>
<thead>
<tr>
<th>y</th>
<th>0</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
<th>0.6</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\frac{1}{k}) = 16</td>
<td>u(y)</td>
<td>1.3</td>
<td>.645</td>
<td>1.301</td>
<td>1.882</td>
<td>1.835</td>
<td>6</td>
</tr>
<tr>
<td>(\sigma B_0^2) = 16</td>
<td>u(y)</td>
<td>1.3</td>
<td>.645</td>
<td>1.301</td>
<td>1.882</td>
<td>1.835</td>
<td>6</td>
</tr>
<tr>
<td>(\frac{1}{k} + \frac{\sigma B_0^2}{\mu}) = 32</td>
<td>u(y)</td>
<td>3.246</td>
<td>4.726</td>
<td>6.592</td>
<td>8.473</td>
<td>9.276</td>
<td>6</td>
</tr>
</tbody>
</table>

© 2020 Scholars Journal of Physics, Mathematics and Statistics | Published by SAS Publishers, India | 152
Graph of table-1
Table for skin friction: \( P = 9, \ U = 6, \ \mu = h = .5, \ \frac{\nu_0}{2\nu} = 6 \ \& \ \frac{1}{k} = \frac{\sigma B_0^2}{\mu} = 16, \sqrt{\left(\frac{\nu_0}{2\nu}\right)^2 - \left(\frac{1}{k} + \frac{\sigma B_0^2}{\mu}\right)} = 2 \)

<table>
<thead>
<tr>
<th></th>
<th>( y )</th>
<th>0</th>
<th>.1</th>
<th>.2</th>
<th>.3</th>
<th>.4</th>
<th>.5</th>
<th>.6</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{1}{k} = 16 )</td>
<td>( \sigma_{sv} )</td>
<td>1.899</td>
<td>-4.685</td>
<td>-10.304</td>
<td>-22.184</td>
<td>-47.038</td>
<td>14.123</td>
<td>-205.03</td>
</tr>
<tr>
<td>( \frac{\sigma B_0^2}{\mu} = 16 )</td>
<td>( \sigma_{sv} )</td>
<td>1.899</td>
<td>-4.685</td>
<td>-10.304</td>
<td>-22.184</td>
<td>-47.038</td>
<td>14.123</td>
<td>-205.03</td>
</tr>
<tr>
<td>( \frac{1}{k} + \frac{\sigma B_0^2}{\mu} = 32 )</td>
<td>( \sigma_{sv} )</td>
<td>6.632</td>
<td>8.572</td>
<td>9.845</td>
<td>8.138</td>
<td>-2.436</td>
<td>-36.07</td>
<td>-126.012</td>
</tr>
</tbody>
</table>

Graph of table-2
Table for velocity: \( P = 9, U = 6, \mu = h = .5, \frac{v_0}{2\nu} = 6, \sqrt{\left(\frac{v_0}{2\nu}\right)^2 - \left(\frac{1}{k} + \frac{\sigma B_0^2}{\mu}\right)^2} = 2 \) when \( \frac{1}{k} < \frac{\sigma B_0^2}{\mu} \)

<table>
<thead>
<tr>
<th>( \frac{1}{k} = 11 )</th>
<th>( \frac{\sigma B_0^2}{\mu} = 21 )</th>
<th>( \frac{1}{k} + \frac{\sigma B_0^2}{\mu} = 32 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>( u(y) )</td>
<td>( u(y) )</td>
</tr>
<tr>
<td>0</td>
<td>1.074</td>
<td>.617</td>
</tr>
<tr>
<td>.1</td>
<td>1.384</td>
<td>1.283</td>
</tr>
<tr>
<td>.2</td>
<td>1.776</td>
<td>2.087</td>
</tr>
<tr>
<td>.3</td>
<td>2.359</td>
<td>2.818</td>
</tr>
<tr>
<td>.4</td>
<td>3.456</td>
<td>2.829</td>
</tr>
<tr>
<td>.5</td>
<td>6.0</td>
<td>6.0</td>
</tr>
<tr>
<td>.6</td>
<td>12.858</td>
<td>6.0</td>
</tr>
</tbody>
</table>

Graph of table-3

© 2020 Scholars Journal of Physics, Mathematics and Statistics | Published by SAS Publishers, India 155
Table for skin friction: \( P = 9, \ U = 6, \ \mu = h = .5, \ \frac{v_0}{2v} = 6, \left(\frac{v_0}{2v}\right)^2 - \left(\frac{1}{k} + \frac{\sigma B_0^2}{\mu}\right) = 2 \) when \( \frac{1}{k} < \frac{\sigma B_0^2}{\mu} \)

Table-4: (for skin friction)

