

Magnetized Anisotropic Dark Energy Cosmological Model in $f(R)$ Gravity Theory Using Special Law of Variation

Kailas R. Borgade^{1*}, Dr. S. R. Bhoyar², Dr. Vineeta Basotia³

¹Research Scholar of Shri Jagdishprasad Jhabarmal Tibrewala University, Vidyanagri, Jhunjhunu, Rajasthan, India-333001

²Department of Mathematics, Phulsing Naik College, Pusad District Yavatmal, Maharashtra, India-445204

³Department of Mathematics, Shri Jagdishprasad Jhabarmal Tibrewala University, Vidyanagri, Jhunjhunu, Rajasthan, India

DOI: [10.36347/sjpm.2021.v08i03.003](https://doi.org/10.36347/sjpm.2021.v08i03.003)

| Received: 24.02.2021 | Accepted: 21.03.2021 | Published: 30.03.2021

*Corresponding author: Kailas R. Borgade

Abstract

Original Research Article

We formulate $f(R)$ gravity plane symmetric cosmological models of the universe corresponding to the magnetized anisotropic dark energy momentum tensor using special law of variation with Lagrangian be the arbitrary function of Ricci scalar. The energy momentum tensor is having with anisotropic dark energy EoS $\rho = p/\omega$ with a magnetized energy density. Some physical and kinematical aspects of the presented models have been discussed.

Keywords: Magnetized dark energy, Bianchi Type-I model, $f(R)$ Gravity.

Copyright © 2021 The Author(s): This is an open-access article distributed under the terms of the Creative Commons Attribution 4.0 International License (CC BY-NC 4.0) which permits unrestricted use, distribution, and reproduction in any medium for non-commercial use provided the original author and source are credited.

INTRODUCTION

The hypothetical investigational confirmation [1-4] has recognized that our universe experiencing a late-time accelerating. The proposals that have been put forward to explain this observed phenomenon can basically be classified into two categories. First the growth dominated by a component with negative pressure, dubbed as dark energy and second is by changing the gravity law through the modification of action in general theory of relativity (GTR). The result obtained from Wilkinson Microwave Anisotropy Probe (WMAP) [5] shows that dark energy occupies 73%, dark matter occupy 23% and the usual baryon matter which can be designated by our known particle theory conquers only about 4% of the total energy in the Universe. The simplest interpretation for this dark energy is the introduction of a cosmological constant corresponding to equation of state parameter $\omega = -1$. Also, in the collected works spaced out from the cosmological constant there are other candidates of dark energy which is identified with the energy density of a dynamical scalar field such as quintessence ($\omega > -1$) [6, 7]. Phantom field ($\omega < -1$) [8, 9] and Quinton (that can across from phantom region to quintessence region), [10, 11] Chaplygin gas [12], k-essence [13-16], Tachyon field, Holographic and Age graphic dark energy. In the face of these attempts dark energy is still

one of the most important undeveloped interrogations in theoretical physics. Dark energy and matter refers to the unseen energy and matter parts of the Universe. Matter is invisible, non-baryonic matter hypothesized to elucidate phenomena as well as attraction lensing and galactic rotation curves. Dark energy is believed to permeate the Universe and, despite its low energy-density, is believed to be accountable for the fast growth of the Universe. In recent years, many authors studied dark energy and magnetized dark energy models of the universe. Mishra *et al.* [17] investigated the behavior of the skew-ness parameters for an anisotropic universe in the framework of general relativity. Goswami *et al.* [18] searched the existence of the late time acceleration of the Universe filled with cosmic fluid and uniform magnetic field as source of matter in anisotropic Heckmann-Schucking space-time. Das and Sultana [19] presented in this paper a LRS Bianchi type I cosmological model with dark matter and anisotropic ghost dark energy in presence of magnetic field. Santhi *et al.* [20] considered spatially homogeneous and anisotropic Bianchi type-III space-time filled with matter and anisotropic modified holographic Ricci dark energy in GTR. Artymowski and Lalak [21] obtained a scalar potential with non-zero value of residual vacuum energy, which may be a source of Dark Energy. Katore and Rane [22] studied Bianchi Type-III magnetized Cosmological model when the field of gravitation is governed by either a

Citation: Kailas R. Borgade *et al.* Magnetized Anisotropic Dark Energy Cosmological Model in $f(R)$ Gravity Theory Using Special Law of Variation. Sch J Phys Math Stat, 2021 Mar 8(3): 68-81.

perfect fluid or a cosmic string in Rosen's biometric theory of gravitation. Kandalkar *et al.* [23] investigated locally rotationally symmetric Bianchi Type-I cosmological model filled with dark energy from a wet dark fluid in the presence and absence of magnetic field.

As, there are so many modification of action in GTR are obtained, such as $f(R)$, $f(R,T)$ $f(T)$ $f(R,G)$ $f(T,B)$ etc. These reasonably different attraction theories are an endeavor to construct a semi-classical theme within which GTR and most of its self-made options are often recovered. An undemanding and easy modification to GTR is that the $f(R)$ theory of gravity. During this theory, the Ricci scalar R of Einstein–Hilbert action is replaced by operate of R . Till now several models of $f(R)$ theory are planned.

Abebe *et al.* [24] investigated a class of shear-free, homogeneous but anisotropic cosmological models with imperfect matter sources in the context of $f(R)$ gravity. Astashenok [25] found the Neutron star models in perturbative $f(R)$ gravity with realistic equation of state and he considered FPS and Sly equation of state. Hasmani and Ahmed [26] attempted to study spatially homogeneous Bianchi type-I cosmological models in $f(R)$ theory of gravity. Reddy *et al.* [27] studied the vacuum solutions of Bianchi Type-I and V Space time in $f(R)$ gravity theory. Banik *et al.* [28] applied the dynamical systems approach to investigate the spatially homogeneous and anisotropic Bianchi type-V models for the Palatini version of $f(R)$ gravity. Saiedy [29] studied time-dependent wormhole space times in the radiation background in $f(R)$ gravity and also discussed thermo dynamical properties of the time dependent wormhole in $f(R)$ gravity. Martino *et al.* [30] predicted a various aspects of $f(R)$ gravity at extragalactic and cosmological levels. Bamba *et al.* [31, 32] investigated the oscillating effective Equation of State of the universe around the Phantom divide in $f(R)$ gravity. Also they investigated the possible antigravity regions in $f(R)$ gravity theory corresponding to the Weyl's invariant scalar field theory. Adhav [33] found the exact solution of the field equations for a Kantowski-Sachs space-time filled with cosmic strings in $f(R)$ gravity theory. Akbar and Rong-Gen Cai [34] shown that by applying the first law of thermodynamics to the apparent horizon of an FRW universe and assuming the geometric entropy given by a quarter of the of the apparent horizon area ,he can derive the Friedmann equations describing the dynamics of the universe with any spatial e ; using the entropy formula for the static spherically symmetric black holes in Gauss–Bonnet gravity and in more general Lovelock gravity, where the entropy is not proportional to the horizon area, he can also obtained

