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Predicting the Number of Goals in Football Matches with the Poisson distribution: Example of Spain La Liga

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Abstract **Original Research Article**

In this study, the distribution of the number of goals in the football matches played in the Spanish La Liga was examined, and the probabilities of the number of goals of the teams were calculated according to the appropriate probability distribution. Weekly goal numbers were recorded for a period including 4 seasons between the 2017-2018 season and the 2020-2021 season. The number of goals of Real Madrid, Barcelona and Atletico Madrid, which are among the most successful teams in La Liga, was analyzed during the 4 seasons in question. Using the chi-square test, it became clear that the number of team goals corresponded to the Poisson distribution. Poisson distribution λ parameter values were calculated as 1.934, 2.368 and 1.52, respectively, according to the number of goals of Real Madrid, Barcelona and Atletico Madrid teams. According to the parameter λ calculated, the probabilities of the goals scored by the teams were determined. It was concluded that the Poisson probability distribution is a suitable distribution for estimating the number of goals and probabilities in football matches.

Keywords: Poisson distribution, fit test, number of goals.

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1. INTRODUCTION

Administered by the Liga Nacional de Fútbol Profesional, is contested by 20 teams in LaLiga, with the three lowest-placed teams at the end of each season relegated to the Segunda División and replaced by the top two teams and a play-off winner in that division [1].

A total of 62 teams have competed in La Liga since its beginning. Nine teams have been crowned champions, during this time. With Real Madrid winning the title a record 34 times and Barcelona 26 times. During the 1940s Valencia, Atlético Madrid and Barcelona emerged as the strongest clubs, winning several titles. Real Madrid and Barcelona dominated the championship in the 1950s. During the 1960s and 1970s Real Madrid dominated La Liga, winning 14 titles, with Atlético Madrid winning four [2]. During the 1980s and 1990s Real Madrid was prominent in La Liga. From the 1990s onward, Barcelona has dominated La Liga winning 16 titles to date [3]. Although Real Madrid has been prominent, winning nine titles, La Liga has also seen other champions, including Atlético Madrid, Valencia, and Deportivo La Coruña.

The number of championships of the teams that became champions in La Liga is presented in Table 1 [4].

Table-1: Number of championships of teams

Team	Winners
Real Madrid	34
Barcelona	26
Atletico Madrid	11
Atletic Bilbao	8
Valencia	6
Real Sociedad	1
Deportivo La Coruna	1
Sevilla	1
Real Betis	1

According to UEFA's league coefficient rankings, La Liga has been the top league in Europe in each of the seven years from 2013 to 2019 (calculated using accumulated figures from five preceding seasons), and has led Europe for 22 of the 60 ranked years up to 2019, more than any other country. It has also produced the continent's top-rated club more times (22) than any other league in that period, including the top club in 10 of the 11 seasons between 2009 and 2019; each of these pinnacles was achieved by either Barcelona or Real Madrid. La Liga clubs have won the most UEFA Champions League (18), UEFA Europa League (13), UEFA Super Cup (15), and FIFA Club World Cup (7) titles, and its players have accumulated the highest number of Ballon d'Or awards (23), The Best FIFA Men's Player awards including FIFA World Player of the Year (19), and UEFA Men's Player of the Year awards including UEFA Club Footballer of the Year (11).

The number of goals scored by the teams competing in a football match has the Poisson distribution and the Poisson variables of these teams depend on the offensive power of a team and the defensive power of the opposing team [5]. The result of the match does not only depend on the offensive and defensive abilities of the teams. For this reason, the function used to calculate the Poisson variables should be more complex and include these variables [6, 7].

Dixon and Coles [8] have assessed the profitability of betting companies by applying the Poisson regression model to the English Premier League. Karlis and Ntzoufras have assessed the two-variable Poisson models for football and water polo competitions [9]. Rue and Sarvesen [6] and Cowder *et al*. [10] have tried to predict the results of football matches by using time series models with the Poisson distribution.

This study aimed to check the compatibility of the number of goals scored by the teams in football matches with the Poisson distribution and to predict the number of goals of the teams with the Poisson distribution.

2. MATERIAL AND METHODS

The material of the study consisted of the weekly goal numbers of Real Madrid, Barcelona and Atletico Madrid teams of the La Liga from the 2017-2018 season to the 2020-2021 season. To compile this information, the weekly goal numbers on www.sahadan.com, www.mackolik.com and www.transfermarkt.com websites were assessed [11-22].