<table>
<thead>
<tr>
<th>( \frac{1}{k} = 11 )</th>
<th>( \sigma_{xy} )</th>
<th>0</th>
<th>.1</th>
<th>.2</th>
<th>.3</th>
<th>.4</th>
<th>.5</th>
<th>.6</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sigma_{xy} )</td>
<td>1.421</td>
<td>1.706</td>
<td>2.294</td>
<td>3.764</td>
<td>7.85</td>
<td>19.76</td>
<td>55.14</td>
<td></td>
</tr>
<tr>
<td>( \frac{\sigma B_0^2}{\mu} = 21 )</td>
<td>( \sigma_{xy} )</td>
<td>-1.515</td>
<td>-3.923</td>
<td>-9.372</td>
<td>-21.269</td>
<td>-46.78</td>
<td>5.998</td>
<td>-213.56</td>
</tr>
<tr>
<td>( \frac{1}{k} + \frac{\sigma B_0^2}{\mu} = 32 )</td>
<td>( \sigma_{xy} )</td>
<td>6.632</td>
<td>8.572</td>
<td>9.845</td>
<td>8.138</td>
<td>-2.436</td>
<td>-36.07</td>
<td>-126.012</td>
</tr>
</tbody>
</table>

Graph of table-4
Table for velocity: \( P = 9, \ U = 6, \ \mu = h = .5, \ \frac{v_0}{2 \nu} = 6, \ \sqrt{\left(\frac{v_0}{2 \nu}\right)^2 - \left(\frac{1}{k} + \frac{\sigma B_0^2}{\mu}\right)} = 2 \) when \( \frac{1}{k} > \frac{\sigma B_0^2}{\mu} \)

<table>
<thead>
<tr>
<th>( \frac{1}{k} = 21 )</th>
<th>( y )</th>
<th>( 0 )</th>
<th>( .1 )</th>
<th>( .2 )</th>
<th>( .3 )</th>
<th>( .4 )</th>
<th>( .5 )</th>
<th>( .6 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( u(y) )</td>
<td>.617</td>
<td>1.283</td>
<td>2.087</td>
<td>2.818</td>
<td>2.829</td>
<td>6</td>
<td>- 8.339</td>
<td></td>
</tr>
<tr>
<td>( \frac{\sigma B_0^2}{\mu} = 11 )</td>
<td>( u(y) )</td>
<td>1.074</td>
<td>1.384</td>
<td>1.776</td>
<td>2.359</td>
<td>3.45</td>
<td>6</td>
<td>12.858</td>
</tr>
<tr>
<td>( \frac{1}{k} + \frac{\sigma B_0^2}{\mu} = 32 )</td>
<td>( u(y) )</td>
<td>3.246</td>
<td>4.726</td>
<td>6.592</td>
<td>8.473</td>
<td>9.276</td>
<td>6</td>
<td>- 8.82</td>
</tr>
</tbody>
</table>

Graph of table-5
Table for Skin friction: \( P = 9, U = 6, \mu = h = .5, \frac{v_0}{2u} = 6, \sqrt{\left(\frac{v_0}{2u}\right)^2 - \left(\frac{1}{k} + \frac{\sigma B_0^2}{\mu}\right)} = 2 \) when \( \frac{1}{k} > \frac{\sigma B_0^2}{\mu} \)

<table>
<thead>
<tr>
<th>( \frac{1}{k} )</th>
<th>( \sigma_{xy} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>21</td>
<td>-1.515</td>
</tr>
<tr>
<td>11</td>
<td>1.421</td>
</tr>
<tr>
<td>32</td>
<td>6.632</td>
</tr>
</tbody>
</table>

Table-6: (for skin friction)

<table>
<thead>
<tr>
<th>( \frac{1}{k} )</th>
<th>( \sigma_{xy} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 .1 .2 .3 .4 .5 .6</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>1.706</td>
</tr>
<tr>
<td>19.76</td>
<td></td>
</tr>
<tr>
<td>55.14</td>
<td></td>
</tr>
<tr>
<td>32</td>
<td>8.572</td>
</tr>
<tr>
<td>9.845</td>
<td></td>
</tr>
<tr>
<td>-36.07</td>
<td></td>
</tr>
<tr>
<td>-126.012</td>
<td></td>
</tr>
</tbody>
</table>

Graph of table-6
CONCLUSION AND DISCUSSION

In this paper, we have investigated the velocity by the graph of table-1 of equation (5). The velocity in porous medium & magnetic field at \( \frac{1}{k} = \frac{\sigma B_0^2}{\mu} = 16 \) is less than the corresponding value of velocity in porous with magnetic field at \( \frac{1}{k} = \frac{\sigma B_1^2}{\mu} = 32 \) in the interval \( 0 \leq y \leq 0.4 \) & equal \( \{u(y) = 6\} \) in all medium at \( y = 0.5 \). The value of velocity in porous medium and magnetic field at \( \frac{1}{k} = \frac{\sigma B_0^2}{\mu} = 16 \) is negatively less than the correspondingly negative value of velocity in porous medium with magnetic field at \( \frac{1}{k} = \frac{\sigma B_0^2}{\mu} = 32 \) at \( y = 0.6 \).

