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The Existence of Pullback Attractors for Non-autonomous Stochastic Plate Equations with Linear Memory

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Abstract **Review Article**

Consider the existence of a non-autonomous two-dimensional stochastic plate equation with linear memory term pullback the attractor on R^2. Apply the Ornstein-Uhlenbeck process to deal with the random term, transform the original equation into a deterministic equation containing random variables, and then estimate its consistency by replacing the system solution with variables, and prove that the random dynamic system corresponding to the original system equation pullback the absorption set Existence, and finally proves the system's pullback asymptotically compaction , which leads to the existence of the pullback attractor of the original system.

Keywords: Memory term, plate equation, pull-back attractor, non-autonomous system, unbounded domain.

1. INTRODUCTION

The Plate equation is derived from the elastic vibration equations established by Woinowsky-Krieger [7] in 1950 and Berger [8] in 1955. There have been many studies on the gradual progress of the deterministic nonlinear plate equation. Among them, Carbone [9] explored the singular non-autonomous plate equation with damping term on $\Omega \subset R^n$, and verified that when the nonlinear term dissipates. There is an attractor; Baowei Feng *et al*. [11] considered a class of plate equations with time-varying delays in internal feedback. The main result is the long-term dynamics of the system. The pseudo-stability properties of the system are established and the exponential attractor is proved to be Existence. In the literature [1], the existence of the global attractor in the bounded region of the two-dimensional plate equation with linear memory term is verified.

For the random plate equation, Ma W *et al*. in the literature [12] have the existence of the attractor of the damping plate equation that can add noise; Shen X *et al*. in the literature [13] explored the randomness with memory terms and noise that can be added. Plate equation; Yao *et al*. discussed the long-term morphology of a class of non-autonomous stochastic plate equations on unbounded regions in literature [14]. What this article discusses is a kind of non-autonomous two-dimensional plate equation with linear memory term [1] after introducing random addable noise, it randomly pullback the existence of attractor on R^2.

2. PRELIMINARIES

Set (X,d) be a complete separable metric space with Borel $\sigma-$algebra $\mathrm{B}(X)$,let Ω_1 be a nonempty set, and $(\Omega_2,\mathrm{F}_2,P)$ be a probability space. Suppose there are two mappings: $\{\theta_1(t)\}_{t\in R}$ and $\{\theta_2(t)\}_{t\in R}$ acting on the Ω_1 and Ω_2 respectively. For convenience, we abbreviate $\theta_1(t)$ and $\theta_2(t)$ as $\theta_{1,t}$ and $\theta_{2,t}$, and we call both $\left(\Omega_1,\{\theta_1(t)\}_{t\in R}\right)$ and $\left(\Omega_2,\mathrm{F}_2,P,\{\theta_2(t)\}_{t\in R}\right)$ a parametric dynamical system.

Definition 1.1[2]: $\left(\Omega_1,\left\{\theta_1(t)\right\}_{t\in R}\right)$ and $\left(\Omega_2,\mathrm{F}_2,P,\left\{\theta_2(t)\right\}_{t\in R}\right)$ be a parametric dynamical system, A mapping $\Phi: R^+\times\Omega_1\times\Omega_2\times X\to X$ is called a continuous cocycle on X over $\left(\Omega_1,\left\{\theta_1(t)\right\}_{t\in R}\right)$ and $\left(\Omega_2,\mathrm{F}_2,P,\left\{\theta_2(t)\right\}_{t\in R}\right)$ if for all $\omega_1\in\Omega_1,\omega_2\in\Omega_2$ and $t,\tau\in R^+$ satisfy the following conditions:

(i). $\Phi(\cdot,\omega_1,\cdot,\cdot): R^+\times\Omega_2\times X\to X$ is $\left(\mathrm{B}(R^+)\times\mathrm{F}_2\times\mathrm{B}(X),\ \mathrm{B}(X)\right)$ measurable;

(ii). $\Phi(0,\omega_1,\omega_2,\cdot)$ is the identity on X ;

(iii). $\Phi(t+\tau,\omega_1,\omega_2,\cdot)=\Phi(t,\theta_{1,\tau}\omega_1,\theta_{2,\tau}\omega_2,\cdot)\circ\Phi(\tau,\omega_1,\omega_2,\cdot)$;

(iv). $\Phi(t,\omega_1,\omega_2,\cdot): X\to X$ is continuous.

Definition 1.2: let B and D be e two families of subsets of X which are parametrized by $(\omega_1,\omega_2)\in\Omega_1\times\Omega_2$. B = D equivalent to if $B(\omega_1,\omega_2)=D(\omega_1,\omega_2)$ for any $\omega_1\in\Omega_1,\omega_2\in\Omega_2$ .In the following, we use D to represent some non-empty subset families of X : $\mathrm{D}=\left\{D=\left\{\varnothing\neq D(\omega_1,\omega_2)\subseteq X:\omega_1\in\Omega_1,\omega_2\in\Omega_2\right\}\right\}$

Definition 1.3: $B=\left\{B(\omega_1,\omega_2):\omega_1\in\Omega_1,\omega_2\in\Omega_2\right\}$ be some families of subsets of X ,for all $\omega_1\in\Omega_1,\omega_2\in\Omega_2$ let

$$\Omega\left(B,\omega_1,\omega_2\right)=\bigcap_{\tau\geq 0}\overline{\bigcup_{t\geq\tau}\Phi(t,\theta_{1,-t}\omega_1,\theta_{2,-t}\omega_2,B(\theta_{1,-t}\omega_1,\theta_{2,-t}\omega_2,))}$$

Then call the $\left\{\Omega\left(B,\omega_1,\omega_2\right):\omega_1\in\Omega_1,\omega_2\in\Omega_2\right\}$ $\Omega-$limit set of B, write as $\Omega\left(B\right)$.

Definition 1.4: D be some families of subsets of X , $S=\left\{S(\omega_1,\omega_2):\omega_1\in\Omega_1,\omega_2\in\Omega_2\right\}\in\mathrm{D}$. We call S as $\mathrm{D}-$ pullback absorbing set for Φ if for all $\omega_1\in\Omega_1,\omega_2\in\Omega_2$, $B\in\mathrm{D}$, there exsists $T=T(B,\omega_1,\omega_2)>0$, for all $t\geq T$ such that

$$\Phi(t,\theta_{1,-t}\omega_1,\theta_{2,-t}\omega_2,B(\theta_{1,-t}\omega_1,\theta_{2,-t}\omega_2,))\subseteq S(\omega_1,\omega_2)$$

Definition 1.5: D be some families of subsets of X , we can say Φ is said to be $\mathrm{D}-$pullback asymptotically compact in X ,if for all $\omega_1\in\Omega_1,\omega_2\in\Omega_2$, the sequence $\{\Phi(t_n,\theta_{1,-t_n}\omega_1,\theta_{2,-t_n}\omega_2,x_n)\}_{n=1}^{\infty}$, when $t_n\to\infty$, $x_n\in B(\theta_{1,-t}\omega_1,\theta_{2,-t}\omega_2,)$ has a convergent subsequence in X .

Definition 1.6: D be some families of subsets of X , $\mathrm{A}\equiv\left\{\mathrm{A}\left(\omega_1,\omega_2\right):\omega_1\in\Omega_1,\omega_2\in\Omega_2\right\}$, A can be called a $\mathrm{D}-$pullback attractor for Φ if A fulfill the following conditions (i) - (iii) :

(i). A is measurable in Ω_2 , and $\mathrm{A}\left(\omega_1,\omega_2\right)$ is compact for all $\omega_1\in\Omega_1,\omega_2\in\Omega_2$;

(ii). A is invariant, that is, for every $\omega_1\in\Omega_1,\omega_2\in\Omega_2$, $\Phi\left(t,\omega_1,\omega_2,\mathrm{A}\left(\omega_1,\omega_2\right)\right)=\mathrm{A}(t,\theta_{1,t}\omega_1,\theta_{2,t}\omega_2)$ when $t\geq 0$.