the corresponding Friedmann equations in each gravity. Sharif and Shamir [35] discussed non-vacuum Bianchi Type- I and V in $f(R)$ gravity theory. Shamir [36] studied the exact vacuum solutions of Bianchi type I, III and Kantowski-Sachs space-times in the metric version of $f(R)$ gravity. Sharif and Shamir [37] used the Landau-Lifschitz energy momentum complex in the framework of $f(R)$ gravity to evaluate the energy density of plane symmetric solutions for some $f(R)$ gravity models. Capone and Ruggiero [38] reviewed that the dynamical equivalence between $f(R)$ Gravity in the metric formalism and scalar-tensor gravity and use this equivalence to deduce the past-Newtonian parameters γ and β for $f(R)$ gravity. Moraes *et al.* [39] studied LRS Bianchi Type-I Space-time in $f(R)$ gravity within the phantom energy dominated era. Shri Ram [40] investigated a spatially homogeneous and anisotropic Bianchi type I model filled with perfect fluid in $f(R)$ gravity theory. Hiwarkar *et al.* [41] proposed exact solutions of N-dimensional Bianchi Type-V space-time in $f(R)$ theory of gravity. Odintsov and Oikonomou [42-44] investigated cosmological dynamical system of $f(R)$ gravity, by constructing it in such a way so that is rendered autonomous. They also found the occurrence of future cosmological finite time singularities in the dynamical system corresponding to two cosmological theories that of vacuum $f(R)$ gravity and that of three fluids. Also they introduced a bottom-up $f(R)$ gravity reconstruction technique, in which we fix the observational indices and they seek for the $f(R)$ gravity which may realize them. Soroushfar *et al.* [45] considered three types of black holes in $f(R)$ gravity. Lee [46] found the method for reconstruction of $f(R)$ gravity models both from the background evaluations observations and from the large scale structured measurements. Capozziello *et al.* [47] investigated $f(R)$ gravity can; in general, give rise to cosmological viable models compatible with a matter-dominated epoch evolving into a late accelerated phase. Sotiriou and Faraoni [48] studied the various aspects of $f(R)$ gravity theory. Tan Liu *et al.* [49] formulated parameterized past Newtonian parameters γ and β the effective gravitational constant G_{eff} and the effective cosmological constant Λ_{eff} in the $f(R)$ gravity. Chirde and Shekh [50] found plane symmetric cosmological model with quadratics equation of state in metric version of $f(R)$ gravity. Aditya and Reddy [51] devoted to study with spatially homogeneous and anisotropic locally rotationally symmetric (LRS) Bianchi type-I universe with cosmic string source in the framework of $f(R)$ theory of gravity. In this present context we are devoted to the study of Magnetized Anisotropic Dark Energy Cosmological Model in $f(R)$

Gravity Theory Using Special Law of Variation. Also discuss some kinematical and physical properties of the universe.

Metric, Field equation and Energy momentum Tensor

We consider a Bianchi type-I space-time of the form

$$ds^2 = dt^2 - A^2 dx^2 - B^2 (dy^2 + dz^2), \tag{2.1}$$

Where, A and B are the functions of cosmic time t only. Various researchers and authors who have contemplated the above said model due to its physical attention that it is homogeneous and anisotropic, from which the process of isotropization of the universe is studied through the passage of time also play a vital role in understanding and description of the early stages evolution of the universe whereas the theoretical argument and the modern experimental data from CMB radiation and the LSS consideration, support the existence of an anisotropic phase which turns into an isotropic one. Therefore, the model annihilator anisotropic background that approaches to isotropy at late times is more correct for the characterization of entire evolution of the universe. Several features of plane symmetric cosmological models for different context of use have been discussed by researchers.

Pradhan and Pandey [52] studied the Bianchi Type-I magnetized Cosmological models in the presence of bulk viscous fluid. Santhi *et al.* [53] investigated Bianchi Type-I bulk viscous string model in $f(R)$ gravity.

The corresponding field equations are found by varying the action

$$S = \int \sqrt{-g} \left(\frac{1}{16\pi G} f(R) + L_m \right) d^4x \tag{2.2}$$

Where, $f(R)$ is a general function of the Ricci scalar and L_m is the matter Lagrangian. Variation of action (2.12) with respect to metric gives the following field equations:

$$F(R)R_{ij} - \frac{1}{2}f(R) - \nabla_i \nabla_j F(R) + g_{ij} \square F(R) = T_{ij}^{(M)}, \tag{2.3}$$

Where, $\square = \nabla_i \nabla_j$; $F(R) = \frac{d}{dR} f(R)$.

Bhojar *et al.* [54] investigated Bianchi Type-I space-time with quadratics equation of state in the $f(R)$ gravity theory. Mahanta and Sarma [55] studied the isotropic perfect fluid solutions of Bianchi Type-III space time in the metric version of $f(R)$ gravity. Ntahompagaze *et al.* [56] studied correspondence between $f(R)$ gravity and scalar-tensor theories is revisited since $f(R)$ gravity is a subclass of Brans-

Dicke models with vanishing coupling constant ($\omega=0$). They got solutions to Klein-Gordon equation for $f(R)$ toy models. Arnold *et al.* [57] presented a set of cosmological hydro dynamical simulations that follow galaxy formation in $f(R)$ modified gravity models and are dedicated to finding observational signatures to help distinguish general relativity from alternatives using this information. Geng *et al.* [58] studied thermodynamics in $f(R)$ gravity with informal transformations. Shah and Samanta [59] studied stability analysis for $f(R)$ gravity using dynamical system analysis. Sahoo and Bhattacharjee [60] found the coincidence problem in $f(R)$ gravity and investigated the efficiency and model independency. Gogoi and Goswami [61] introduced a new $f(R)$ gravity model as an attempt to have a model with more parametric control, so that the model can be used to explain the existing problems as well as to explore new directions in physics of gravity, by properly constraining it with recent observational data.

In this system, we are dealing with magnetized anisotropy dark energy fluid whose energy momentum tensor is as:

$$T_{ij} = (-p - \rho_B)u_i u_j - p g_{ij}, \tag{2.4}$$

where u^i is the four-velocity vector of the fluid satisfying $u^i = (-1, -1, -1, \rho)$ and satisfying $u^i u_i = 1, \rho$ be energy density of the fluid and ρ_B energy density of magnetized fluid respectively and p is the pressure along the directional state of parameters $\omega_x, \omega_y, \omega_z$ and A_m be the anisotropy parameter which satisfying the general form of the EoS.

$$p = \rho \omega \tag{2.5}$$

With the given EoS we consider the condition which is nothing but the relation between EoS parameter ω and the skewness parameter δ and is given by $\omega + \delta = 0$.

With respect to magnetized dark energy, the energy momentum tensor (2.4) for the given system is derived by the equation given below:

$$\begin{aligned} T_j^i &= \text{diag}[T_1^1, T_2^2, T_3^3, T_4^4] = \text{diag}[-p_x, -p_y, -p_z, \rho] \\ T_j^i &= \text{diag}[-p_x - \rho_B, -p_y - \rho_B, -p_z - \rho_B, \rho + \rho_B] \\ T_j^i &= \text{diag}[-\omega_x p_x - \rho_B, -\omega_y p_y - \rho_B, -\omega_z p_z - \rho_B, \rho + \rho_B] \\ T_j^i &= \text{diag}[-\omega p - \rho_B, (\omega + \delta)p - \rho_B, (\omega + \delta)p - \rho_B, \rho + \rho_B] \end{aligned} \tag{2.6}$$

Various authors and researchers have been studied the magnetized anisotropic dark energy cosmological model of the universe as mention below:

Bhoyar *et al.* [62] studied non-static plane symmetric cosmological model with magnetized anisotropic dark energy by hybrid expansion law in $f(R,T)$ Gravity. Shaikh and Wankhade [63] investigated Homogenous - Hyper surface magnetized dark energy models with constant deceleration parameter. Shamir and Asad [64] Ali studied anisotropic universe in the presence of magnetized dark energy. Bianchi type-V cosmological model is considered for this purpose. The energy-momentum tensor consists of anisotropic fluid with uniform magnetic field of energy density ρ_B . Farnes [65]

proposed a unifying theory of dark energy and dark matter: Negative masses and matter creation within a modified Λ CDM framework. Zhitnitsky [66] proposed cosmological magnetic field and dark energy as two sides of the same coin. Ray *et al.* [67] constructed an anisotropic dark energy cosmological model in a two-fluid situation, such as the usual dark energy and the magnetized fluid.