Again by the graph of table-3 of equation (5), the value of the velocity in porous medium at \( \frac{1}{k} = 11 \) increases in the interval \( 0 \leq y \leq 0.6 \), velocity in magnetic field at \( \frac{\sigma B_0^2}{\mu} = 21 \) increases in the interval \( 0 \leq y \leq 0.5 \) & the velocity in porous medium with magnetic field at \( \frac{1}{k} = \frac{\sigma B_0^2}{\mu} = 32 \) increases in the interval \( 0 \leq y \leq 0.4 \).

The velocity is equal \( \{u(y) = 6\} \) in all medium at \( y = 0.5 \) & velocity in porous medium at \( \frac{1}{k} = 11 \) is positive while the velocity in magnetic field at \( \frac{\sigma B_0^2}{\mu} = 21 \) & porous medium with magnetic field at \( \frac{1}{k} = \frac{\sigma B_0^2}{\mu} = 32 \) is negative i.e. flow is in opposite direction at \( y = 0.6 \).

Again by the graph of table-5 of equation (5), the value of the velocity in magnetic field at \( \frac{\sigma B_1^2}{\mu} = 11 \) increases in the interval \( 0 \leq y \leq 0.6 \), velocity in porous medium at \( \frac{1}{k} = 21 \) increases in the interval \( 0 \leq y \leq 0.5 \) & the velocity in porous medium with magnetic field at \( \frac{1}{k} = \frac{\sigma B_1^2}{\mu} = 32 \) increases in the interval \( 0 \leq y \leq 0.4 \).

The velocity is equal \( \{u(y) = 6\} \) in all medium at \( y = 0.5 \) & velocity in magnetic field at \( \frac{\sigma B_0^2}{\mu} = 11 \) is positive while the velocity in porous medium at \( \frac{1}{k} = 21 \) & porous medium with magnetic field at \( \frac{1}{k} = \frac{\sigma B_1^2}{\mu} = 32 \) is negative i.e. flow is in opposite direction at \( y = 0.6 \).

Again we have investigated the skin friction by the graph of table-2, of equation (6). The skin friction in porous medium & magnetic field at \( \frac{1}{k} = \frac{\sigma B_0^2}{\mu} = 16 \) is positive at \( y = 0 \) & .5 while the skin friction in porous medium & magnetic field at \( \frac{1}{k} = \frac{\sigma B_0^2}{\mu} = 16 \) is negative at other values of \( y \). Skin friction in porous medium with magnetic field at \( \frac{1}{k} = \frac{\sigma B_0^2}{\mu} = 32 \) is positive in the interval \( 0 \leq y \leq 0.3 \) while the skin friction in porous medium with magnetic field at \( \frac{1}{k} = \frac{\sigma B_0^2}{\mu} = 32 \) is negative in the interval \( 0.4 \leq y \leq 0.6 \).

Again by the graph of table-4 of equation (6), the skin friction in porous medium at \( \frac{1}{k} = 11 \) is positive & increases in the interval \( 0 \leq y \leq 0.6 \) while skin friction in magnetic field at \( \frac{\sigma B_0^2}{\mu} = 21 \) is negative excepts only at \( y = .5 \). The skin friction in porous medium with magnetic field at \( \frac{1}{k} = \frac{\sigma B_1^2}{\mu} = 32 \) in the interval \( 0 \leq y \leq 0.3 \) is positive while the skin friction in porous medium with magnetic field at \( \frac{1}{k} = \frac{\sigma B_1^2}{\mu} = 32 \) is negative in the interval \( 0.4 \leq y \leq 0.6 \).

Again by the graph of table-6 of equation (6), the skin friction in magnetic field at \( \frac{\sigma B_1^2}{\mu} = 11 \) is positive & increases in the interval \( 0 \leq y \leq 0.6 \) while the skin friction in porous medium at \( \frac{1}{k} = 21 \) is negative excepts only at \( y = .5 \). The skin friction in porous medium with magnetic field at \( \frac{1}{k} = \frac{\sigma B_1^2}{\mu} = 32 \) in the interval \( 0 \leq y \leq 0.3 \) is positive while the skin friction in porous medium with magnetic field at \( \frac{1}{k} = \frac{\sigma B_1^2}{\mu} = 32 \) is negative in the interval \( 0.4 \leq y \leq 0.6 \).

Also we have investigated skin frictions, average velocity, the volumetric flow, drag coefficients & stream lines by the equations (7), (8), (9), (10), (11), (12), (13) & (14) respectively.
REFERENCES


Author Acknowledgement:
Dr. Anand Swrup Sharma is professor of Applied Sciences (Mathematics) at the Future University, Bareilly (India). He has a keen interest in fluid Mechanics. My Academic Qualification is M.Sc, M.Phil, & PhD in Mathematics & D.Sc (Scholar)
Email: sharma.as09@gmail.com