(iii). A attracts every member of D , that is for $B=\left\{B(\omega_1,\omega_2):\omega_1\in\Omega_1,\omega_2\in\Omega_2\right\}\in\mathrm{D}$, for all $\omega_1\in\Omega_1,\omega_2\in\Omega_2$,there is $\lim_{t\to\infty}d\left(\Phi(t,\theta_{1,-t}\omega_1,\theta_{2,-t}\omega_2,B(\theta_{1,-t}\omega_1,\theta_{2,-t}\omega_2,)),\mathrm{A}\left(\omega_1,\omega_2\right)\right)=0.$

Suppose now $\Omega_1=R$, Define operator family $\left\{\theta_1(t)\right\}_{t\in R}$ of shift operators by, for all $t,h\in R$, $\theta_{1,t}\left(h\right)=t+h$ 。

Proposition 1.1: D be some families of subsets of X , and Φ is a a continuous cocycle on X over $\left(R,\left\{\theta_1(t)\right\}_{t\in R}\right)$ and $\left(\Omega_2,\mathrm{F}_2,P,\left\{\theta_2(t)\right\}_{t\in R}\right)$. Φ has a $\mathrm{D}-$pullback attractor A in D if and only if Φ be D-pullback asymptotically compact , has a closed measurable $\mathrm{D}-$pullback absorbing set S in D and for any $\tau \in R$, $\omega_2 \in \Omega_2$, there exsist the unique attractor A in X : $\mathrm{A}(\tau,\omega)=\Omega(S,\tau,\omega)=\bigcup_{B\in \mathrm{D}}\Omega(B,\tau,\omega)$.

Lemma 1.1: Gagliardo-Nirenberg Inequality

Let Ω be an open, bounded domain of the lipschitz class in R^n. Suppose that $1 \le p,q \le \infty, 1 \le r, 0 < \theta \le 1$, if

$$k-\frac{n}{p} \le \theta(m-\frac{n}{q})+(1-\theta)\frac{n}{r}$$

Then the following inequality holds

$$\|u\|_{k,p} \le c(\Omega)\|u\|_r^{1-\theta}\|u\|_{m,p}^{\theta}.$$

Lemma 1.2: Gronwall's lemma

Suppose that $u(t)$, $h(t)$ and $g(t)$ are three locally integrable function over $[t_0,+\infty]$, If the differential equation satisfies the following

$$\frac{du}{dt}+\gamma u(t) \le h(t),$$

Then for all $\gamma \ge 0$

$$u(t_1) \le \exp(-\gamma(t_1-t_0))u(t_0)+\int_{t_0}^{t_1}\exp(-\gamma(t-s))h(s)ds.$$

3. COCYCLES FOR NON-AUTONOMOUS STOCHASTIC PLATE EQUATIONS WITH LINEAR MEMORY ON R^2

Consider the plate equation defined on R^2 with a linear memory term and white noise

$$\rho u_{tt}+r_1r_2u_t+\phi(0)\Delta^2u-(N_1+\beta|u_x|^2)u_{xx}-(N_2+\beta|u_y|^2)u_{yy}+\int_0^\infty \phi'(s)\Delta^2u(t-s)ds$$
$$=r_1f(x,y,t)+\rho h(x,y)\frac{d\omega(t)}{dt} \tag{2.1}$$

Initial value conditions

$$u(x,y,\tau)=u^0(x,y)$$
$$u_t(x,y,\tau)=u^0{}_t(x,y) \tag{2.2}$$

Where, $r_1,r_2,N_1,N_2,\rho,\beta$ are non-negative constants , $\phi(0),\phi(\infty)>0,\phi'(s)<0,\forall s\in R^+$, In order to facilitate the processing of memory term, without loss of generality, there is $\phi(\infty)=1$, $h(x,y)$ is known functions in $H^2(R^2)$, random variables $\omega(t)$ is a two-sided real-valued Wiener process on a probability space.

Next, we define the relevant continuous stochastic dynamic system for the plate equation in (2.1). Consider the probability space (Ω,F,P) , which $\Omega=\{\omega\in C(R,R):\omega(0)=0\}$.Let F is the Borel $\sigma-$algebra induced by the compact-open topology of Ω , P the corresponding Wiener measure on (Ω,F).

Definite a group $\{\theta_{2,t}\}_{t\in R}$ acting on a probability space (Ω,F,P) , and the time shift by

$$\theta_{2,t}\omega(\cdot)=\omega(\cdot+t)-\omega(t),\omega\in\Omega,t\in R$$

Then $(\Omega,\mathrm{F},P,\{\theta_{2,t}\}_{t\in R})$ is a parametric dynamical system.

For all $\omega\in\Omega$, we consider the Ornstein-Uhlenbeck equation:

$$dz+zdt=d\omega$$

And we have $dz(\theta_{2,t}\omega)+z(\theta_{2,t}\omega)dt=d\omega$, Then we can easy to check that the random variable z have a stationary solution denote by

$$z(\omega)=-\int_{-\infty}^{0}e^{\tau}\omega(\tau)d\tau.$$

Then there exists set Ω which is $\theta_{2,t}$ invariant set of full P measure,, and the random variable $z(\theta_{2,t}\omega)$ is continuous in t for any $\omega\in\Omega$, then it is found in reference [4] [5] [6] , the variable $|z(\omega)|$ is tempered . For the sake of simplicity, we don't distinguish between Ω and Ω .

Let $\mu(s)=-\phi'(s)$, and define the following transformation

$$\eta(x,y,s)=u(x,y,t)-u(x,y,t-s)$$

Then (2.1) can be transformed into the following equations

$$\begin{cases}\rho u_{tt}+r_1r_2u_t+\Delta^2u-(N_1+\beta|u_x|^2)u_{xx}-(N_2+\beta|u_y|^2)u_{yy}+\int_0^\infty\mu(s)\Delta^2\eta ds\\ =r_1f(x,y,t)+\rho h(x,y)\dfrac{d\omega(t)}{dt}\\ \eta_t+\eta_s=u_t\end{cases}\tag{2.3}$$

Then introducing variables $m=u_t+\delta u, v=m-hz(\theta_{2,t}\omega)$, (2.1) can be transformed into the following deterministic equation with random variables by the Ornstein-Uhlenbeck Transformation

$$\begin{cases}v=u_t+\delta u-hz\\ \eta_t+\eta_s=u_t\\ \rho(v_t+\delta^2u-\delta v-\delta hz)+r_1r_2(v+hz-\delta u)+\Delta^2u-(N_1+\beta|u_x|^2)u_{xx}-(N_2+\beta|u_y|^2)u_{yy}\\ +\int_0^\infty\mu(s)\Delta^2\eta ds=r_1f(x,y,t)+\rho hz\end{cases}\tag{2.4}$$

If the initial time of the system is τ , then the initial value condition of equation (2.4) is

$$\begin{cases}u(x,y,\tau)=u^0(x,y)\\ u_t(x,y,\tau)=u^0{}_t(x,y)\\ \eta^\tau(x,y,s)=u(x,y,\tau)-u(x,y,\tau-s)=\eta^0(x,y,s)\end{cases}\tag{2.5}$$

now, $m^0=u^0{}_t(x,y)+\delta u^0(x,y)$, $v^0=m^0-hz(\tau)$。

In this paper, we make the following assumptions about the memory kernel μ function:

$(H1)$ $\mu\in C^1(R^+)\bigcap L^1(R^+), \mu'(s)\le 0$, for any $s\in R^+$;

$(H2)$ $\int_0^\infty\mu(s)ds=M>0;$

$(H3)$ $\mu'(s)+\alpha\mu'(s)\le 0$, for any $s\in R^+,\alpha>0$.