Field Equations and Solutions

In the existence of magnetized anisotropic dark energy fluid source given in equation (2.6), the field equations (2.3) corresponding to the metric (2.1) lead to the following set of linearly independent differential equations of the form:

$$\left[\frac{\ddot{A}}{A} + 2\frac{\dot{A}\dot{B}}{AB} \right] F - \frac{1}{2}f - 2\left[\frac{\dot{B}}{B} \right] \dot{F} - \ddot{F} = -\omega p - \rho_B \tag{3.1}$$

$$\left[\frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}^2}{B^2} \right] F - \frac{1}{2}f - \left[\frac{\dot{A}}{A} + \frac{\dot{B}}{B} \right] \dot{F} - \ddot{F} = (\omega + \delta)p - \rho_B \tag{3.2}$$

$$\left[\frac{\ddot{A}}{A} + 2\frac{\dot{B}}{B} \right] F - \frac{1}{2}f - \left[\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B} \right] \dot{F} = \rho + \rho_B \tag{3.3}$$

Here the overhead dot denotes differentiation with respect to t . The system of highly non-linear differential equations are consistent consists of three equations with six unknowns. Hence, to solve one can introduce more conditions either by an assumptions corresponding to some physical situations or an arbitrary mathematical supposition.

Using $\omega = p/\rho$ in equation (3.1) and also we use $\omega + \delta = 0$ afterword multiply the resulting equation by -1, we get

$$-\left[\frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}^2}{B^2} \right] F + \frac{1}{2}f + \left[\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B} \right] \dot{F} + \ddot{F} = \rho_B \tag{3.4}$$

Now from (3.3) and (3.4) we get,

$$\left[\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} - \frac{\dot{A}\dot{B}}{AB} - \frac{\dot{B}^2}{B^2} \right] F - \left[\frac{\dot{B}}{B} \right] \dot{F} + \ddot{F} = \rho \tag{3.5}$$

Using equation (3.5) in (3.3), we get

$$\left[\frac{\ddot{B}}{B} + \frac{\dot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}^2}{B^2} \right] F - \frac{1}{2}f + \left[\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B} \right] \dot{F} - \ddot{F} = \rho_B \tag{3.6}$$

As we know that, the density is never negative here, so the equation (3.4) is just for convenience.

As we have, $\omega = p/\rho$ using this in (3.1) we get,

$$\left[\frac{\ddot{A}}{A} + 2\frac{\dot{A}\dot{B}}{AB} \right] F - \frac{1}{2}f - 2\left[\frac{\dot{B}}{B} \right] \dot{F} - \ddot{F} = -\omega^2 \rho - \rho_B \tag{3.7}$$

Using the (3.4) and (3.5) in (3.7) we get,

$$\left\{ \left[\frac{\ddot{A} B}{A \dot{B}} - \frac{\ddot{B}}{B} + \frac{\dot{A}}{A} - \frac{\dot{B}}{B} \right] \frac{F}{\dot{F}} - \left[\frac{\ddot{A}}{A} - \frac{\ddot{B}}{B} + \frac{\dot{A} \dot{B}}{A B} - \frac{\dot{B}^2}{B^2} \right] \frac{F}{\ddot{F}} - \left\langle \frac{\dot{A}}{A} \left(2 - \frac{B}{\dot{B}} - \frac{B^2}{\dot{B}^2} \right) - \frac{A \dot{B}}{\dot{A} B} + \frac{\dot{B}}{\dot{B}} \right\rangle \frac{\dot{F}}{F} - \left[\frac{\dot{A}}{A} + \frac{\dot{B}}{B} \right] \frac{\dot{F}}{\dot{F}} \right\}^{1/2} = \omega \quad (3.8)$$

Now using (3.7) and (3.8) in (3.1), we get

$$\frac{\left(f + \left[\frac{\dot{A}}{A} + 4 \frac{\dot{B}}{B} \right] \dot{F} + 2 \ddot{F} - \left[\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + 3 \frac{\dot{A} \dot{B}}{A B} + \frac{\dot{B}^2}{B^2} \right] F \right)}{\left(\left[\frac{\ddot{A} B}{A \dot{B}} - \frac{\ddot{B}}{B} + \frac{\dot{A}}{A} - \frac{\dot{B}}{B} \right] \frac{F}{\dot{F}} - \left[\frac{\ddot{A}}{A} - \frac{\ddot{B}}{B} + \frac{\dot{A} \dot{B}}{A B} - \frac{\dot{B}^2}{B^2} \right] \frac{F}{\ddot{F}} - \left\langle \frac{\dot{A}}{A} \left(2 - \frac{B}{\dot{B}} - \frac{B^2}{\dot{B}^2} \right) - \frac{A \dot{B}}{\dot{A} B} + \frac{\dot{B}}{\dot{B}} \right\rangle \frac{\dot{F}}{F} - \left[\frac{\dot{A}}{A} + \frac{\dot{B}}{B} \right] \frac{\dot{F}}{\dot{F}} \right)^{1/2}} = p \quad (3.9)$$

We can use the special law of variation which is nothing but the relation between F and a this gives us,

$$F = ba^n \quad (3.10)$$

Without loss of generality we choose $b = 1$, so

$$F = (e^{\beta kt} - 1)^{n/\beta} = a^n \quad (3.11)$$

Now for $k=1$, we get the scale factor in the form

$$F = (e^{\beta t} - 1)^{n/\beta} = a^n \quad (3.12)$$

Katore and Shaikh [68] investigated cosmological model with strange quark matter attached to the string cloud in general theory of gravitation for Axially Symmetric space time. They obtained model with the help of special law of variation for Hubble parameter proposed by Bermann (Nuovo Cimento B 74:182, 1983).

As we know that the special volume is,

$$V = \sqrt{-g} = AB^2 \quad (3.13)$$

$$\text{But } a = V^{1/3} = (AB^2)^{1/3} \quad (3.14)$$

In 1983 Berman proposed the special law of variation of Hubble's parameter yields a constant value of deceleration parameter of the universe, according to Berman's law:

$$q = \frac{d}{dt} \left(\frac{1}{H} \right) - 1 \quad (3.15)$$

Where, H is the Hubble's Parameter?

$$H = \frac{\dot{a}}{a} \quad (3.16)$$

Where a is the average scale factor which given by the following equation and the graphical nature of average scale factor versus cosmic time is as shown in Fig. 1.

$$a = (e^{\beta t} - 1)^{1/\beta} \quad (3.17)$$

Behavior of scale factor of magnetized anisotropic dark energy cosmological model versus cosmic time with the appropriate choice of constants as shown in the following Fig. 1. While observing the graphical nature of the scale factor which is the function of time; it has been concluded that the scale factor is increases exponentially with infinite time interval.

