Scholars Journal of Physics, Mathematics and Statistics
OPEN ACCESS
Abbreviated Key Title: Sch J Phys Math Stat
ISSN 2393-8056 (Print) | ISSN 2393-8064 (Online)
Journal homepage: https://saspublishers.com

Defective and Repairable Items Unsteady Deterioration Rate Inventory Model Under Three Tired Prices and Time Dependent Demand

D.M. Patel[1*], R.D. Patel[2]

[1]Department of Commerce, Narmada College of Science and Commerce, Zadeshwar, Bharuch, India
[2]Visiting Professor, M. Sc. (IT) Program, Veer Narmad South Gujarat University, Surat, India

DOI: 10.36347/sjpms.2021.v08i05.002 | **Received:** 05.04.2021 | **Accepted:** 17.05.2021 | **Published:** 22.05.2021

***Corresponding author:** D.M. Patel

Abstract **Review Article**

Lot received or units produced are not all perfect items. A time and price dependent demand inventory model is formulated when items produced or lot received are of defective and repairable nature. Three tired pricing is considered. For different situations, expression for total profit is derived to derive optimal solution. For parameter, post-optimality computations are also done.

Keywords: Inventory model, Varying deterioration, Price dependent demand, Time dependent demand, Three tire pricing, Defective items, Repairable items, Time varying holding cost.

INTRODUCTION

Everywhere inventories are essential. Absence of inventory make customer uncomfortable and it may affect goodwill or profit of future of the organization. Also certain items are such that their value reduces during their storage period and we say these items as deteriorating items. In last few decays much attention has been given towards developing decaying items models. A stock model under fixed deterioration rate was proposed by Ghare and Schrader [1963]. Subsequently, taking into account changeable deterioration rate, model was expanded by Covert and Philip [1973]. Under selling price dependent demand and Weibull decay rate, Aggarwal and Goel [1984] obtained an inventory model. Mukhopadhyay et al. [2004] obtained a stock model for decaying items under price dependent demand. A stock with selling price related inventory model for decaying units was formulated by Teng and Chang [2005]. Under changeable storage cost and stock dependent demand, a stock model was constructed by Alfares [2007]. A time and selling price dependent inventory model was obtained by Mathew [2013]. A stock level model under stock size and price fluctuating demand with varying deterioration was constructed by Patel and Sheikh [2015]. Under trade credit situation for non-instantaneous deteriorating items, a stock level model was formulated by Tsao et al. [2017].

Imperfect quality of goods affects management of inventory. Therefore this characteristic is to be taken into consideration. Several academicians studied and analysed difficulties associated with defective production process of an item. Lee and Rosenblatt [1985] proposed an imperfect quality items stock model for obtaining optimal order policy. An EOQ model that contains known proportion of defective units in received lot and to remove these items costs incurs of fixed and variable inspection nature was obtained by Schwaller [1988]. When production process is not perfect, an inventory model was formulated by Cheng [1991]. It was assumed that demand of item depends on unit production cost. An inventory model when received items are not 100% perfect was obtained by Salameh and Jaber [2000]. Imperfect units are separated after 100% screening and their selling will be done at discounted price. An EPQ model under known proportion of defective units in lot produced following a uniform distribution was obtained by Hayek and Salameh [2001]. A defective items production inventory model was obtained by Goyal and Barron [2002]. A no shortage defective items inventory model was constructed by Papachristos and Konstantaras [2006] in which at cycle end defective units has been removed. Under effect of learning, an EPQ model having defective units was obtained by Jaber et al. [2008]. Under stock out situation, Hsu and Hsu [2012] obtained a defective units stock model. An EPQ with rework to determine selling price, stock size and shipment size collectively was formulated by Taleizadeh et al. [2015]. A deteriorating items production inventory model for defective items was considered by Shukla et al. [2016]. Naik and Patel [2017] constructed a time and price related stock model for defective units under unsteady deterioration.

The term repairing/remanufacturing in inventory modelling was first introduced by Schrady [1967]. By considering multi-item system sharing the same repair facility and stock-out service level constraints, Mabini et al. [1992] expanded the model. A linear demand inventory model for defective items in which some items can be repairable, was considered by Yadav and Kumar [2014]. Gothi et al. [2017] formulated a linear demand and exponential type deterioration of items inventory model in which received items having defects but some of them can be repairable. Naik and Patel [2018] constructed a price and time dependent demand stock model for defective and repairable items under unsteady deterioration.

A varying deterioration inventory model for defective and repairable items is developed. Three tire pricing policy is adopted. Three tire price and time related demand function is considered. Stock outs are not permitted. Model is justified with numerical example and post-optimality computations.