In this paper, we denote by $H=L^2(R^2), V=H_0^2(R^2)$ endowed with the inner product $(\cdot,\cdot)$, $((\cdot,\cdot))$ respectively, the norm $\|\cdot,\cdot\|$, $\|\cdot,\cdot\|_{H^2}$ respectively, and denote the $\|\cdot\|_p$ as the norm of $L^p(R^2)$.

Where, $(u,v)=\int_{R^2} u(x,y)v(x,y)dxdy$, $((u,v))=\int_{R^2} \Delta u(x,y)\Delta v(x,y)dxdy$, and we define $D(A)=\{v\in V, Av\in H\}$, $A=\Delta^2$, the operator A is assumed to be: $D(A)\to H$ are isomorphism , then there exists $\alpha>0$,such that $(Au,u)\geq\alpha\|u\|_{H^2}^2$ for all $u\in V$ .From the Poincare inequality, we have: $\|v\|_{H^2}\geq\lambda_1\|v\|$, $\forall v\in V$, where λ_1 is the first eigenvalue of $A^{1/2}$, and in order to determine the phase space, regarding the memory kernel function μ , the following weighted Hilbert space is introduced:

Let $L^2_\mu(R^+,H_0^2)$ be the Hilbert space of H_0^2 fuctions on R^+, and the inner product and norm are defined as follows

$$(\phi,\psi)_{\mu,V}=\int_0^\infty \mu(s)(\Delta\phi(s),\Delta\psi(s))ds$$

$$\|\phi\|_{\mu,V}^2=(\phi,\phi)_{\mu,V}=\int_0^\infty \mu(s)\|\phi\|_{H^2}^2\,ds$$

We denote $H_0=V\times H\times L^2_\mu(R^+,V)$, using the classic Faedo-Galerkin method [3], the solution of equation (2.4) can be well-posed in phase space H_0 under the above assumptions, that is, equation (2.4) has a continuous weak solution $w(t)=w(t+\tau,\tau,\theta_{2,-\tau}\omega,w_0):\left(u(t,\tau,\omega),v(t,\tau,\omega),\eta^{(t,\tau,\omega)}\right)$, in phase space that depends on the initial value $w_0:\left(u^0,v^0,\eta^0\right)$.

Next we can define a cocycle $\Phi:R^+\times R\times\Omega\times H_0\to H_0$, and we let

$$\Phi(t,\tau,\omega,w_0)=w(t+\tau,\tau,\theta_{2,-\tau}\omega,w_0) \tag{2.6}$$

Let B be a random bounded subset of H_0 , and denote by $\|B\|=\sup_{\varphi\in B}|\varphi|$. Let $D=\{D(\tau,\omega):\tau\in R,\omega\in\Omega\}$ is a family of subsets as B , and satisfying to be inclusion-closed and tempered, that is

$$\lim_{s\to-\infty} e^{\lambda s}\|D(t+s,\theta_{2,s}\omega)\|_{H_0}^2=0 \tag{2.7}$$

Where λ is a non-negative constant. Let $D_\lambda=\{D=\{D(\tau,\omega):\tau\in R,\omega\in\Omega\}\}$. It can be seen D_λ is tempered from literature [2]. For the external force term, when we deriving uniform estimate of solution, the following condition will be satisfied for any $\tau\in R$, there is

$$\int_{-\infty}^{\tau} e^{\lambda s}|f(\cdot,\cdot,s)|^2\,ds<\infty \tag{2.8}$$

Then easy to get from (2.8)

$$\lim_{k\to\infty}\int_{-\infty}^{\tau}\int_{x^2+y^2\geq k^2} e^{\lambda s}|f(x,y,s)|^2\,dxdyds=0 \tag{2.9}$$

4. UNIFORM ESTIMATES OF SOLUTIONS

Next, we estimate the uniform of the solution of the equation. In this paper, we use the symbol c to represent a non-negative constant, whose value is different in different places and can be determined by context.

Lemma 3.1: If the above assumptions are true, then for any $\tau\in R,\omega\in\Omega$, $D=\{D(\tau,\omega):\tau\in R,\omega\in\Omega\}\in D_\lambda$, and there exist time $T=T(\tau,\omega,D)>0$, for the solution of equation (2.4) for all $t>T$, there are

$$\begin{aligned}&\|v(\tau,\tau-t,\theta_{2,-\tau}\omega,v_{\tau-t})\|^2+\|u(\tau,\tau-t,\theta_{2,-\tau}\omega,u_{\tau-t})\|_{H^2}^2+\|\eta(\tau,\tau-t,\theta_{2,-\tau}\omega,\eta_{\tau-t})\|_{\mu,V}^2\leq R(\omega)\\&\int_{\tau-t}^{\tau} e^{c_0(s-\tau)}E(s)ds\leq R(\omega)\end{aligned} \tag{3.1}$$

Where $E(s)=\|v(s)\|^2+\|\Delta u(s)\|^2+\|u_x(s)\|^2+\|u_y(s)\|^2+\|u_x(s)\|_4^4+\|u_y(s)\|_4^4+\|\eta\|_{\mu,V}^2$, $R(\omega)$ is tempered.

Proof. Taking $\delta\in(0,\delta_0),\delta_0=\min\left\{\frac{r_1r_2}{4\rho},\frac{\lambda_1^2}{2r_1r_2},\frac{r_1r_2}{\rho+1}\right\}$.Taking the inner product of the third formula in equation (2.4) with v in $L^2(R^2)$, we can get that

$$\begin{aligned}&\frac{1}{2}\frac{d}{dt}\left\{\rho\|v\|^2+\|\Delta u\|^2+N_1\|u_x\|^2+N_2\|u_y\|^2+\frac{\beta}{6}\|u_x\|_4^4+\frac{\beta}{6}\|u_y\|_4^4\right\}+I_1+N_1\delta\|u_x\|^2+N_2\delta\|u_y\|^2\\&+\frac{\beta\delta}{3}\|u_x\|_4^4+\frac{\beta\delta}{3}\|u_y\|_4^4+(\eta,v)_{\mu,V}-(\Delta u,\Delta hz(\theta_{2,t}\omega))-N_1(u_x,h_xz(\theta_{2,t}\omega))-N_2(u_y,h_yz(\theta_{2,t}\omega))\\&-\frac{\beta}{3}(|u_x|^3,z(\theta_{2,t}\omega)h_x)-\frac{\beta}{3}(|u_y|^3,h_yz(\theta_{2,t}\omega))=r_1(f,v)\end{aligned}\tag{3.2}$$

Where $I_1=-\delta\rho\|v\|^2+r_1r_2\|v\|^2+\delta\|\Delta u\|^2-(r_1r_2\delta-\delta^2\rho)(u,v)+(r_1r_2-\delta\rho-\rho)(hz(\theta_{2,t}\omega),v).$

Then take the value of δ into the calculation to get

$$\begin{aligned}I_1&=(r_1r_2-\delta\rho)\|v\|^2+\delta\|\Delta u\|^2-(r_1r_2\delta-\delta^2\rho)(u,v)+(r_1r_2-\delta\rho-\rho)(hz(\theta_{2,t}\omega),v)\\&\ge\left(r_1r_2-\delta\rho-\frac{r_1r_2-\delta\rho}{4}-\frac{\delta\rho}{4}\right)\|v\|^2+\left(\delta-\delta^2\frac{r_1r_2-\delta\rho}{\lambda_1^2}\right)\|\Delta u\|^2-\frac{(r_1r_2-\delta\rho-\rho)^2}{\delta\rho}\|hz(\theta_{2,t}\omega)\|^2\\&\ge\frac{r_1r_2}{2}\|v\|^2+\frac{\delta}{2}\|\Delta u\|^2-\frac{(r_1r_2-\delta\rho-\rho)^2}{\delta\rho}\|hz(\theta_{2,t}\omega)\|^2.\end{aligned}\tag{3.3}$$