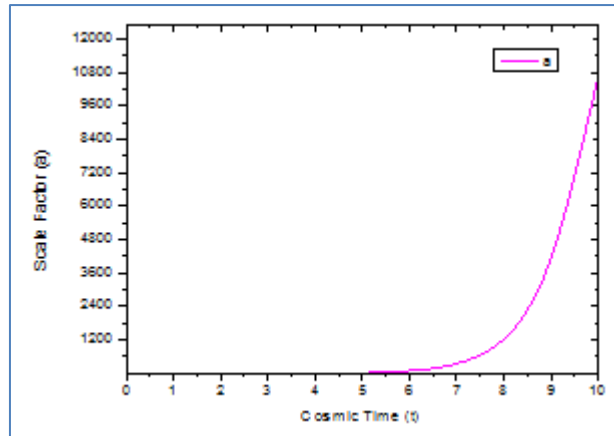


Fig-1: Behavior of scale factor of magnetized anisotropic dark energy cosmological model versus cosmic time with the appropriate choice of constants $\beta = 0.2$ and $n = 0.1$.

From (3.13), (3.16) and (3.17), the metric potential comes out to be and we take $A = B^n; n \neq 0$ with this we get the values of A and B as follow

$$B = (e^{\beta t} - 1)^{\frac{3}{\beta(n+2)}} \tag{3.18}$$

$$A = (e^{\beta t} - 1)^{\frac{3n}{\beta(n+2)}} \tag{3.19}$$

The potential function obtained in equation (3.18 and 3.19) are the exponential and power functions of cosmic time t and it will be vanishes $t = 0$ and obtained the constant model.

Plane symmetric space-time with parameterize EoS in the metric version of $f(R)$ gravity becomes:

$$ds^2 = dt^2 - \left[(e^{\beta t} - 1)^{\frac{3n}{\beta(n+2)}} \right]^2 dx^2 - \left[(e^{\beta t} - 1)^{\frac{3}{\beta(n+2)}} \right]^2 (dy^2 + dz^2) \tag{3.20}$$

In the model (3.19), it is observed that both metric potentials are the exponential and power term. Here we observed 2.

Physical Properties of the Model

The Ricci Scalar with the values of metric potentials A and B , we have

$$R = 2 \left[\frac{\ddot{A}}{A} + 2 \frac{\ddot{B}}{B} + 2 \frac{\dot{A}}{A} \frac{\dot{B}}{B} + \frac{\dot{B}^2}{B} \right] \tag{4.1}$$

By considering the potential function (3.18) and (3.19) and also preposition of the field equations (3.1-3.9) with Ricci scalar (4.1) and its derivative with respect to cosmic time t ; the physical characteristics of the model say energy density, magnetized energy density, pressure, magnetized and also an-isotropy parameter of the model have been calculated as follows:

The Ricci scalar of the model is establish to be

$$R = 6 \left[\left(\frac{3n^2 - \beta(n+2)^2 + 12}{(n+2)^2} \right) \frac{e^{2\beta t}}{(e^{\beta t} - 1)^2} + \frac{\beta e^{\beta t}}{e^{\beta t} - 1} \right] \tag{4.2}$$

Equation (4.2) represents the function of the Ricci scalar of the model, it is clear that the function of the Ricci scalar is positive and decreasing function of time. The equation of energy density (4.3) and the graphical behavior of energy density with cosmic time are as shown below:

$$\rho = \left[\left\{ \frac{3[6 - 3n^2 - \beta(n+2)(n+1) - 3n]}{(n+2)^2} - \frac{3n}{n+2} + n(n-\beta) \right\} \frac{e^{2\beta t}}{(e^{\beta t} - 1)^2} + \left\{ \frac{3\beta(n+1)}{n+2} + n\beta \right\} \frac{e^{\beta t}}{e^{\beta t} - 1} \right] (e^{\beta t} - 1)^{n/\beta} \quad (4.3)$$

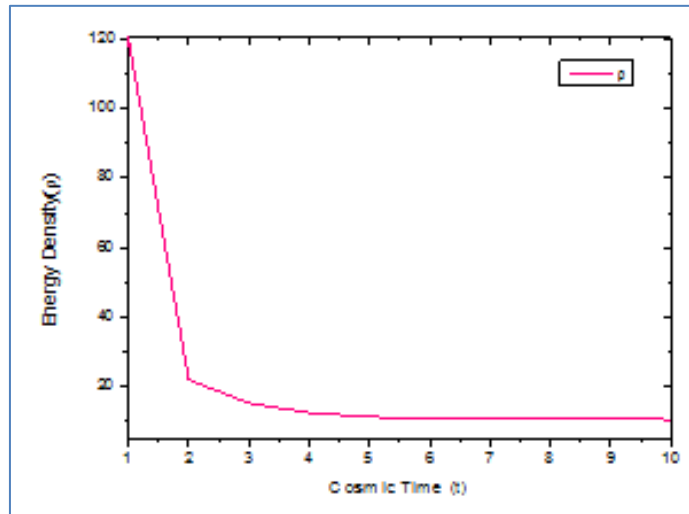


Fig-2: Behavior of energy density of magnetized anisotropic dark energy cosmological model versus cosmic time with the appropriate choice of constants $n = 0.1$ and $\beta = 0.2$.

Also energy density of magnetized fluid (4.4) is given by the following expression:

$$\rho_B = \left[\left\{ \frac{(n+2)[3(n^2+n) + (n-\beta)(n+2)(n+2) - \beta] + 9n}{(n+2)^2} \right\} \frac{e^{2\beta t}}{(e^{\beta t} - 1)^2} - \beta \left\{ \frac{n(n+2) - 3}{n+2} \right\} \frac{e^{\beta t}}{e^{\beta t} - 1} \right] (e^{\beta t} - 1)^{n/\beta} \quad (4.4)$$

$$- \left[\frac{9\beta[\beta(n+2)^2 + 2n + 8]}{(n-\beta)(n+2)^2} \right] \frac{1}{e^{\beta t} - 1} - \left[\frac{6\beta[\beta(n+2)^2 - 3n + 12]}{(n-2\beta)(n+2)^2} \right] \frac{1}{(e^{\beta t} - 1)^2}$$

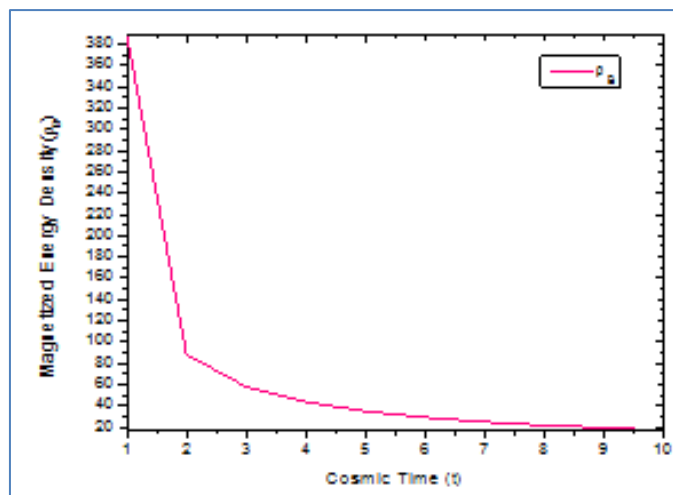


Fig-3: Behavior of magnetized energy density of fluid in magnetized anisotropic dark energy cosmological model versus cosmic time with the appropriate choice of constants, $\beta = 0.2$, and $n = 0.1$.