ASSUMPTIONS AND NOTATIONS

NOTATIONS

Notations used in modelare

$D(p_i,t)$: Price and time dependent demand ($a_i+b_it-\rho_ip_i$, $a>0$, $0<b_i<1$, $\rho_i>0$, $i=1,2,3$ in different intervals)

c : Unit cost of purchasing of item

p_i : Selling price per unit of item in different intervals in a cycle

d : Imperfect articles (%)

$1 - d$: % of perfect articles

d_1 : Items for repairing (in %)

d : % of perfect articles

d_1 : Items for repairing (in %)

λ : Rate of screening

SR : Revenue from sales

A : Per order cost of replenishment

z : Screening cost of one item

p_d : Selling value of imperfect quality units

$h(t)$: Storage cost of item ($x + y\,t$, $x > 0$, $0 < y < 1$)

m : Per unit cost of transportation of repairable items

μ_1 : Period of screening

T : Inventory cycle time

I(t) : At time t, inventory size

Q : Quantity required in a cycle

θ : Rate of deterioration in $t_1 \le t \le t_2$, $0 < \theta < 1$

θt : Rate of deterioration in $t_2 \le t \le T$, $0 < \theta < 1$

π : Items per unit profit.

ASSUMPTIONS

Model is based on assumptions

- Demand of item is function of time and prices.
- Product has infinite and instantaneous replenishment rate.
- There is zero lead time.
- There are no shortages of items.
- During screening process, demand occurs but is less than screening rate (λ) i.e. $(a_i+b_it-\rho_ip_i) < \lambda$.
- Deterioration of items and defective items are independent.
- Some defective items are repairable items.
- In each cycle, no repairing or replacement of deteriorated items.
- Only one item is taken for analysis.
- Varying holding cost is considered.
- Screening machine takes very less time for inspection of items for verification means we say that screening rate (λ) is sufficiently large.

THE MATHEMATICAL MODEL AND ANALYSIS

Items of amount Q are received at the starting of cycle. Out of Q units d% of items are defective items and out of these defective items, repairable items are d_1%. These units, at rate of λ per unit time as shown in figure below go through screening process during time 0 to μ_1. Items which are found to be perfect are separated and demand occurred during 0 to μ_1 will be fulfilled from these perfect quality items. Moreover from these defective items repairable items are separated and sent for repairing to manufacturer and remaining non-repairable units are sold at end of cycle at reduced price as a single batch. One cycle time is divided as $(0, t_1)$, (t_1, t_2) and (t_2, T). In period $(0, t_1)$ there is no deterioration and price is p_1, in period (t_1, t_2) deterioration rate is θ and price is p_2, in period (t_2,T) deterioration rate is θt and price is p_3, (where $p_1 > p_2 > p_3$). At end of a cycle because of deterioration and demand, level of inventory reaches to zero.

Here $\mu_1 = \dfrac{Q}{\lambda}$ (1)

And we put restriction for defective percentage (d) as:

$$d \le 1 - \frac{(a_i + b_it - \rho_ip_i)}{\lambda}. \quad (2)$$

During cycle time ($0 \le t \le T$) inventory size is as shown below:

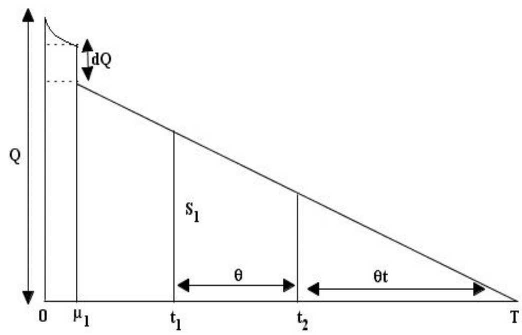

Fig-1

For period (0,T), differential equations considered are:

$$\frac{dI(t)}{dt} = -(a_1+b_1t-\rho_1p_1), \qquad 0 \le t \le t_1 \quad (3)$$

$$\frac{dI(t)}{dt} + \theta I(t) = -(a_2+b_2t-\rho_2p_2), \quad t_1 \le t \le t_2 \quad (4)$$

$$\frac{dI(t)}{dt} + \theta t I(t) = -(a_3+b_3t-\rho_3p_3), \qquad t_2 \le t \le T \quad (5)$$

conditions initially taken as: $I(0) = Q$, $I(t_1) = S_1$, $I(T) = 0$.

Respective solutions are:

$$I(t)=Q-(a_1t-\rho_1p_1t+\frac{1}{2}b_1t^2) \qquad (6)$$

$$I(t) = \begin{bmatrix} a_2(t_1-t)-\rho_2p_2(t_1-t)+\frac{1}{2}a_2\theta\left(t_1^2-t^2\right) \\ -\frac{1}{2}\rho_2p_2\theta\left(t_1^2-t^2\right)+\frac{1}{2}b_2\left(t_1^2-t^2\right)+\frac{1}{3}b_2\theta\left(t_1^3-t^3\right) \\ -a_2\theta t\left(t_1-t\right)+\rho_2p_2\theta t\left(t_1-t\right)-\frac{1}{2}b_2\theta t\left(t_1^2-t^2\right) \end{bmatrix} \qquad (7)$$

$$+S_1\left[1+\theta(t_1-t)\right]$$

$$I(t)=\begin{bmatrix} a_3(T-t)-\rho_3p_3(T-t)+\frac{1}{2}b_3\left(T^2-t^2\right) \\ +\frac{1}{6}a_3\theta\left(T^3-t^3\right)-\frac{1}{6}\rho_3p_3\theta\left(T^3-t^3\right)+\frac{1}{8}b_3\theta\left(T^4-t^4\right) \\ -\frac{1}{2}a_3\theta t^2(T-t)+\frac{1}{2}\rho_3p_3\theta t^2(T-t)-\frac{1}{4}b_3\theta t^2\left(T^2-t^2\right) \end{bmatrix} \qquad (8)$$

(Higher powers of θ are not considered)

dQ is defective items separated at time μ_1 after screening process.