Poisson Distribution: Let X be a discrete random variable assuming the possible values:
0, 1, . .. , n, ... If

$$P(X=x)=\frac{e^{-\lambda}\lambda^{x}}{x!},\quad x=0,1,2,\dots,n,\dots,$$

It is said that X has a Poisson distribution with parameter $\lambda>0$ [23]. The expected value (E(X)) of the Poisson distribution is calculated as follows.

$$E(X)=\sum_{x=0}^{\infty}x\frac{e^{-\lambda}\lambda^{x}}{x!}=\lambda e^{-\lambda}\sum_{x=1}^{\infty}\frac{\lambda^{x-1}}{(x-1)!}=\lambda$$

For calculating higher moments, one can also use the probability generating function

$$E(X^{n})=\sum_{x=0}^{\infty}e^{-\lambda}\frac{(\lambda x)^{x}}{x!}=e^{-\lambda(1-x)}$$

and then differentiate this identity with respect to x at the place x = 0 [24]. It get then

$$E(X)=\lambda$$
$$E[X(X-1)]=\lambda^{2}$$
$$E(X^{3})=E[X(X-1)(X-2)],\dots$$
$$\text{so that } E(X^{2})=\lambda+\lambda^{2}$$
$$Var(X)=\lambda$$

3. APPLICATION AND FINDINGS

In the Spanish La Liga between the 2017-2018 and 2020-2021 seasons, during the four years, the number of goals scored by Real Madrid, Barcelona and Atletico Madrid in the football matches is considered to be Poisson distributed. Weekly records of the number of goals in the matches played by the mentioned teams, 152 weekly reports each, are given in the table below (Table 2). The accuracy of the suggestion that the number of goals is Poisson distributed can be checked with the Chi-Square Conformance test. The compatibility test will be done separately for the distribution of the number of goals of the 3 teams in the matches.

Table-2: Number of goals by week

	Real Madrid	Barcelona	Atletico Madrid
Number of goals	**Number of weeks**	**Number of weeks**	**Number of weeks**
0	26	13	30
1	34	35	56
2	47	47	40
3	25	21	16
4	13	18	4
5	4	14	5
6	2	3	1
7	1	0	0
8	0	1	0
Total	152	152	152

H_0: Goal points are Poisson distributed
H_1: Goals points are not Poisson distributed

Average number of goals for teams

For Real Madrid,

$$\bar{X} = \frac{0*26+1*34+2*47+3*25+4*13+5*4+6*2+7*1}{152} = \frac{294}{152} = 1.934$$

Similarly, when the same calculation was made, the average of goals per game was 2.368 for Barcelona and 1.520 for Atletico Madrid. The introductory statistics on the number of goals scored in the matches played by the teams in the last 4 years (152 matches) are presented in Table 3.

Table-3: Introductory statistics of the number of goals

	N	Minimum	Maximum	Sum	Mean	Std. Error	Std. Deviation	Variance
Real Madrid	152	0	7	294	1.934	0.116	1.427	2.035
Barcelona	152	0	8	360	2.368	0.127	1.564	2.446
Atletico Madrid	152	0	6	231	1.520	0.101	1.245	1.549

Table 3 demonstrates the total number of goals, the average number of goals, standard error, standard deviation, variance, maximum and minimum values of the teams for the period of 152 matches. As the mean and variance of the teams are very close to each other, it is clear that they converge to the Poisson distribution. The graph showing the number of goals of the teams is also given in Figure 1, Figure 2 and Figure 3.