The EoS parameter for the given magnetized dark energy cosmological model is found out and given by the equation (4.5) and also its graphical nature shown in Fig.4.

$$\omega = \left[\left\{ \frac{3[6-2n^2-5n-\beta(n+2)]}{n\beta(n+2)^2} - \frac{n^3+n^2(6\beta+2)+n+2}{\beta(n+2)} \right\} \frac{e^{\beta t}}{(e^{\beta t}-1)} - \left\{ \frac{3\beta n(n-1)-3}{n(n-\beta)(n+2)} + \frac{n(n+2)}{\beta(n+2)-3} \right\} \left(\frac{e^{\beta t}-1}{e^{\beta t}} \right) \right]^{1/2} - \frac{e^{\beta t}}{\beta} - \left\{ \frac{n^4(2\beta-3)+n^3(2\beta^2-3\beta-7)-n^2(7\beta+4)+n(7\beta^2-\beta-6)+2\beta(\beta+1)}{n(n-\beta)(n+2)^2} \right\} (n+2) \left\{ \frac{\beta^2}{\beta(n+2)-3} + \frac{n^2}{3} + \frac{1}{3n-\beta(n+2)} \right\} \tag{4.5}$$

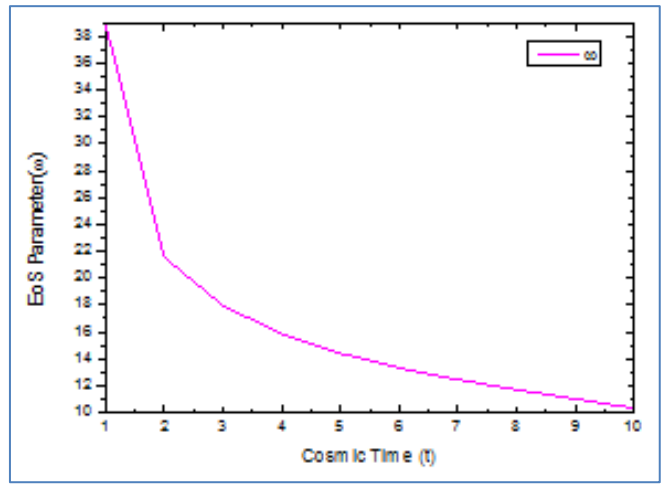


Fig-4: Behavior of EoS parameter of magnetized anisotropic dark energy cosmological model versus cosmic time with the appropriate choice of constants $\beta = 0.2$ and $n = 0.1$.

The Pressure of the universe is given by the equation (4.6) and the graphical nature shown in Fig. 5.

$$p = \left[\frac{\left[- \left\{ \frac{12[\beta(n+2)^2-3n+12]}{n(n-2\beta)(n+2)^2} \right\} \frac{1}{(e^{\beta t}-1)^2} - \left\{ \frac{18\beta[\beta(n+2)^2+2n+8]}{(n-2)(n+2)^2} \right\} \frac{1}{(e^{\beta t}-1)} - \left\{ \frac{\beta[2n(n+2)-3(n+1)]}{(n-2)(n+2)^2} \right\} \left(\frac{e^{\beta t}}{e^{\beta t}-1} \right) \right] (e^{\beta t}-1)^{n/\beta}}{\left[\frac{3[n^2(\beta-3)+n(2\beta+9)+\beta(n+2)]}{(n+2)^2} - \frac{3n^2+2n[(n-\beta)(n+2)-6]}{n+2} \right] \left(\frac{e^{2\beta t}}{e^{\beta t}-1} \right)} \right]^{1/2} - \left[\frac{\left\{ \frac{3[6-2n^2-5n-\beta(n+2)]}{n\beta(n+2)^2} - \frac{n^3+n^2(6\beta+2)+n+2}{\beta(n+2)} \right\} \frac{e^{\beta t}}{(e^{\beta t}-1)} - \left\{ \frac{3\beta n(n-1)-3}{n(n-\beta)(n+2)} + \frac{n(n+2)}{\beta(n+2)-3} \right\} \left(\frac{e^{\beta t}-1}{e^{\beta t}} \right) \right]^{1/2} - \frac{e^{\beta t}}{\beta} - \left\{ \frac{n^4(2\beta-3)+n^3(2\beta^2-3\beta-7)-n^2(7\beta+4)+n(7\beta^2-\beta-6)+2\beta(\beta+1)}{n(n-\beta)(n+2)^2} \right\} (n+2) \left\{ \frac{\beta^2}{\beta(n+2)-3} + \frac{n^2}{3} + \frac{1}{3n-\beta(n+2)} \right\} \tag{4.6}$$

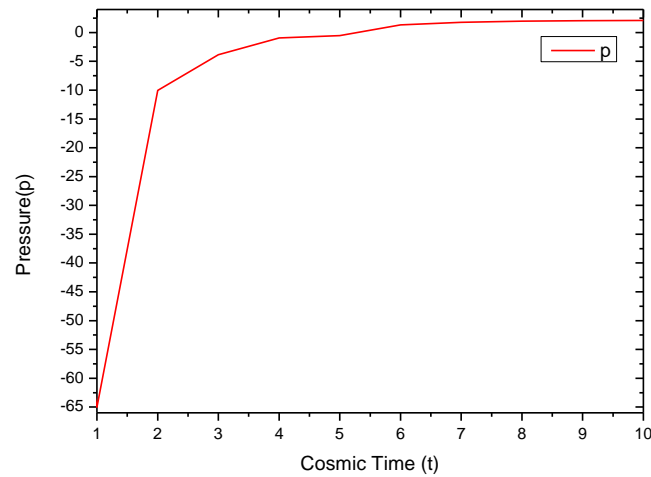


Fig-5: Behavior of pressure of magnetized anisotropic dark energy cosmological model versus cosmic time with the appropriate choice of constants $\beta = 0.2$ and $n = 0.1$.

The skewness parameter for the desired model is found out to be and given by (4.7) and also its graphical behavior shown in Fig.6.

$$\delta = -\omega = \left[\begin{aligned} & - \left\{ \frac{3[6 - 2n^2 - 5n - \beta(n+2)]}{n\beta(n+2)^2} \right\} \left\{ \frac{e^{\beta t}}{(e^{\beta t} - 1)} + \frac{3\beta n(n-1) - 3}{n(n-\beta)(n+2)} \left(\frac{e^{\beta t} - 1}{e^{\beta t}} \right) \right\} \right]^{1/2} \\ & + \frac{e^{\beta t}}{\beta} + \left\{ \frac{n^4(2\beta - 3) + n^3(2\beta^2 - 3\beta - 7) - n^2(7\beta + 4) + n(7\beta^2 - \beta - 6) + 2\beta(\beta + 1)}{n(n-\beta)(n+2)^2} \right\} \\ & - (n+2) \left\{ \frac{\beta^2}{\beta(n+2) - 3} + \frac{n^2}{3} + \frac{1}{3n - \beta(n+2)} \right\} \end{aligned} \right] \tag{4.7}$$

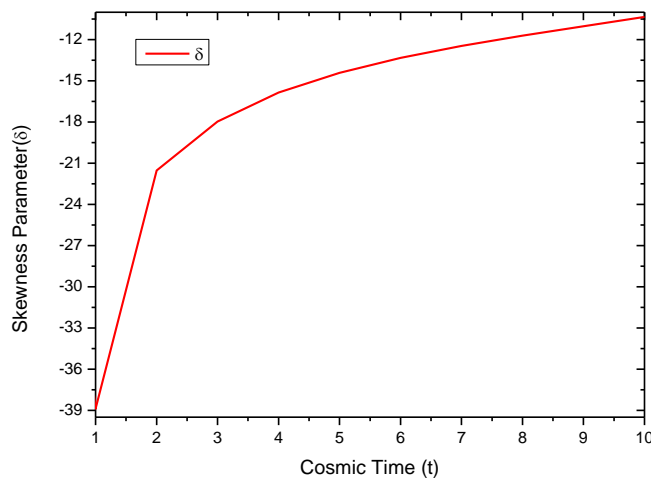


Fig-6: Behavior of skewness parameter of magnetized anisotropic dark energy cosmological model versus cosmic time with the appropriate choice of constants $\beta = 0.2$ and $n = 0.1$.