Therefore between $\mu_1 \le t \le T$, effective inventory is:

$$I_1(t)=Q(1-d)-(a_1t-\rho_1p_1t+\frac{1}{2}b_1t^2). \qquad (9)$$

Substituting $t = t_1$, in (6) gives

$$Q = S_1+\left(a_1t_1-\rho_1p_1t_1+\frac{1}{2}b_1t_1^2\right). \qquad (10)$$

Under the assumption that for repairing, repairable items of amount d_1% sent to manufacturer and before completion of cycle ($t_2 \le t \le T$), we receive back items after repairing. These repaired items before completion of cycle, are sold at original price. For sending to manufacturer and receiving back items repaired causes transportation cost. Level of stock during $t_2 \le t \le T$ is as below:

$$I_3(t) = \begin{bmatrix} a_3(T-t)-\rho_3p_3(T-t)+\frac{1}{2}b_3\left(T^2-t^2\right) \\ +\frac{1}{6}a_3\theta\left(T^3-t^3\right)-\frac{1}{6}\rho_3p_3\theta\left(T^3-t^3\right)+\frac{1}{8}b_3\theta\left(T^4-t^4\right) \\ -\frac{1}{2}a_3\theta t^2(T-t)+\frac{1}{2}\rho_3p_3\theta t^2(T-t)-\frac{1}{4}b_3\theta t^2\left(T^2-t^2\right) \end{bmatrix} \qquad (11)$$

$$+ d_1Q.$$

Taking $t = t_2$ in equations (7) and (11), we get

$$I_2(t_2) = \begin{bmatrix} a_2(t_1 - t_2) - \rho_2 p_2(t_1 - t_2) + \frac{1}{2}a_2\theta(t_1^2 - t_2^2) \\ -\frac{1}{2}\rho_2 p_2\theta(t_1^2 - t_2^2) + \frac{1}{2}b_2(t_1^2 - t_2^2) + \frac{1}{3}b_2\theta(t_1^3 - t_2^3) \\ -a_2\theta t_2(t_1 - t_2) + \rho_2 p_2\theta t_2(t_1 - t_2) - \frac{1}{2}b_2\theta\mu_2(t_1^2 - t_2^2) \end{bmatrix} + S_1\left[1 + \theta(t_1 - t_2)\right] \tag{12}$$

$$I_3(t_2) = \begin{bmatrix} a_3(T - t_2) - \rho_3 p_3(T - t_2) + \frac{1}{2}b_3(T^2 - t_2^2) \\ +\frac{1}{6}a_3\theta(T^3 - t_2^3) - \frac{1}{6}\rho_3 p_3\theta(T^3 - t_2^3) + \frac{1}{8}b_3\theta(T^4 - t_2^4) \\ -\frac{1}{2}a_3\theta t_2^2(T - t_2) + \frac{1}{2}\rho_3 p_3\theta\mu_2^2(T - t_2) - \frac{1}{4}b_3\theta t_2^2(T^2 - t_2^2) \end{bmatrix} + d_1 Q. \tag{13}$$

We get from equations (12) and (13)

$$S_1 = \frac{1}{\left[1 + \theta(t_1 - t_2) - d_1\right]} \begin{bmatrix} a_3(T - t_2) - \rho_3 p_3(T - t_2) + \frac{1}{2}b_3(T^2 - t_2^2) \\ +\frac{1}{6}a_3\theta(T^3 - t_2^3) - \frac{1}{6}\rho_3 p_3\theta(T^3 - t_2^3) + \frac{1}{8}b_3\theta(T^4 - t_2^4) \\ -\frac{1}{2}a_3\theta t_2^2(T - t_2) + \frac{1}{2}\rho_3 p_3\theta\mu_2^2(T - t_2) \\ -\frac{1}{4}b_3\theta t_2^2(T^2 - t_2^2) - a_2(t_1 - t_2) + \rho_2 p_2(t_1 - t_2) \\ -\frac{1}{2}a_2\theta(t_1^2 - t_2^2) + \frac{1}{2}\rho_2 p_2\theta(t_1^2 - t_2^2) \\ -\frac{1}{2}b_2(t_1^2 - t_2^2) - \frac{1}{3}b_2\theta(t_1^3 - t_2^3) \\ +a_2\theta t_2(t_1 - t_2) - \rho_2 p_2\theta t_2(t_1 - t_2) + \frac{1}{2}b_2\theta t_2(t_1^2 - t_2^2) \\ +d_1\left(a_1 t_1 - \rho_1 p_1 t_1 + \frac{1}{2}b_1 t_1^2\right) \end{bmatrix}. \tag{14}$$