For memory term $(\eta,v)_{\mu,V}$

$$(\eta,v)_{\mu,V}=\frac{1}{2}\frac{d}{dt}\|\eta\|^2_{\mu,V}+(\eta,\eta_s)_{\mu,V}+\delta(\eta,u)_{\mu,V}+(\eta,-hz(\theta_{2,t}\omega))_{\mu,V}\tag{3.4}$$

It can be obtained by applying (H1-H3) and young and holder inequalities

$$\begin{aligned}(\eta,\eta_s)_{\mu,V}&=\frac{1}{2}\int_0^\infty\mu(s)\frac{d}{ds}\|\Delta\eta\|^2ds=-\frac{1}{2}\int_0^\infty\mu'(s)\|\Delta\eta\|^2ds\ge\frac{\alpha}{2}\|\eta\|_{\mu,V}^2\\\delta(\eta,u)_{\mu,V}&=\delta\int_0^\infty\mu(s)(\Delta\eta,\Delta u)ds\ge-\delta\left(\int_0^\infty\mu(s)\|\Delta\eta\|^2ds\right)^{1/2}\cdot\left(\int_0^\infty\mu(s)\|\Delta u\|^2ds\right)^{1/2}\\&\ge-\frac{\alpha}{4}\int_0^\infty\mu(s)\|\Delta\eta\|^2ds-\frac{\delta^2}{\alpha}\int_0^\infty\mu(s)\|\Delta u\|^2ds\ge-\frac{\alpha}{4}\|\eta\|_{\mu,V}^2-\frac{M\delta^2}{\alpha}\|\Delta u\|^2\\(\eta,-hz(\theta_{2,t}\omega))_{\mu,V}&=-\int_0^\infty\mu(s)(\Delta\eta,\Delta hz(\theta_{2,t}\omega))ds\ge-\frac{\alpha}{8}\|\eta\|_{\mu,V}^2-\frac{2M}{\alpha}\|\Delta hz(\theta_{2,t}\omega)\|^2\end{aligned}\tag{3.5}$$

Can be obtained from (3.5)

$$(\eta,v)_{\mu,V}\ge\frac{1}{2}\frac{d}{dt}\|\eta\|_{\mu,V}^2+\frac{\alpha}{8}\|\eta\|_{\mu,V}^2-\frac{M\delta^2}{\alpha}\|\Delta u\|^2-\frac{2M}{\alpha}\|\Delta hz(\theta_{2,t}\omega)\|^2\tag{3.6}$$

Similarly, we then use holder and young inequalities to estimate the last few terms on the left side of (3.2) inequality

$$\begin{aligned}
&-(\Delta u, \Delta h z(\theta_{2,t}\omega)) \geq -\frac{1}{\delta}\left\|\Delta h z(\theta_{2,t}\omega)\right\|^2 - \frac{\delta}{4}\|\Delta u\|^2 \\
&-N_1(u_x, h_x z(\theta_{2,t}\omega)) \geq -\frac{N_1}{2\delta}\left\|h_x z(\theta_{2,t}\omega)\right\|^2 - \frac{\delta}{2}N_1\|u_x\|^2 \\
&-N_2(u_y, h_y z(\theta_{2,t}\omega)) \geq -\frac{N_2}{2\delta}\left\|h_y z(\theta_{2,t}\omega)\right\|^2 - \frac{\delta}{2}N_2\|u_y\|^2 \\
&-\frac{\beta}{3}(|u_x|^3, h_x z(\theta_{2,t}\omega)) \geq -\frac{c\beta}{\delta}\left\|h_x z(\theta_{2,t}\omega)\right\|_4^4 - \frac{\delta\beta}{4}\|u_x\|_4^4 - \frac{c\beta}{\delta} \\
&-\frac{\beta}{3}(|u_y|^3, h_y z(\theta_{2,t}\omega)) \geq -\frac{c\beta}{\delta}\left\|h_y z(\theta_{2,t}\omega)\right\|_4^4 - \frac{\delta\beta}{4}\|u_y\|_4^4 - \frac{c\beta}{\delta}
\end{aligned} \tag{3.7}$$

To (3.2) the external force term on the right

$$r_1(f, v) \leq r_1\|f\|\|v\| \leq \frac{r_1}{r_2}\|f\|^2 + \frac{r_1 r_2}{4}\|v\|^2 \tag{3.8}$$

Denote $E(t) = \left\{\rho\|v\|^2 + \|\Delta u\|^2 + N_1\|u_x\|^2 + N_2\|u_y\|^2 + \frac{\beta}{6}\|u_x\|_4^4 + \frac{\beta}{6}\|u_y\|_4^4 + \|\eta\|^2_{\mu,V}\right\}$, then there exist a non-negative constant C be related to $r_1, r_2, N_1, N_2, \delta, \beta, M, \alpha, h$, we take

$$C = \max\left\{\frac{(r_1 r_2 - \delta\rho - \rho)^2}{\delta\rho}\|h\|^2, \frac{2M}{\alpha}\|\Delta h\|^2, \frac{1}{\delta}\|\Delta h\|^2, \frac{N_1}{2\delta}\|h_x\|^2, \frac{N_2}{2\delta}\|h_y\|^2, \frac{c\beta}{\delta}\|h_x\|_4^4, \frac{c\beta}{\delta}\|h_y\|_4^4, \frac{c\beta}{\delta}\right\}$$

We can get the following formula from (3.1)-(3.8):

$$\begin{aligned}
&\frac{d}{dt}E(t) + \frac{r_1 r_2}{2}|v|^2 + \delta\left(\frac{1}{2} - \frac{2M\delta}{\alpha}\right)\|\Delta u\|^2 + \delta N_1\|u_x\|^2 + \delta N_2\|u_y\|^2 + \frac{\delta\beta}{6}\|u_x\|_4^4 + \frac{\delta\beta}{6}\|u_y\|_4^4 + \frac{\alpha}{4}\|\eta\|^2_{\mu,V} \\
&\leq \frac{2r_1}{r_2}\|f\|^2 + C\left(1 + \left\|z\left(\theta_{2,t}\omega\right)\right\|^2 + \left\|z\left(\theta_{2,t}\omega\right)\right\|_4^4\right)
\end{aligned} \tag{3.9}$$

Take the appropriate value of δ so that $\frac{1}{2} - \frac{2M\delta}{\alpha} > \frac{1}{4}$, then we take $c_0 = \frac{1}{2}\min\left\{\frac{\delta}{4}, \frac{r_1 r_2}{2\rho}, \frac{\alpha}{8}\right\}$, there are

$$\frac{d}{dt}E(t) + 2c_0 E(t) \leq \frac{2r_1}{r_2}\|f\|^2 + C\left(1 + \left\|z\left(\theta_{2,t}\omega\right)\right\|^2 + \left\|z\left(\theta_{2,t}\omega\right)\right\|_4^4\right) \tag{3.10}$$