Kinematical Properties of the Model

The kinematical properties which are important in cosmology while discussing the geometrical behavior of the universe. By preposition of the potential function (3.18) and (3.19) and also considering the field equations (3.1-3.9) with Ricci scalar (4.1) and its derivative with respect to cosmic time t (4.2); the kinematical characteristics of the model say; spatial volume, Hubble parameter, expansion scalar, mean parameterized anisotropy parameter, shear scalar, deceleration parameter and overall density parameter have been calculated as follows:

The spatial volume,

$$V = (e^{\beta t} - 1)^{3/\beta} \tag{5.1}$$

The spatial volume is the exponential and power function of time t and the graphical behavior of the spatial volume is increasing exponentially with infinite time interval and at $t = 0$ the spatial volume vanishes. Behavior of spatial volume of magnetized anisotropic dark energy cosmological model versus cosmic time with the appropriate choice of constants is shown in Fig. 7.

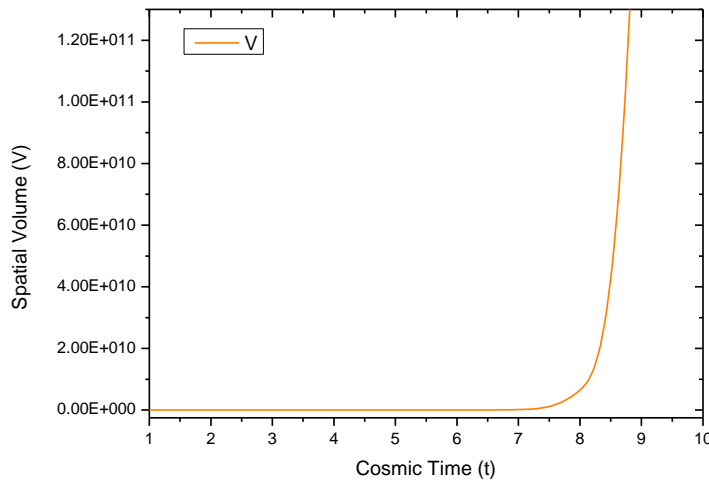


Fig-7: Behavior of spatial volume of magnetized anisotropic dark energy cosmological model versus cosmic time with the appropriate choice of constants $n = 0.1$ and $\beta = 0.2$.

$$H = \left(\frac{e^{\beta t}}{e^{\beta t} - 1} \right) \tag{5.2}$$

Behaviors of Hubble parameter of magnetized anisotropic dark energy cosmological model versus

cosmic time with the appropriate values of constants are shown graphically in Fig. 8.

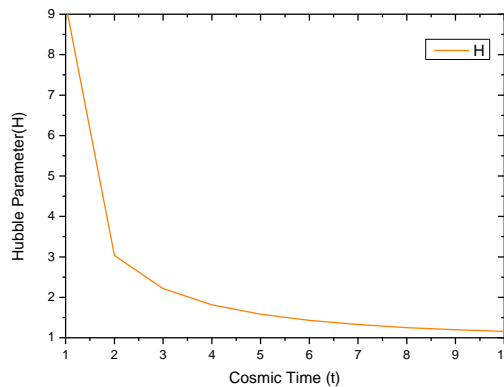


Fig-8: Behavior of Hubble parameter of magnetized anisotropic dark energy cosmological model versus cosmic time with the appropriate choice of constants $n = 0.1$ and $\beta = 0.2$.

The expansion scalar,

$$\theta = 3H = \left(\frac{3e^{\beta t}}{e^{\beta t} - 1} \right) \tag{5.3}$$

Behaviors of expansion scalar which is the rational and positive function of time t of magnetized anisotropic dark energy cosmological model versus cosmic time with the appropriate values of constants are shown graphically in Fig. 9.

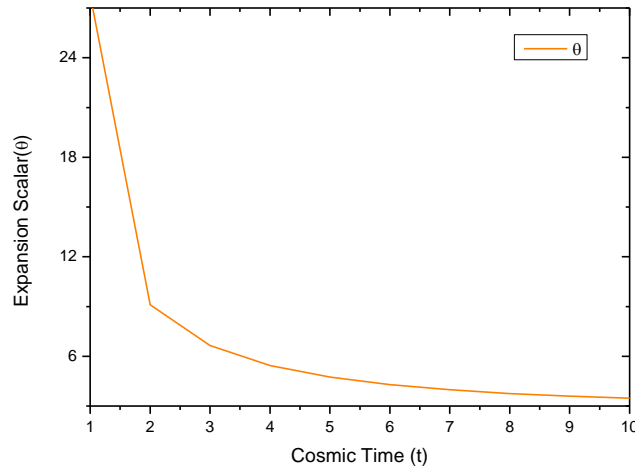


Fig-9: Behavior of expansion scalar of magnetized anisotropic dark energy cosmological model versus cosmic time with the appropriate choice of constants $n = 0.1$ and $\beta = 0.2$.

Mean anisotropy parameter,

$$A_m = \sum_{i=1}^3 \left(\frac{H_i - H}{H} \right)^2 = \left(\frac{2(n-1)^2}{(n+2)} \right) \tag{5.4}$$

Similarly the Shear Scalar is

$$\sigma^2 = \left(\frac{2(n-1)^2}{(n+2)} \right) \frac{e^{2\beta t}}{(e^{\beta t} - 1)^2} \tag{5.5}$$

While introducing the appropriate values of constants in mean parameterized anisotropy parameter it is found that the mean parameterized anisotropy parameter is constant and also not the function of cosmic time t .

After choosing appropriate values of constants the behavior of shear scalar of magnetized anisotropic dark energy cosmological model versus cosmic time is shown in Fig. 10. And it is observed that the shear scalar is the negative function of cosmic time t .

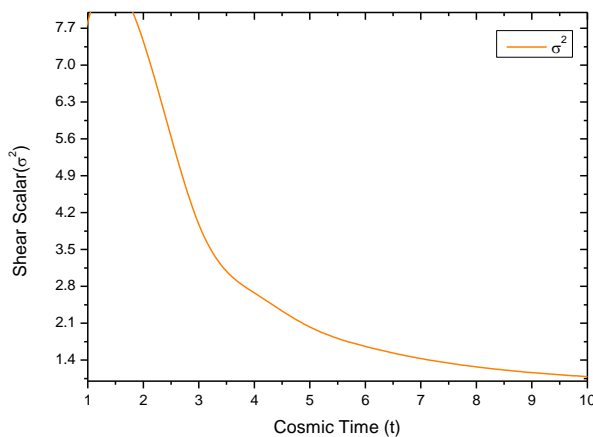


Fig-10: Behavior of shear scalar of magnetized anisotropic dark energy cosmological model versus cosmic time with the appropriate choice of constants $n = 0.1$ and $\beta = 0.2$.

The Deceleration parameter is found to be,

$$q = \frac{d}{dt} \left(\frac{1}{H} \right) - 1 = \frac{e^{\beta t} - \beta}{e^{\beta t}} \tag{5.6}$$

By choosing appropriate values of constants it is observed that the deceleration parameter is the

negative function of cosmic time t . At $t=0$, $q=-1$; which shows that the universe have an accelerated expansion. At the present time the negative worth of deceleration parameter like $q = 0.91 \pm 0.14$ is a lot of favorable per on top of recent empirical knowledge.