We get equation (10) by substituting value of S_1 from equation (14)

$$Q = \frac{1}{\left[1 + \theta(t_1 - t_2) - d_1\right]} \begin{bmatrix} a_3(T - t_2) - \rho_3 p_3(T - t_2) + \frac{1}{2}b_3(T^2 - t_2^2) \\ +\frac{1}{6}a_3\theta(T^3 - t_2^3) - \frac{1}{6}\rho_3 p_3\theta(T^3 - t_2^3) + \frac{1}{8}b_3\theta(T^4 - t_2^4) \\ -\frac{1}{2}a_3\theta t_2^2(T - t_2) + \frac{1}{2}\rho_3 p_3\theta\mu_2^2(T - t_2) \\ -\frac{1}{4}b_3\theta t_2^2(T^2 - t_2^2) - a_2(t_1 - t_2) + \rho_2 p_2(t_1 - t_2) \\ -\frac{1}{2}a_2\theta(t_1^2 - t_2^2) + \frac{1}{2}\rho_2 p_2\theta(t_1^2 - t_2^2) - \frac{1}{2}b_2(t_1^2 - t_2^2) \\ -\frac{1}{3}b_2\theta(t_1^3 - t_2^3) + a_2\theta t_2(t_1 - t_2) - \rho_2 p_2\theta t_2(t_1 - t_2) \\ +\frac{1}{2}b_2\theta t_2(t_1^2 - t_2^2) + d_1\left(a_1 t_1 - \rho_1 p_1 t_1 + \frac{1}{2}b_1 t_1^2\right) \end{bmatrix} + \left(a_1 t_1 - \rho_1 p_1 t_1 + \frac{1}{2}b_1 t_1^2\right). \tag{15}$$

Using (15) in (6), gives

$$I_1(t) = \frac{1}{\left[1+\theta\left(t_1-t_2\right)-d_1\right]}\left[\begin{array}{l}
a_3\left(T-t_2\right)-\rho_3 p_3\left(T-t_2\right)+\frac{1}{2}b_3\left(T^2-t_2^2\right) \\
+\frac{1}{6}a_3\theta\left(T^3-t_2^3\right)-\frac{1}{6}\rho_3 p_3\theta\left(T^3-t_2^3\right)+\frac{1}{8}b_3\theta\left(T^4-t_2^4\right) \\
-\frac{1}{2}a_3\theta t_2^2\left(T-t_2\right)+\frac{1}{2}\rho_3 p_3\theta\mu_2^2\left(T-t_2\right) \\
-\frac{1}{4}b_3\theta t_2^2\left(T^2-t_2^2\right)-a_2\left(t_1-t_2\right)+\rho_2 p_2\left(t_1-t_2\right) \\
-\frac{1}{2}a_2\theta\left(t_1^2-t_2^2\right)+\frac{1}{2}\rho_2 p_2\theta\left(t_1^2-t_2^2\right)-\frac{1}{2}b_2\left(t_1^2-t_2^2\right) \\
-\frac{1}{3}b_2\theta\left(t_1^3-t_2^3\right)+a_2\theta t_2\left(t_1-t_2\right)-\rho_2 p_2\theta t_2\left(t_1-t_2\right) \\
+\frac{1}{2}b_2\theta t_2\left(t_1^2-t_2^2\right)+d_1\left(a_1 t_1-\rho_1 p_1 t_1+\frac{1}{2}b_1 t_1^2\right)
\end{array}\right]$$
$$+\left(a_1 t_1-\rho_1 p_1 t_1+\frac{1}{2}b_1 t_1^2\right)-\left(a_1 t-\rho_1 p_1 t+\frac{1}{2}b_1 t^2\right). \qquad (16)$$

Substituting (15) in (9) gives

$$I_1(t) = \frac{(1-d)}{\left[1+\theta\left(t_1-t_2\right)-d_1\right]}\left[\begin{array}{l}
a_3\left(T-t_2\right)-\rho_3 p_3\left(T-t_2\right)+\frac{1}{2}b_3\left(T^2-t_2^2\right) \\
+\frac{1}{6}a_3\theta\left(T^3-t_2^3\right)-\frac{1}{6}\rho_3 p_3\theta\left(T^3-t_2^3\right)+\frac{1}{8}b_3\theta\left(T^4-t_2^4\right) \\
-\frac{1}{2}a_3\theta t_2^2\left(T-t_2\right)+\frac{1}{2}\rho_3 p_3\theta\mu_2^2\left(T-t_2\right) \\
-\frac{1}{4}b_3\theta t_2^2\left(T^2-t_2^2\right)-a_2\left(t_1-t_2\right) \\
+\rho_2 p_2\left(t_1-t_2\right)-\frac{1}{2}a_2\theta\left(t_1^2-t_2^2\right)+\frac{1}{2}\rho_2 p_2\theta\left(t_1^2-t_2^2\right) \\
-\frac{1}{2}b_2\left(t_1^2-t_2^2\right)-\frac{1}{3}b_2\theta\left(t_1^3-t_2^3\right)+a_2\theta t_2\left(t_1-t_2\right) \\
-\rho_2 p_2\theta t_2\left(t_1-t_2\right)+\frac{1}{2}b_2\theta t_2\left(t_1^2-t_2^2\right) \\
+d_1\left(a_1 t_1-\rho_1 p_1 t_1+\frac{1}{2}b_1 t_1^2\right)
\end{array}\right]$$
$$+(1-d)\left(a_1 t_1-\rho_1 p_1 t_1+\frac{1}{2}b_1 t_1^2\right)-\left(a_1 t-\rho_1 p_1 t+\frac{1}{2}b_1 t^2\right). \qquad (17)$$