Multiply (3.10) by $e^{c_0(t-\tau)}$ and integrate on $(\tau - t, \tau)$

$$\begin{aligned}
&E(\tau) + c_0\int_{\tau-t}^{\tau} e^{c_0(s-\tau)}E(s)ds \leq e^{-c_0 t}E_{\tau-t} + e^{-c_0\tau}\frac{2r_1}{r_2}\int_{-\infty}^{\tau} e^{c_0 s}\|f(s)\|^2 ds \\
&+C\int_{\tau-t}^{\tau} e^{c_0(s-\tau)}\left(1 + \left\|z(\theta_{2,s-\tau}\omega)\right\|^2 + \left\|z(\theta_{2,s-\tau}\omega)\right\|_4^4\right)ds \\
&\leq e^{-c_0 t}E_{\tau-t} + e^{-c_0\tau}\frac{2r_1}{r_2}\int_{-\infty}^{\tau} e^{c_0 s}\|f(s)\|^2 ds + C\int_{-\infty}^{0} e^{c_0 s}\left(1 + \left\|z(\theta_{2,s}\omega)\right\|^2 + \left\|z(\theta_{2,s}\omega)\right\|_4^4\right)ds
\end{aligned} \tag{3.11}$$

Take $E_{\tau-t} \in D(\tau - t, \theta_{2,-t}\omega)$, there is

$$\limsup_{t\to\infty} e^{-c_0 t}\left|E_{\tau-t}\right|^2 \leq \limsup_{t\to\infty} e^{-c_0 t}\left\|D(\tau - t, \theta_{2,-t}\omega)\right\|^2_{H_0} = 0 \tag{3.12}$$

Therefore there exists time $T = T(\tau,\omega,D) > 0$ so that for all $t > T$, $e^{-c_0 t}E_{\tau-t} < 1$, let $R(\omega) = 1 + e^{-c_0 t}E_{\tau-t} + e^{-c_0\tau}\frac{2r_1}{r_2}\int_{-\infty}^{\tau} e^{c_0 s}\|f(s)\|^2 ds + C\int_{-\infty}^{0} e^{c_0 s}(1+\|z(\theta_{2,s}\omega)\|^2 + \|z(\theta_{2,s}\omega)\|_4^4)ds$, Easy to check that $\lim_{t\to-\infty} e^{\lambda t}\|R(\omega)\|_{H_0} = 0$ from (2.8) etc. Then it can be known that $R(\omega)$ is tempered, which completes the proof of Theorem 3.1.

Lemma 3.2: If the above assumptions are true, then for any $\tau \in R, \omega \in \Omega$, $D = \{D(\tau,\omega): \tau \in R, \omega \in \Omega\} \in D_\lambda$, then there exist time $T = T(\tau,\omega,D) > 1$, for the solution of equation (2.4) for all $t > T$ and $\varepsilon > 0$, there are

$$\int_{\sqrt{x^2+y^2}\ge\sqrt{2}k}(|v(\tau,\tau-t,\theta_{2,-\tau}\omega,v_{\tau-t})|^2 + |\Delta u(\tau,\tau-t,\theta_{2,-\tau}\omega,u_{\tau-t})|^2 + \int_0^\infty \mu(s)|\eta(\tau,\tau-t,\theta_{2,-\tau}\omega,\eta_{\tau-t})|^2 ds)dxdy \le \varepsilon$$

Where $(v_{\tau-t},u_{\tau-t},\eta_{\tau-t}) \in D(\tau-t,\theta_{2,-t}\omega)$.

Proof. fistly, let $m(s)$ be a smooth function defined on R^+ so that $0 \le m(s) \le 1$ for all $s \in R^+$,

$$m(s) = \begin{cases} 0 & \text{when } 0 \le s \le 1 \\ 1 & \text{when } s \ge 2 \end{cases} \tag{3.13}$$

Then there exists a positive constant c so that $m'(s) \le c$, $m''(s) \le c$, taking the inner product of the third formula in equation (2.4) with $m\left(\frac{x^2+y^2}{k^2}\right)v$ in $L^2(R^2)$, and denote $c_1 = r_1r_2 - \rho\delta, c_2 = r_1r_2 - \rho\delta - \rho$, then we can get that

$$\begin{aligned}
&\frac{1}{2}\frac{d}{dt}\int_{R^2}\rho m(\frac{x^2+y^2}{k^2})v^2dxdy - \delta c_1\int_{R^2} m(\frac{x^2+y^2}{k^2})uvdxdy + c_1\int_{R^2} m(\frac{x^2+y^2}{k^2})v^2dxdy \\
&+c_2\int_{R^2} m(\frac{x^2+y^2}{k^2})hzvdxdy + \int_{R^2} m(\frac{x^2+y^2}{k^2})v\Delta^2udxdy - N_1\int_{R^2} m(\frac{x^2+y^2}{k^2})u_{xx}vdxdy \\
&-\beta\int_{R^2} m(\frac{x^2+y^2}{k^2})|u_x|^2u_{xx}vdxdy - N_2\int_{R^2} m(\frac{x^2+y^2}{k^2})u_{yy}vdxdy - \beta\int_{R^2} m(\frac{x^2+y^2}{k^2})|u_y|^2u_{yy}vdxdy \\
&\int_0^\infty \mu(s)\int_{R^2} m(\frac{x^2+y^2}{k^2})v\Delta^2\eta dxdyds = r_1\int_{R^2} m(\frac{x^2+y^2}{k^2})fvdxdy
\end{aligned} \tag{3.14}$$

For similarity and (3.3), the left side of (3.14) can be estimated

$$\begin{aligned}
\left|-\delta c_1\int_{R^2} m(\frac{x^2+y^2}{k^2})uvdxdy\right| &\ge -c_1\delta^2\int_{R^2} m(\frac{x^2+y^2}{k^2})u^2dxdy - \frac{c_1}{4}\int_{R^2} m(\frac{x^2+y^2}{k^2})v^2dxdy \\
&\ge -\frac{c_1\delta^2}{\lambda_1^2}\int_{R^2} m(\frac{x^2+y^2}{k^2})(\Delta u)^2dxdy - \frac{c_1}{4}\int_{R^2} m(\frac{x^2+y^2}{k^2})v^2dxdy
\end{aligned} \tag{3.15}$$

$$\left|c_2\int_{R^2} m(\frac{x^2+y^2}{k^2})hzvdxdy\right| \ge -\frac{c_2^2}{\delta\rho}\int_{R^2} m(\frac{x^2+y^2}{k^2})(hz)^2dxdy - \frac{\delta\rho}{4}\int_{R^2} m(\frac{x^2+y^2}{k^2})v^2dxdy \tag{3.16}$$

$$\int_{R^2} m(\frac{x^2+y^2}{k^2})v\Delta^2udxdy = \int_{R^2} m(\frac{x^2+y^2}{k^2})\Delta v\Delta udxdy + \int_{R^2} m''(\frac{x^2+y^2}{k^2})\frac{2}{k^2}v\Delta udxdy \tag{3.17}$$

For (3.17), among them , it can be seen from the expression of v and the setting of the smooth function m

$$\left|\int_{R^2} m''(\frac{x^2+y^2}{k^2})\frac{2}{k^2}v\Delta udxdy\right| \ge -\left|\int_{R^2} c\frac{2}{k^2}v\Delta udxdy\right| \ge -\frac{c}{k^2}\|\Delta u\|^2 - \frac{c}{k^2}\|v\|^2 \tag{3.18}$$

$$\begin{aligned}&\int_{R^2} m(\frac{x^2+y^2}{k^2})\Delta v\Delta u dxdy=\int_{R^2} m(\frac{x^2+y^2}{k^2})\Delta(u_t+\delta u-hz)\Delta u dxdy\\&=\frac{1}{2}\frac{d}{dt}\int_{R^2} m(\frac{x^2+y^2}{k^2})(\Delta u)^2dxdy+\delta\int_{R^2} m(\frac{x^2+y^2}{k^2})(\Delta u)^2dxdy-\int_{R^2} m(\frac{x^2+y^2}{k^2})\Delta u\Delta hzdxdy\\&\geq\frac{1}{2}\frac{d}{dt}\int_{R^2} m(\frac{x^2+y^2}{k^2})(\Delta u)^2dxdy+\delta\int_{R^2} m(\frac{x^2+y^2}{k^2})(\Delta u)^2dxdy-\int_{R^2}\frac{\delta}{4} m(\frac{x^2+y^2}{k^2})(\Delta u)^2 dxdy\\&-\int_{R^2}\frac{1}{\delta} m(\frac{x^2+y^2}{k^2})(\Delta hz)^2 dxdy\end{aligned}\tag{3.19}$$