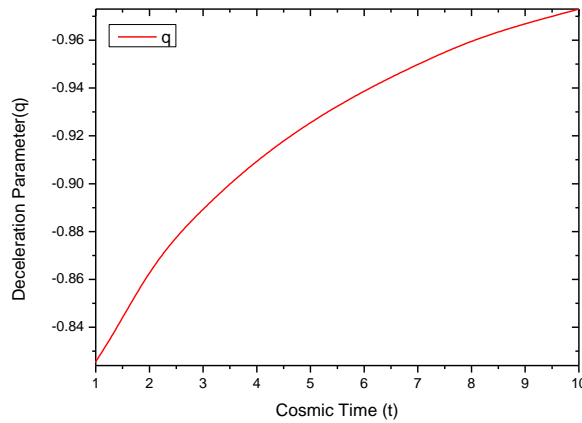


Fig-11: Behavior of deceleration parameter of magnetized anisotropic dark energy cosmological model versus cosmic time with the appropriate choice of constants $n = 0.1$ and $\beta = 0.2$.

The overall density parameter is,

$$\Omega = \frac{\rho}{3H^2}$$

$$\Omega = \left[\left\{ \frac{\beta(n+1)}{n+2} + \frac{n\beta}{3} \right\} \frac{e^{\beta t} - 1}{e^{\beta t}} + \left\{ \frac{6 - 3n^2 - \beta(n+2)(n+1) - 3n}{(n+2)^2} - \frac{n}{n+2} + \frac{n(n-\beta)}{3} \right\} \right] (e^{\beta t} - 1)^{n/\beta} \tag{5.7}$$

The behavior of overall density parameter of magnetized anisotropic dark energy cosmological model versus cosmic time with the appropriate choice of constants is shown in Fig. 12. It has been observed

that the overall density parameter is positive function with an infinite time interval. At $t = 0$ the overall density parameter vanishes.

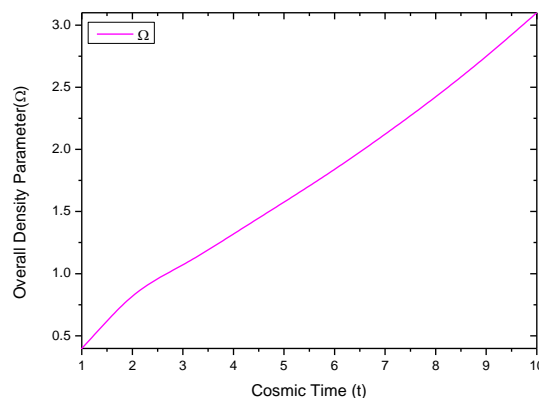


Fig-12: Behavior of overall energy density parameter of magnetized anisotropic dark energy cosmological model versus cosmic time with the appropriate choice of constants $n = 0.1$ and $\beta = 0.2$.

CONCLUSION

In this context with parameterized equation of state (EoS), in the metric version of $f(R)$ gravity has been investigated. We concluded that the model of the

universe with anisotropic magnetized dark energy is the accelerating, expanding and hence does not shows isotropy. An exact solution of the field equations correspond to special law of variation which provides singular model. We've got evaluated some necessary

cosmological physical and kinematical quantities for this model along with his graphical behavior.. The energy density and kinematical parameters H, θ and σ^2 all are infinite at this point but the volume scale factor vanishes. Also, with the expansion, expansion scalar and shear scalar are decreases.

ACKNOWLEDGEMENT

We are immensely grateful to the honorary referee and also the editor for the illuminating suggestions that have considerably enhanced our work in terms of investigation, standard and presentation.

REFERENCES

- Riess AG, Filippenko AV, Challis P, Clocchiatti A, Diercks A, Garnavich PM, Gilliland RL, Hogan CJ, Jha S, Kirshner RP, Leibundgut BR. Observational evidence from supernovae for an accelerating universe and a cosmological constant. *The Astronomical Journal*. 1998 Sep;116(3):1009.
- Perlmutter S, Turner MS, White M. Constraining dark energy with type Ia supernovae and large-scale structure. *Physical Review Letters*. 1999 Jul 26;83(4):670.
- Spergel DN, Verde L, Peiris HV, Komatsu E, Nolte MR, Bennett CL, Halpern M, Hinshaw G, Jarosik N, Kogut A, Limon M. First-year Wilkinson Microwave Anisotropy Probe (WMAP)* observations: determination of cosmological parameters. *The Astrophysical Journal Supplement Series*. 2003 Sep;148(1):175.
- Spergel S. Wilkinson Microwave Anisotropy Probe (WMAP) Three Year Results: Implications for Cosmology *Astrophys. J*. 2007; 170 377
- Bennet CL. First-Year Wilkinson Microwave Anisotropy Probe (WMAP)* Observations: Preliminary Maps and Basic Results. *Astrophys. J. Suppl. Ser*. 2003; 148 1,
- Ratra B, Peebles J. Cosmological consequences of a rolling homogeneous scalar field. *Phys. Rev. D*, 1988; 37 321.
- Wetterich C. Quintessence and the cosmological constant. *Nucl. Phys. B*.1998; 302 668.
- Caldwell R. A Phantom Menace? Cosmological consequences of a dark energy component with super-negative equation of state *Phys. Lett. B*. 2002; 545 23.
- Wei Y, Tian Y. Hesseence: a new view of quintom dark energy. *Class. Quantum Gravity*, 2004; 21 5347.
- Elizalde E. Late-time cosmology in a (phantom) scalar-tensor theory: Dark energy and the cosmic speed-up. *Phys. Rev. D*. 2004; 70 043539.
- Nojiri S. Properties of singularities in the (phantom) dark energy universe. *Phys. Rev. D*; 2005: 71 063004.
- Kamenshchik A. An alternative to quintessence. *Phys. Lett. B*. 2001:511 -265.
- Chiba T. Kinetically driven quintessence. *Phys. Rev. D*; 2000: 62- 023511,
- Armendariz-Picon C. Dynamical Solution to the Problem of a Small Cosmological Constant and Late-Time Cosmic Acceleration *Phys. Rev. Lett*. 2000. 85 -4438,
- Scherrer RJ. Purely kinetic k-essence as unified dark matter. *Phys. Revs. Lett*. 2004; 93 -011301
- Capozziello S. Deriving the mass of particles from Extended Theories of Gravity in LHC era. *Phys. Lett. B*; 2010: 693 198,
- Mishra B, Tripathy SK, Tarai S. Cosmological models with a hybrid scale factor in an extended gravity theory. *Modern Physics Letters A*. 2018 Mar 21;33(09):1850052.
- Das K, Sultana T. Magnetized anisotropic ghost dark energy cosmological model. *Astrophysics and Space Science*. 2015 Jun;357(2):1-7.
- Santhi MV, Rao VU, Aditya Y. Anisotropic magnetized holographic Ricci dark energy cosmological models. *Canadian Journal of Physics*. 2017;95(4):381-92.
- Artymowski M, Lalak Z. Inflation and dark energy from f (R) gravity. *Journal of Cosmology and Astroparticle Physics*. 2014 Sep 22;2014(09):036.
- Katore SD, Rane RS. Magnetized cosmological models in bimetric theory of gravitation. *Pramana*. 2006 Aug;67(2):227-37.
- Kandalkar SP. *Int. J. of Inov. Res. In Sci., Engi. And Tech*. 2014; 3(2): 9450-9455,
- Abebe A. Shear-free anisotropic cosmological models in f(R) gravity *Gen Relativ Gravit*. 2016: 48:49.
- Astashenok AV. *AIP Conference Proceedings*. 2014; 1606, 99.
- Hasmani AH, Al-Haysah AM. *AAM: Int. J*. 2019; 14(1): 334 – 348.
- Reddy DR, Kumar RS, Kumar TP. Bianchi type-III dark energy model in f (R, T) gravity. *International Journal of Theoretical Physics*. 2013 Jan;52(1):239-45.
- Banik DK, Banik SK, Bhuyan K. Bianchi V cosmological model in Palatini f (R) gravity. *General Relativity and Gravitation*. 2018 Jan 1;50(1):13.
- Saiedi H. Modified \$ f (R) \$ Gravity and Thermodynamics of Time-Dependent Wormholes at Event Horizon. *arXiv preprint arXiv:1604.01071*. 2016 Apr 4.
- De Martino I, De Laurentis M, Capozziello S. Constraining f (R) Gravity by the Large-Scale Structure. *Universe*. 2015 Sep;1(2):123-57.
- Bamba K. Possible antigravity regions in F(R) theory? *Physics Letters B*.2014; 730: 136–140.
- Bamba K, Geng CQ. *Pro. Of Theo. Phys.*, 2009; 122(5): 1267-1276.
- Adhav KS. Kantowski-Sachs string cosmological model in f(R) theory of gravity *Canadian Journal of Physics*. 2012; 90(2); 119-123.