We get equation (7) by substituting value of S_1 from equation (13)

$$I_2(t) = \frac{\left[1+\theta\left(t_1-t\right)\right]}{\left[1+\theta\left(t_1-t_2\right)-d_1\right]}\left[\begin{array}{l}
a_3\left(T-t_2\right)-\rho_3 p_3\left(T-t_2\right)+\frac{1}{2}b_3\left(T^2-t_2^2\right) \\
+\frac{1}{6}a_3\theta\left(T^3-t_2^3\right)-\frac{1}{6}\rho_3 p_3\theta\left(T^3-t_2^3\right)+\frac{1}{8}b_3\theta\left(T^4-t_2^4\right) \\
-\frac{1}{2}a_3\theta t_2^2\left(T-t_2\right)+\frac{1}{2}\rho_3 p_3\theta\mu_2^2\left(T-t_2\right) \\
-\frac{1}{4}b_3\theta t_2^2\left(T^2-t_2^2\right)-a_2\left(t_1-t_2\right)+\rho_2 p_2\left(t_1-t_2\right) \\
-\frac{1}{2}a_2\theta\left(t_1^2-t_2^2\right)+\frac{1}{2}\rho_2 p_2\theta\left(t_1^2-t_2^2\right)-\frac{1}{2}b_2\left(t_1^2-t_2^2\right) \\
-\frac{1}{3}b_2\theta\left(t_1^3-t_2^3\right)+a_2\theta t_2\left(t_1-t_2\right)-\rho_2 p_2\theta t_2\left(t_1-t_2\right) \\
+\frac{1}{2}b_2\theta t_2\left(t_1^2-t_2^2\right)+d_1\left(a_1 t_1-\rho_1 p_1 t_1+\frac{1}{2}b_1 t_1^2\right)
\end{array}\right]$$
$$+\left[\begin{array}{l}
a_2\left(t_1-t\right)-\rho_2 p_2\left(t_1-t\right)+\frac{1}{2}a_2\theta\left(t_1^2-t^2\right) \\
-\frac{1}{2}\rho_2 p_2\theta\left(t_1^2-t^2\right)+\frac{1}{2}b_2\left(t_1^2-t^2\right)+\frac{1}{3}b_2\theta\left(t_1^3-t^3\right) \\
-a_2\theta t\left(t_1-t\right)+\rho_2 p_2\theta t\left(t_1-t\right)-\frac{1}{2}b_2\theta t\left(t_1^2-t^2\right)
\end{array}\right] \qquad (18)$$

Total profit (π) consists of:

(i) Cost of ordering (OC) = A (19)

(ii) Cost of screening (SrC) = z Q (20)

(iii) Transportation cost (TC) = md_1Q (21)

(iv) $HC = \int_0^T (x + y t) I(t) dt$

$$= \int_0^{\mu_1} (x + y t) I_1(t) dt + \int_{\mu_1}^{t_1} (x + y t) I_1(t) dt + \int_{t_1}^{t_2} (x + y t) I_2(t) dt + \int_{t_2}^{T} (x + y t) I_3(t) dt \quad (22)$$

(v) $$DC = c\left(\int_{t_1}^{t_2} \theta I_2(t) dt + \int_{t_2}^{T} \theta t I_3(t) dt\right) \quad (23)$$

(vi) SR = Revenue generated during the cycle
+ Revenue from imperfect quality items
+ Revenue from repaired items

$$= \begin{pmatrix} p_1\int_0^{t_1}(a_1+b_1t-\rho_1p_1)dt \\ +p_2\int_{t_1}^{t_2}(a_2+b_2t-\rho_2p_2)dt \\ +p_3\int_{t_2}^{T}(a_3+b_3t-\rho_3p_3)dt \\ +p_d(d-d_1)Q-p(d-d_1)Q \end{pmatrix}. \quad (24)$$