For terms containing N_1 in (3.14)

$$-N_1\int_{R^2} m(\frac{x^2+y^2}{k^2})u_{xx}vdxdy=N_1\int_{R^2} m(\frac{x^2+y^2}{k^2})u_xv_xdxdy+N_1\int_{R^2} m'(\frac{x^2+y^2}{k^2})\frac{2x}{k^2}vu_xdxdy\tag{3.20}$$

For （3.20）

$$\left|N_1\int_{R^2} m'(\frac{x^2+y^2}{k^2})\frac{2x}{k^2}vu_xdxdy\right|\geq-\frac{N_1c}{k}\int_{k\leq\sqrt{x^2+y^2}\leq\sqrt{2}k} vu_xdxdy\geq-\frac{N_1c}{k}\left(\|v\|^2+\|u_x\|^2\right)\tag{3.21}$$

$$\begin{aligned}&N_1\int_{R^2} m(\frac{x^2+y^2}{k^2})u_xv_xdxdy=N_1\int_{R^2} m(\frac{x^2+y^2}{k^2})u_x(u_t+\delta u-hz)_xdxdy\\&=\frac{1}{2}\frac{d}{dt}\int_{R^2} N_1m(\frac{x^2+y^2}{k^2})(u_x)^2dxdy+N_1\delta\int_{R^2} m(\frac{x^2+y^2}{k^2})(u_x)^2dxdy-N_1\int_{R^2} m(\frac{x^2+y^2}{k^2})u_xh_xzdxdy\\&\geq\frac{1}{2}\frac{d}{dt}\int_{R^2} N_1m(\frac{x^2+y^2}{k^2})(u_x)^2dxdy+N_1\delta\int_{R^2} m(\frac{x^2+y^2}{k^2})(u_x)^2dxdy-\int_{R^2}\frac{N_1\delta}{2} m(\frac{x^2+y^2}{k^2})(u_x)^2 dxdy\\&-\int_{R^2}\frac{N_1}{2\delta} m(\frac{x^2+y^2}{k^2})(h_xz)^2 dxdy.\end{aligned}\tag{3.22}$$

Similar to for terms containing N_2 in (3.14)

$$-N_2\int_{R^2} m(\frac{x^2+y^2}{k^2})u_{yy}vdxdy=N_2\int_{R^2} m(\frac{x^2+y^2}{k^2})u_yv_ydxdy+N_1\int_{R^2} m'(\frac{x^2+y^2}{k^2})\frac{2y}{k^2}vu_ydxdy\tag{3.23}$$

$$\left|N_2\int_{R^2} m'(\frac{x^2+y^2}{k^2})\frac{2y}{k^2}vu_ydxdy\right|\geq-\frac{N_2c}{k}\int_{k\leq\sqrt{|x|^2+|y|^2}\leq\sqrt{2}k} vu_ydxdy\geq-\frac{N_2c}{k}\left(\|v\|^2+\|u_y\|^2\right)\tag{3.24}$$

$$\begin{aligned}&N_2\int_{R^2} m(\frac{x^2+y^2}{k^2})u_yv_ydxdy=N_2\int_{R^2} m(\frac{x^2+y^2}{k^2})u_y(u_t+\delta u-hz)_ydxdy\\&=\frac{1}{2}\frac{d}{dt}\int_{R^2} N_2m(\frac{x^2+y^2}{k^2})(u_y)^2dxdy+N_2\delta\int_{R^2} m(\frac{x^2+y^2}{k^2})(u_y)^2dxdy-N_2\int_{R^2} m(\frac{x^2+y^2}{k^2})u_yh_yzdxdy\\&\geq\frac{1}{2}\frac{d}{dt}\int_{R^2} N_2m(\frac{x^2+y^2}{k^2})(u_y)^2dxdy+N_2\delta\int_{R^2} m(\frac{x^2+y^2}{k^2})(u_y)^2dxdy-\int_{R^2}\frac{N_2\delta}{2} m(\frac{x^2+y^2}{k^2})(u_y)^2 dxdy\\&-\int_{R^2}\frac{N_2}{2\delta} m(\frac{x^2+y^2}{k^2})(h_yz)^2 dxdy.\end{aligned}\tag{3.25}$$

And for (3.14) the fourth item from the left has

$$\begin{aligned}&-\beta\int_{R^2} m(\frac{x^2+y^2}{k^2})|u_x|^2 u_{xx} v dxdy\\&=\frac{\beta}{3}\int_{R^2} m(\frac{x^2+y^2}{k^2})|u_x|^3 v_x dxdy+\frac{\beta}{3}\int_{R^2} m'(\frac{x^2+y^2}{k^2})\frac{2x}{k^2}|u_x|^3 vdxdy\end{aligned} \tag{3.26}$$

For（3.26）

$$\left|\frac{\beta}{3}\int_{R^2} m'(\frac{x^2+y^2}{k^2})\frac{2x}{k^2}|u_x|^3 vdxdy\right|\ge-\left|\frac{\beta c}{k}\int_{k\le\sqrt{x^2+y^2}\le\sqrt{2}k}|u_x|^3 vdxdy\right|\ge-\frac{\beta c}{k}\|u_x\|_6^3-\frac{\beta c}{k}\|v\|^2 \tag{3.27}$$

For (3.27) by Gagliardo-Nirenberg, holder, young inequality, there are

$$-\frac{\beta c}{k}\|u_x\|_6^3-\frac{\beta c}{k}\|v\|^2\ge-\frac{\beta c}{k}\|\Delta u\|^3-\frac{\beta c}{k}\|v\|^2\ge-\frac{\beta c}{k}\|\Delta u\|^4-c-\frac{\beta c}{k}\|v\|^2 \tag{3.28}$$

$$\begin{aligned}&\frac{\beta}{3}\int_{R^2} m(\frac{x^2+y^2}{k^2})|u_x|^3 v_x dxdy=\frac{\beta}{3}\int_{R^2} m(\frac{x^2+y^2}{k^2})|u_x|^3(u_t+\delta u-hz)_x dxdy\\&=\frac{1}{2}\frac{d}{dt}\int_{R^2}\frac{\beta}{6}m(\frac{x^2+y^2}{k^2})(u_x)^4 dxdy+\frac{\beta\delta}{3}\int_{R^2} m(\frac{x^2+y^2}{k^2})(u_x)^4 dxdy-\frac{\beta}{3}\int_{R^2} m(\frac{x^2+y^2}{k^2})|u_x|^3 h_x zdxdy\\&\ge\frac{1}{2}\frac{d}{dt}\int_{R^2}\frac{\beta}{6}m(\frac{x^2+y^2}{k^2})(u_x)^4 dxdy+\frac{\beta\delta}{3}\int_{R^2} m(\frac{x^2+y^2}{k^2})(u_x)^4 dxdy-\int_{R^2}\frac{\beta\delta}{4}m(\frac{x^2+y^2}{k^2})(u_x)^4 dxdy\\&-\int_{R^2}\frac{c\beta}{\delta}m(\frac{x^2+y^2}{k^2})(h_x z)^4 dxdy.\end{aligned} \tag{3.29}$$