33. Akbar M. Generalized second law of thermodynamics in extended theories of gravity. *International Journal of Theoretical Physics*. 2009 Sep;48(9):2672-8.
34. Sharif M, Shamir MF. Non-vacuum Bianchi types I and V in $f(R)$ gravity. *General Relativity and Gravitation*. 2010 Nov 1;42(11):2643-55.
35. Shamir MF. Some Bianchi type cosmological models in $f(R)$ gravity. *Astrophysics and Space Science*. 2010 Nov;330(1):183-9.
36. Sharif M, Shamir MF. Energy distribution in $f(R)$ gravity. *General Relativity and Gravitation*. 2010 Jun;42(6):1557-69.
37. Capone M, Ruggiero ML. IOP Publishing. 2010; 27(12):125006.
38. Moraes PH, Sahoo PK, Taori B, Sahoo P. Phantom energy-dominated universe as a transient stage in $f(R)$ cosmology. *International Journal of Modern Physics D*. 2019 Jul 21;28(10):1950124.
39. Ladke LS, Jaiswal VK, Hiwarkar RA. *International journal of engineering sciences & management. Int. J. of Engg. Sci. & Mgmt.(IJESM)*. 2014 Jul;4(3):66-74.
40. Odintsov SD, Oikonomou VK. Autonomous dynamical system approach for $f(R)$ gravity. *Physical Review D*. 2017 Nov 28;96(10):104049.
41. Odintsov SD, Oikonomou VK. Dynamical systems perspective of cosmological finite-time singularities in $f(R)$ gravity and interacting multifluid cosmology. *Physical Review D*. 2018 Jul 6;98(2):024013.
42. Odintsov SD, Oikonomou VK. Reconstruction of slow-roll $F(R)$ gravity inflation from the observational indices. *Annals of Physics*. 2018 Jan 1;388:267-75.
43. Soroushfar S. *Eur. Phys. J.C*. 2016; 76:476,.
44. Lee S. ar Xiv: 1710.11581v1 [astro-ph.CO] (2017).
45. Capozziello S, Nojiri SI, Odintsov SD, Troisi A. Cosmological viability of $f(R)$ -gravity as an ideal fluid and its compatibility with a matter dominated phase. *Physics Letters B*. 2006 Aug 10;639(3-4):135-43.
46. Sotiriou TP, Faraoni V. $f(R)$ theories of gravity. *Reviews of Modern Physics*. 2010 Mar 1;82(1):451.
47. Liu T, Zhang X, Zhao W. Constraining $f(R)$ gravity in solar system, cosmology and binary pulsar systems. *Physics Letters B*. 2018 Feb 10;777:286-93.
48. Chirde VR, Shekh SH. *IJTPC*. 2017; 13
49. Aditya Y, Reddy DR. Locally rotationally symmetric Bianchi type-I string cosmological models in $f(R)$ theory of gravity. *International Journal of Geometric Methods in Modern Physics*. 2018 Sep 7;15(09):1850156.
50. Pradhan A, Pandey OP. Bianchi type I anisotropic magnetized cosmological models with varying Λ . *International Journal of Modern Physics D*. 2003 Aug;12(07):1299-314.
51. Santhi MV. *Int. J. of Dynamical Systems & Geometric Theories*; 2019, 17: 23–38.
52. Bhoyar SR, Chirde VR, Shekh SH. Non-static plane symmetric cosmological model with magnetized anisotropic dark energy by hybrid expansion law in $f(r, t)$ gravity. *Inter. J. of Adv. Resea.* 2015;3(9):492-500.
53. Mahanta CR, Sarma N. Bianchi Type-III Cosmological Model in $f(R)$ Theory of Gravity. *Prespacetime Journal*. 2017; 8(1): 33-39.
54. Ntahompagaze J, Abebe A, Mbonye M. On $f(R)$ gravity in scalar-tensor theories. *International Journal of Geometric Methods in Modern Physics*. 2017 Jul 10;14(07):1750107.
55. Arnold C, Leo M, Li B. Realistic simulations of galaxy formation in $f(R)$ modified gravity. *Nature Astronomy*. 2019 Oct;3(10):945-54.
56. Geng CQ, Hsu WC, Lu JR, Luo LW. Thermodynamics of $f(R)$ Gravity with Disformal Transformation. *Entropy*. 2019 Feb;21(2):172.
57. Shah P, Samanta GC. Stability analysis for cosmological models in $f(R)$ gravity using dynamical system analysis. *The European Physical Journal C*. 2019 May;79(5):1-9.
58. Sahoo PK, Bhattacharjee S. Revisiting the coincidence problem in $f(R)$ gravitation. *New Astronomy*. 2020 May 1;77:101351.
59. Gogoi DJ, Goswami UD. A new $f(R)$ gravity model and properties of gravitational waves in it. *The European Physical Journal C*. 2020 Dec;80(12):1-21.
60. Bhoyar SR, Chirde VR, Shekh SH. Physical Aspects of Bianchi Type-I Space-time with Quadratic Equation of State in $f(R)$ Gravity. *Prespacetime Journal*. 2016 Mar 10;7(3).
61. Shaikh AY, Wankhade KS. Magnetized Dark Energy Cosmological Models with Constant Deceleration Parameter. *Phys Astron Int J*. 2017 Oct;1(4):00020.
62. Shamir MF, Ali A. Bianchi type-V cosmology with magnetized anisotropic dark energy. *Canadian Journal of Physics*. 2017;95(3):274-82.
63. Farnes JS. A unifying theory of dark energy and dark matter: Negative masses and matter creation within a modified Λ CDM framework. *Astronomy & Astrophysics*. 2018 Dec 1;620:A92.
64. Zhitnitsky AR. Cosmological magnetic field and dark energy as two sides of the same coin. *Physical Review D*. 2019 May 16;99(10):103518.
65. Ray PP. *Inte. J. of Mod. Phys. D*. 2019; 28,07.
66. Katore SD, Shaikh AY. Cosmological model with strange quark matter attached to cosmic string for axially symmetric space-time. *International Journal of Theoretical Physics*. 2012 Jun;51(6):1881-8.