(by not considering higher powers of θ)

$$\pi = \frac{1}{T}\left[SR - OC - SrC - TC - HC - DC\right] \quad (25)$$

Putting value in equation (25) from equations (19) to (24) provides overall unit profit. Moreover, it can be obtained in terms of p_1, p_2, p_3 and T using $t_1=v_1T$, $t_2=v_2T$ in (25). Taking derivative with respect to p_1, p_2, p_3, T and equating it to zero, in equation (25), gives

$$\frac{\partial\pi}{\partial p_1}=0,\ \frac{\partial\pi}{\partial p_2}=0,\ \frac{\partial\pi}{\partial p_3}=0,\ \frac{\partial\pi}{\partial T}=0, \quad (26)$$

Moreover it has to satisfy the condition

$$\begin{vmatrix} \frac{\partial\pi^2}{\partial p_1^2} & \frac{\partial\pi^2}{\partial p_1\partial p_2} & \frac{\partial\pi^2}{\partial p_1\partial p_3} & \frac{\partial\pi^2}{\partial p_1\partial T} \\ \frac{\partial\pi^2}{\partial p_2\partial p_1} & \frac{\partial\pi^2}{\partial^2 p_2^2} & \frac{\partial\pi^2}{\partial p_2\partial p_3} & \frac{\partial\pi^2}{\partial p_2\partial T} \\ \frac{\partial\pi^2}{\partial p_3\partial p_1} & \frac{\partial\pi^2}{\partial p_3\partial p_2} & \frac{\partial\pi^2}{\partial^2 p_3^2} & \frac{\partial\pi^2}{\partial p_3\partial T} \\ \frac{\partial\pi^2}{\partial T\,\partial p_1} & \frac{\partial\pi^2}{\partial T\,\partial p_2} & \frac{\partial\pi^2}{\partial T\,\partial p_3} & \frac{\partial\pi^2}{\partial^2 T^2} \end{vmatrix} > 0. \quad (27)$$

NUMERICAL EXAMPLE

Considering A= Rs.100, a_1=500, a_2=490, a_3=480, b_1=0.04, b_2=0.05, b_3=0.06, c=25, p_d = 15, d = 0.05, d_1 = 0.03, z = 0.40, m = 70, θ = 0.05, x = 5, y = 0.05, ρ_1=4, ρ_2=4.5, ρ_3=5, v_1 = 0.30, v_2 = 0.50, in suitable units. Optimal values are: p_1*=62.4150, p_2*=56.6567, p_3*=50.5989, T*=0.3921, Profit*=11750.6414 and Q* = 92.6572.

Equation (27) is also satisfied. Graphs for prices and profit are also shown below.

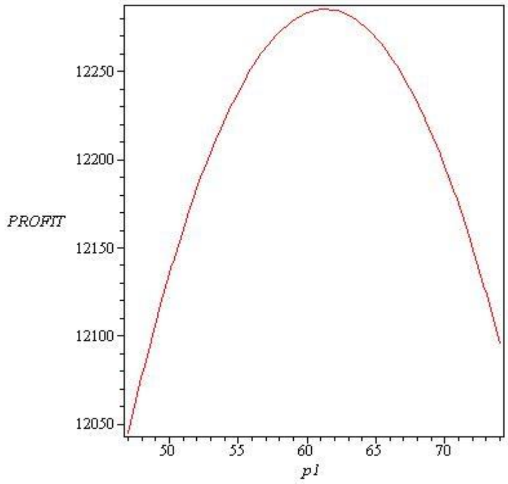

Fig-2: p_1 and Profit

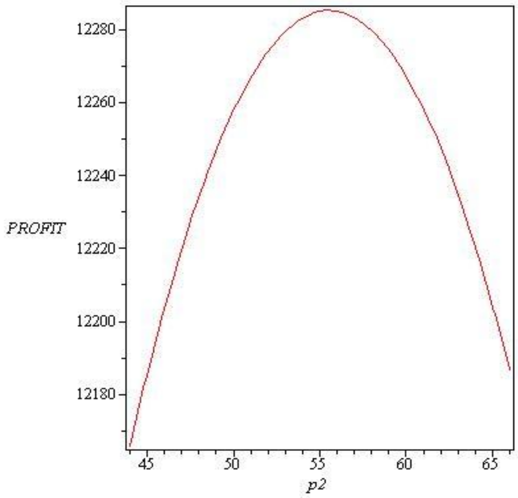

Fig-3: p_2 and Profit

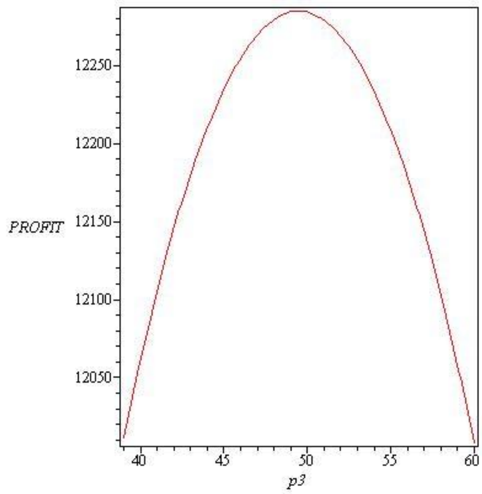

Fig-4: p_3 and Profit

POST-OPTIMALITY ANALYSIS

Study of one parameter at a time, table below gives post-optimality computations.