Similarly,

$$\begin{aligned}&-\beta\int_{R^2} m(\frac{x^2+y^2}{k^2})|u_y|^2 u_{yy} vdxdy\\&=\frac{\beta}{3}\int_{R^2} m(\frac{x^2+y^2}{k^2})|u_y|^3 v_y dxdy+\frac{\beta}{3}\int_{R^2} m'(\frac{x^2+y^2}{k^2})\frac{2y}{k^2}|u_y|^3 vdxdy\end{aligned} \tag{3.30}$$

$$\left|\frac{\beta}{3}\int_{R^2} m'(\frac{x^2+y^2}{k^2})\frac{2y}{k^2}|u_y|^3 vdxdy\right|\ge-\frac{\beta c}{k}\int_{k\le\sqrt{x^2+y^2}\le\sqrt{2}k}|u_y|^3 vdxdy\ge-\frac{\beta c}{k}\|\Delta u\|^4-c-\frac{\beta c}{k}\|v\|^2 \tag{3.31}$$

$$\begin{aligned}&\frac{\beta}{3}\int_{R^2} m(\frac{x^2+y^2}{k^2})|u_y|^3 v_y dxdy=\frac{\beta}{3}\int_{R^2} m(\frac{x^2+y^2}{k^2})|u_y|^3(u_t+\delta u-hz)_y dxdy\\&=\frac{1}{2}\frac{d}{dt}\int_{R^2}\frac{\beta}{6}m(\frac{x^2+y^2}{k^2})(u_y)^4 dxdy+\frac{\beta\delta}{3}\int_{R^2} m(\frac{x^2+y^2}{k^2})(u_y)^4 dxdy-\frac{\beta}{3}\int_{R^2} m(\frac{x^2+y^2}{k^2})|u_y|^3 h_y zdxdy\\&\ge\frac{1}{2}\frac{d}{dt}\int_{R^2}\frac{\beta}{6}m(\frac{x^2+y^2}{k^2})(u_y)^4 dxdy+\frac{\beta\delta}{3}\int_{R^2} m(\frac{x^2+y^2}{k^2})(u_y)^4 dxdy-\int_{R^2}\frac{\beta\delta}{4}m(\frac{x^2+y^2}{k^2})(u_y)^4 dxdy\\&-\int_{R^2}\frac{c\beta}{\delta}m(\frac{x^2+y^2}{k^2})(h_y z)^4 dxdy\end{aligned} \tag{3.32}$$

For memory term

$$\begin{aligned}&\int_0^\infty \mu(s)\int_{R^2} m(\frac{x^2+y^2}{k^2})v\Delta^2\eta dxdyds\\&=\int_0^\infty \mu(s)\int_{R^2} m(\frac{x^2+y^2}{k^2})\Delta v\Delta\eta dxdyds+\int_0^\infty \mu(s)\int_{R^2} m''(\frac{x^2+y^2}{k^2})\frac{2}{k^2}v\Delta\eta dxdyds\end{aligned} \tag{3.33}$$

For (3.33), similar to the previous processing methods， there are

$$\begin{aligned}&\left|\int_0^\infty \mu(s)\int_{R^2} m''(\frac{x^2+y^2}{k^2})\frac{2}{k^2}v\Delta\eta dxdyds\right|\ge-\frac{c}{k^2}\left|\int_0^\infty \mu(s)\int_{R^2} v\Delta\eta dxdyds\right|\\&\ge-\frac{c}{k^2}\left|\int_0^\infty \mu(s)\int_{R^2}(v)^2+(\Delta\eta)^2dxdyds\right|\ge-\frac{c}{k^2}\|\eta\|_{\mu,V}^2-\frac{cM}{k^2}\|v\|^2\end{aligned} \tag{3.34}$$

$$\begin{aligned}&\int_0^\infty \mu(s)\int_{R^2} m(\frac{x^2+y^2}{k^2})\Delta v\Delta\eta dxdyds=\int_0^\infty \mu(s)\int_{R^2} m(\frac{x^2+y^2}{k^2})\Delta(\eta_t+\eta_s+\delta u-hz)\Delta\eta dxdyds\\&=\frac{1}{2}\frac{d}{dt}\int_0^\infty \mu(s)\int_{R^2} m(\frac{x^2+y^2}{k^2})(\Delta\eta)^2dxdyds+\int_0^\infty \mu(s)\frac{1}{2}\frac{d}{ds}\int_{R^2} m(\frac{x^2+y^2}{k^2})(\Delta\eta)^2dxdyds\\&+\int_0^\infty \mu(s)\int_{R^2} m(\frac{x^2+y^2}{k^2})\delta\Delta u\Delta\eta dxdyds-\int_0^\infty \mu(s)\int_{R^2} m(\frac{x^2+y^2}{k^2})\Delta hz\Delta\eta dxdyds\\&\ge\frac{1}{2}\frac{d}{dt}\int_0^\infty \mu(s)\int_{R^2} m(\frac{x^2+y^2}{k^2})(\Delta\eta)^2dxdyds+\frac{\alpha}{8}\int_0^\infty \mu(s)\int_{R^2} m(\frac{x^2+y^2}{k^2})(\Delta\eta)^2dxdyds\\&-\frac{M\delta^2}{2}\int_{R^2} m(\frac{x^2+y^2}{k^2})(\Delta u)^2dxdy-\frac{2M}{\alpha}\int_{R^2} m(\frac{x^2+y^2}{k^2})(\Delta hz)^2dxdy\end{aligned} \tag{3.35}$$

For external force term

$$r_1\int_{R^2} m(\frac{x^2+y^2}{k^2})fvdxdy\le\frac{r_1}{r_2}\int_{R^2} m(\frac{x^2+y^2}{k^2})(f)^2dxdy+\frac{r_1r_2}{4}\int_{R^2} m(\frac{x^2+y^2}{k^2})(v)^2dxdy \tag{3.36}$$

It can be seen from (3.14)-(3.36) that there are non-negative constants C_1, C_2 related to $N_1, N_2, M, \delta, c, \beta, k$, such that

$$\begin{aligned}&\frac{1}{2}\frac{d}{dt}\int_{R^2} m(\frac{x^2+y^2}{k^2})\left(\rho|v|^2+|\Delta u|^2+N_1|u_x|^2+N_2|u_y|^2+\frac{\beta}{6}|u_x|^4+\frac{\beta}{6}|u_y|^4+\int_0^\infty\mu(s)(\Delta\eta)^2ds\right)dxdy\\&+\int_{R^2} m(\frac{x^2+y^2}{k^2})\{\left(c_1-c_1\delta-c_2-\frac{r_1r_2}{4}\right)|v|^2+\left(\delta-\frac{c_1\delta}{4\lambda_1^2}-\frac{\delta}{4}-\frac{M\delta^2}{2}\right)|\Delta u|^2+\frac{N_1\delta}{2}|u_x|^2+\frac{N_2\delta}{2}|u_y|^2\\&+\frac{\beta}{12}|u_x|^4+\frac{\beta}{12}|u_y|^4+\frac{\alpha}{8}\int_0^\infty\mu(s)(\Delta\eta)^2ds\}dxdy\le C_1(\|v\|^2+\|\Delta u\|^2+\|u_x\|^2+\|u_y\|^2+\|\eta\|_{\mu,V}^2)\\&+C_1\|\Delta u\|^4+C_2\left(1+\|z\|^2+\|z\|_4^4\right)+\frac{r_1}{r_2}\int_{R^2} m(\frac{x^2+y^2}{k^2})(f)^2dxdy\end{aligned} \tag{3.37}$$