Table-1: Post-optimality Analysis

Parameter	%	T	p_1	p_2	p_3	Profit	Q
a_1	+20%	0.3886	74.9173	56.7822	50.7212	13690.0944	97.4917
	+10%	0.3903	68.6661	56.7195	50.6600	12674.4407	95.0750
	-10%	0.3938	56.1638	56.5940	50.5378	10918.6967	90.1902
	-20%	0.3956	49.9126	56.5314	50.4767	10178.6067	87.7206
a_2	+20%	0.3857	62.3259	67.5378	50.5845	12881.2137	94.9846
	+10%	0.3889	62.3704	62.0972	50.5916	12289.2306	93.8370
	-10%	0.3954	62.4595	51.2163	50.6063	11265.4469	91.4686
	-20%	0.3988	62.5042	45.7760	50.6139	10833.6480	90.2695
a_3	+20%	0.3647	62.1906	56.6243	60.1384	14161.9612	95.1199
	+10%	0.3776	62.3024	56.6397	55.3670	12898.3408	93.8598
	-10%	0.4083	62.5284	56.6758	45.8347	10718.9489	91.4739
	-20%	0.4269	62.6430	56.6974	41.0754	9803.3686	90.3936
x	+20%	0.3613	62.4350	56.6969	50.6710	11706.2453	85.2680
	+10%	0.3757	62.4252	56.6773	50.6357	11727.9807	88.7222
	-10%	0.4107	62.4041	56.6352	50.5604	11774.3537	97.1223
	-20%	0.4323	62.3926	56.6125	50.5200	11799.2753	102.3096
θ	+20%	0.3904	62.4134	56.6609	50.6045	11747.9225	92.3013
	+10%	0.3912	62.4142	56.6588	50.6017	11749.2812	92.4675
	-10%	0.3929	62.4157	56.6547	50.5960	11752.0031	92.8231
	-20%	0.3938	62.4165	56.6526	50.5932	11753.3664	93.0123
A	+20%	0.4291	62.4326	56.6982	50.6784	11701.9334	101.3246
	+10%	0.4110	62.4240	56.6779	50.6395	11725.7389	97.0861
	-10%	0.3721	62.4055	56.6344	50.5562	11776.8103	87.9667
	-20%	0.3511	62.3954	56.6109	50.5111	11804.4634	83.0382
ρ_1	+20%	0.3919	52.2312	56.5513	50.4929	11020.1836	92.6280
	+10%	0.3920	56.8601	56.5992	50.5411	11352.1816	92.6491
	-10%	0.3922	69.2047	56.7270	50.6695	12237.7246	92.6480
	-20%	0.3924	77.6925	56.8149	50.7579	12846.6777	92.6385
ρ_2	+20%	0.3923	62.4186	47.5828	50.5994	11306.8780	92.5458
	+10%	0.3922	62.4168	51.7073	50.5991	11508.5486	92.6016
	-10%	0.3920	62.4131	62.7060	50.5986	12046.6306	92.7129
	-20%	0.3918	62.4113	70.2678	50.5983	12416.7270	92.7447
ρ_3	+20%	0.3937	62.4272	56.6586	42.6021	10793.9478	92.5121
	+10%	0.3929	62.4211	56.6577	46.4368	11228.6539	92.3682
	-10%	0.3913	62.4089	56.6558	55.9306	12389.0033	92.7277
	-20%	0.3905	62.4028	56.6549	62.5958	13187.3791	92.7966

Calculations of Table 1 shows that increase or decrease in value of profit and order quantity occur, when parameters 'a_1', 'a_2', 'a_3' increase/decrease.

Also when parameters 'θ' and 'x' increase/ decrease then total profit and order quantity decrease/ increase. Moreover, when increase/decrease in parameters 'A' and 'ρ_1' occurs, then profit also shows decrease/increase and order quantity shows increase/ decrease. Also when parameters 'ρ_2' and 'ρ_3' increase/ decrease then profit and order quantity decrease/ increase.

SPECIAL CASE

Taking d=0, d_1=0, p_d=0, z=0, m=0 gives
T^* =0.3908, p_1^* = 62.6469, p_2^* = 54.8515, p_3^* = 48.8029, Profit*= Rs. 12606.6509.
The result corresponds with result obtained by Patel and Patel [2021].

CONCLUSION

Price and time dependent demand for imperfect quality and repairable items under three tire pricing inventory model is developed. For major parameters, post-optimality analysis is done. There will be variations in profit and order quantity with variations in parameter values.

REFERENCES

1. Aggarwal, S.P. and Goel, V.P. (1984): Order level inventory system with demand pattern for deteriorating items; Eco. Comp. Econ. Cybernet, Stud. Res., Vol. 3, pp. 57-69.
2. Alfares, H.(2007): Inventory model with stock level dependent demand rate and variable holding cost; International J. Production Economics, Vol. 108, pp. 259-265.
3. Cheng, T.C.E. (1991): An economic quantity model with demand dependent unit production cost

and imperfect production process; IIE Transactions, Vol. 23, pp. 23-28.
4. Covert, R.P. and Philip, G.C. (1973): An EOQ model for items with Weibull distribution deterioration; American Institute of Industrial Engineering Transactions, Vol. 5, pp. 323-328.
5. Ghare, P.N. and Schrader, G.F. (1963): A model for exponentially decaying inventories; J. Indus. Engg., Vol. 15, pp. 238-243.
6. Gothi, U.B., Joshi, M. and Parmar, K. (2017): An inventory model of repairable items with exponential deterioration and linear demand rate; IOSR Journal of Mathematics, Vol. 13, No. 3, pp. 75-82.
7. Goyal, S.K. and Barron, L.E.C. (2002): Note on economic production quantity model for item with imperfect quality – a practical approach; Int. J. Prod. Eco., Vol. 77, pp. 85-87.
8. Hayek, P.A., Salameh, M.K., 2001. Production lot sizing with the reworking of imperfect quality items produced. Production Planning and Control 12, 584–590.
9. Hsu, J.T. and Hsu, L.F. (2012): A note on optimal inventory model for items with imperfect quality and shortage backlogging; Int. J. Ind. Engg. Comp., Vol. 3, pp. 939-948.
10. Jaber, M.Y., Goyal, S.K. and Imran, M. (2008): Economic production quantity model for items with imperfect quality subject to learning effects; International J. Prod. Eco., Vol. 115, pp. 143-150.
11. Lee, H.L. and Rosenblatt, M.J. (1985): Optimal inspection and ordering policies for products with imperfect quality; IIE Transactions, Vol. 17, No. 3, pp. 284-289.
12. Mabini, M.C., Pintelon, L.M., & Gelders, L.F. (1992): EOQ type formulation for controlling repairable inventories; International Journal of Production Economics, Vol. 28, pp. 21-33.
13. Mathew, R.J. (2013): Perishable inventory model having mixture of Weibull lifetime and demand as function of both selling price and time; International J. of Scientific and Research Publication, Vol. 3(7), pp. 1-8.
14. Mukhopadhyay, R.N., Mukherjee, R.N. and Chaudhary, K.S. (2004): Joint pricing and ordering policy for deteriorating inventory; Computers and Industrial Engineering, Vol. 47, pp. 339-349.
15. Naik, B.T. and Patel, R. (2017): Inventory model with different deterioration rates for imperfect quality items under price and time dependent demand; Global Journal of Pure and Applied Mathematics, Vol. 13, No. 9, pp. 6907-6918.
16. Naik, B.T. and Patel, R. (2018): Imperfect Quality And Repairable Items Inventory Model With Different Deterioration Rates Under Price And Time Dependent Demand; International Journal Of Engineering Research And Development, Vol. 14, No. 7, pp. 41-48.
17. Papachristos, S. and Konstantaras, I. (2006): Economic ordering quantity models for items with imperfect quality; International J. Prod. Eco., Vol. 100(1), pp. 148-154.
18. Patel, D. M. and Patel, R.D. (2021) : Deteriorating items inventory model under different deterioration with three tired prices and time dependent demand; International J. of Science Technology and Management, Vol. 10, No. 4, pp. 148-156.
19. Patel, R. and Sheikh, S.R. (2015): Inventory Model with Different Deterioration Rates under Linear Demand and Time Varying Holding Cost; International J. Mathematics and Statistics Invention, Vol. 3, No. 6, pp. 36-42.
20. Salameh, M.K. and Jaber, M.Y. (2000): Economic production quantity model for items with imperfect quality; J. Production Eco., Vol. 64, pp. 59-64.
21. Schrady, D.A. (1967): A deterministic Inventory model for repairable items; Naval Research Logistics Quarerly, Vol. 14, No. 3, pp. 391–398.
22. Schwaller, R.L., 1988: EOQ under inspection costs. Production and Inventory Management 29, 22–24.
23. Shukla, H.S., Tripathi, R.P. and Sang, N. (2016): Economic production quantity model for defective items under deterioration; Uncertain Supply Chain Management, Vol. 4, pp. 221-232.
24. Taleizadeh, A.A., Kalantari, S.S.and Cárdenas-Barrón, L.E. (2015): Determining optimal price, replenishment lot size and number of shipments for an EPQ model with rework and multiple shipments; J. of Industrial and Management Optimization, Vol. 11, pp. 1059-1071.
25. Teng, J.T. and Chang, H.T. (2005): Economic production quantity model for deteriorating items with price and stock dependent demand; Computers and Oper. Res., Vol. 32, pp. 279-308.
26. Tsao, Y.C., Zhang, Q., Fang, H.P. and Lee, P.L. (2017): Two tired pricing and ordering for non-instantaneous deteriorating items under trade credits: Oper. Res. Int. J., DOI 10.1007/s12351-017-0306-9.
27. Yadav, R.K. and Kumar, R. (2014): An inventory model for repairable items with linear demand; IOSR Journal of Mathematics, Vol. 10, No. 6, pp. 1-6.