From Lemma 3.1 and (3.10), it can be seen that there exist positive constant $c_0>0$ such that

$$\begin{aligned}&\frac{d}{dt}\int_{R^2} m(\frac{x^2+y^2}{k^2})\left(\rho|v|^2+|\Delta u|^2+N_1|u_x|^2+N_2|u_y|^2+\frac{\beta}{6}|u_x|^4+\frac{\beta}{6}|u_y|^4+\int_0^\infty\mu(s)(\Delta\eta)^2ds\right)dxdy\\&+c_0\int_{R^2} m(\frac{x^2+y^2}{k^2})\left(\rho|v|^2+|\Delta u|^2+N_1|u_x|^2+N_2|u_y|^2+\frac{\beta}{6}|u_x|^4+\frac{\beta}{6}|u_y|^4+\int_0^\infty\mu(s)(\Delta\eta)^2ds\right)dxdy\\&\le C_1(\|v\|^2+\|\Delta u\|^2+\|u_x\|^2+\|u_y\|^2+\|\eta\|_{\mu,V}^2)+C_1\|\Delta u\|^4+C_2\left(1+\|z\|^2+\|z\|_4^4\right)+\frac{r_1}{r_2}\int_{R^2} m(\frac{x^2+y^2}{k^2})(f)^2dxdy\end{aligned} \tag{3.38}$$

Similar to (3.11), apply Gronwall's lemma to (3.38), and then from Theorem 4.1, there are

$$\begin{aligned}
&\int_{R^2} m(\frac{x^2+y^2}{k^2})\left(\rho|v|^2+|\Delta u|^2+N_1|u_x|^2+N_2|u_y|^2+\frac{\beta}{6}|u_x|^4+\frac{\beta}{6}|u_y|^4+\int_0^\infty \mu(s)(\Delta\eta)^2\,ds\right)dxdy \\
&\le e^{-c_0 t}\int_{\tau-t}^{\tau} E_{\tau-s}ds + C_1 e^{-c_0 t}\int_{\tau-t}^{\tau} e^{c_0 s}(\|v\|^2+\|\Delta u\|^2+\|u_x\|^2+\|u_y\|^2+\|\eta\|_{\mu,V}^2)ds + C_1 e^{-c_0 t}\int_{\tau-t}^{\tau} e^{c_0 s}\|\Delta u\|^4\,ds \\
&+C_2 e^{-c_0 t}\int_{-\infty}^{\tau} e^{c_0 s}\left(1+\|z\|^2+\|z\|_4^4\right)ds+\frac{2r_1 e^{-c_0 t}}{r_2}\int_{-\infty}^{\tau} e^{c_0 s}\int_{\sqrt{x^2+y^2}\ge k}|f|^2\,dxdyds \\
&\le e^{-c_0 t}\int_{\tau-t}^{\tau} E_{\tau-s}ds + C_1 e^{-c_0 t}\int_{\tau}^{t} e^{c_0 s}R(\omega)ds + C_1 e^{-c_0 t}\int_{\tau}^{t} e^{c_0 S}R(\omega)^2 ds \\
&+C_2 e^{-c_0 t}\int_{-\infty}^{\tau} e^{c_0 s}\left(1+\|z\|^2+\|z\|_4^4\right)ds+\frac{2r_1}{r_2}\int_{-\infty}^{\tau} e^{c_0(s-t)}\int_{\sqrt{x^2+y^2}\ge k}|f|^2\,dxdyds
\end{aligned} \tag{3.39}$$

Then when t is large enough, from (2.9) (3.12) we can see that the above formula tends to 0, therefore exist time $T(\tau,\omega,D)$ for $t\ge T$ and any positive constant ε, such that

$$\begin{aligned}
&\int_{R^2} m(\frac{x^2+y^2}{k^2})\left(\rho|v(\tau,\tau-t,\theta_{2,-\tau}\omega,v_{\tau-t})|^2+|\Delta u(\tau,\tau-t,\theta_{2,-\tau}\omega,u_{\tau-t})|^2+\int_0^\infty \mu(s)|\eta(\tau,\tau-t,\theta_{2,-\tau}\omega,\eta_{\tau-t})|^2\,ds\right)dxdy \\
&\le \int_{R^2} m(\frac{x^2+y^2}{k^2})\left(\rho|v|^2+|\Delta u|^2+N_1|u_x|^2+N_2|u_y|^2+\frac{\beta}{6}|u_x|^4+\frac{\beta}{6}|u_y|^4+\int_0^\infty \mu(s)(\Delta\eta)^2\,ds\right)dxdy\le\varepsilon
\end{aligned} \tag{3.40}$$

At this point, we have completed the proof of Theorem 3.2.

5. EXISTENCE OF PULLBACK ATTRACTORS FOR STOCHASTIC PLATE EQUATIONS

Next we will prove that Φ is $D-$pullback asymptotically compact in H_0.

Lemma 4.1: From assumptions of the front, Φ is $D-$pullback asymptotically compact in H_0. That is, for all $\tau\in R,\omega\in\Omega,D\in D_\lambda$, $\Phi(\tau-t_n,\theta_{2,-t_n}\omega,w_{0,n})$ has a convergent subsequence in H_0 when $t_n\to\infty, w_{0,n}\in D(\tau-t_n,\theta_{2,-t_n}\omega,w_{0,n})$.

Proof. To prove the asymptotically compact of Φ, then it needs to be explained that for any $\varepsilon>0$, there is a finite sphere coverage with a radius not exceeding ε for the sequence $\Phi(\tau-t_n,\theta_{2,-t_n}\omega,w_{0,n})$. And we denote $Q_k=\{x,y\in R^2:|x|+|y|\le k\}$, where $k>0$, and let $Q_k^c=R^2/Q_k$. Then it can be known from Theorem 3.2, there exist $K=K\{\tau,\omega,\varepsilon\}\ge\sqrt{2}k$ and $N_1=N_1\{\tau,\omega,D,\varepsilon\}\ge 1$ so that for all $n\ge N_1$, there is

$$\left|\Phi(\tau-t_n,\theta_{2,-t_n}\omega,w_{0,n})\right|_{H_0(Q_k^c)}\le\frac{\varepsilon}{2} \tag{4.1}$$

According to lemma 3.1, there exist $N_2=N_2\{\tau,\omega,D,\varepsilon\}\ge N_1$ so that for all $n\ge N_2$, there is

$$\left|\Phi(\tau-t_n,\theta_{2,-t_n}\omega,w_{0,n})\right|_{H_0(Q_k)}\le c(\tau,\omega) \tag{4.2}$$

Where $c(\tau,\omega)$ is constant controlled by the right side of（3.11. Therefore Φ is precompact in $H_0(Q_k)$ so that there exist a finite sphere coverage with a radius not exceeding $\frac{\varepsilon}{2}$ in the $H_0(Q_k)$, and according to （4.1), $\Phi(\tau-t_n,\theta_{2,-t_n}\omega,w_{0,n})$ has a finite sphere coverage with a radius not exceeding ε in the $H_0(R^2)$, now we proved that Φ is $D-$pullback asymptotically compact in H_0.

Lemma 4.2 From assumptions of the front, the cocycle Φ determined by（2.1）-（2.2）has a unique $D-$pullback attractor $\mathrm{A}\in\mathrm{D}_\lambda$ in H_0.

Proof. According to the lemma 3.1, we know that Φ has a closed measurable $D-$pullback absorbing set, and Φ is $D-$pullback asymptotically compact in H_0 by lemma 4.1. hence allow from Proposition 2.1, we can know that the cocycle Φ has the unique $D-$pullback attractor $\mathrm{A} \in \mathrm{D}_\lambda$ in H_0.

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