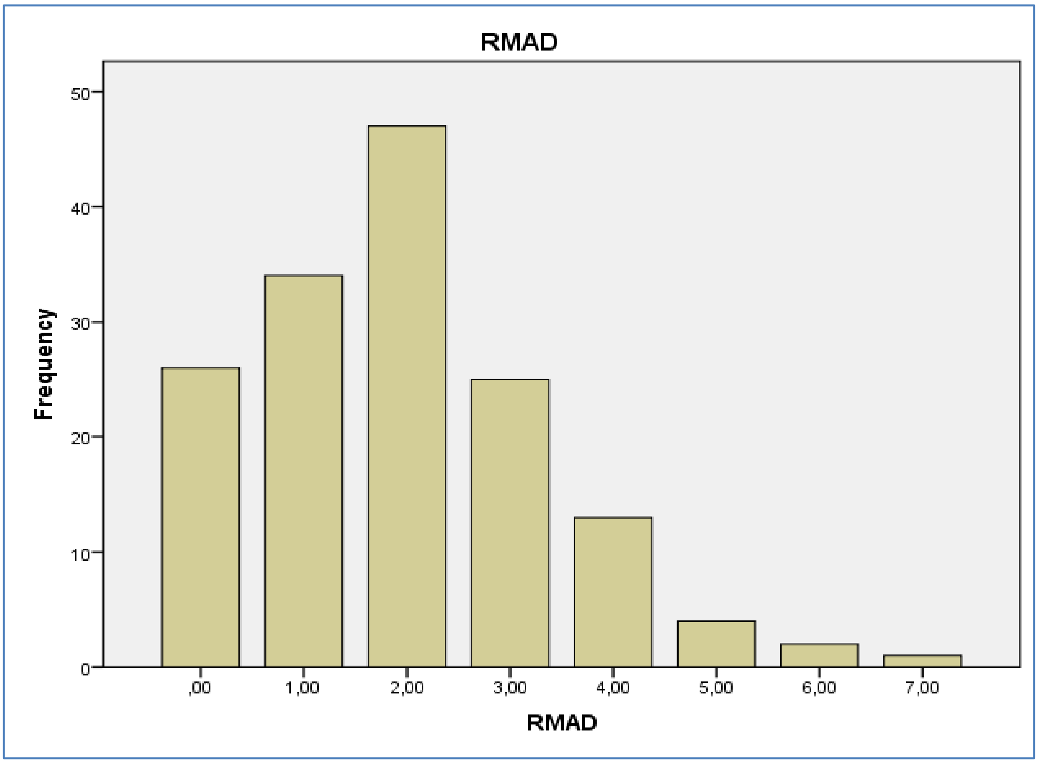

Fig-1: Real Madrid's goal numbers graph (RMAD: Real Madrid)

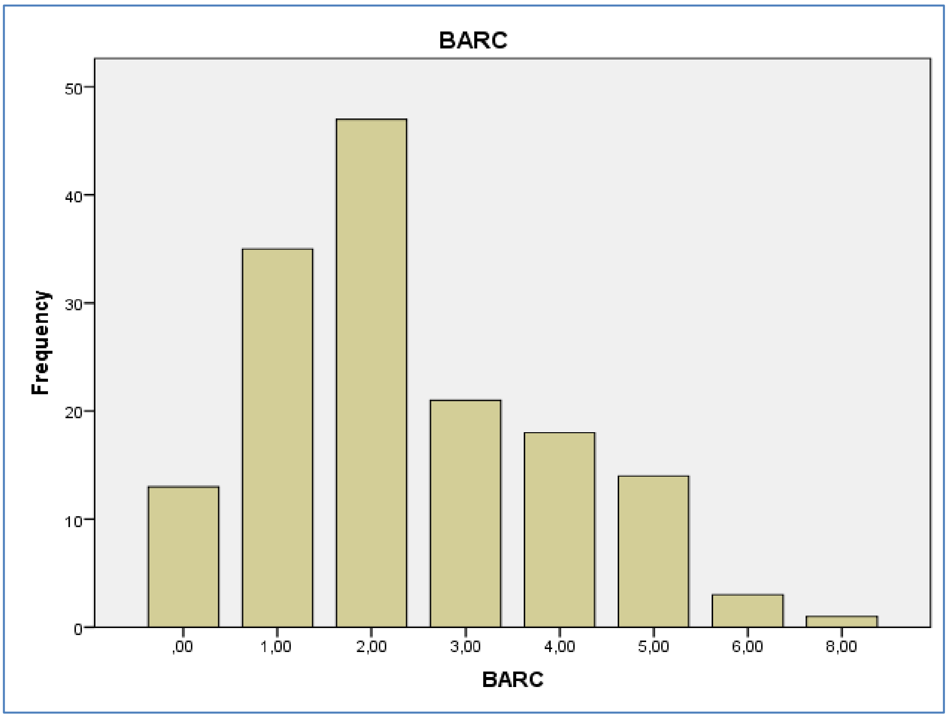

Fig-2: Barcelona's goal numbers graph (BARC: Barcelona)

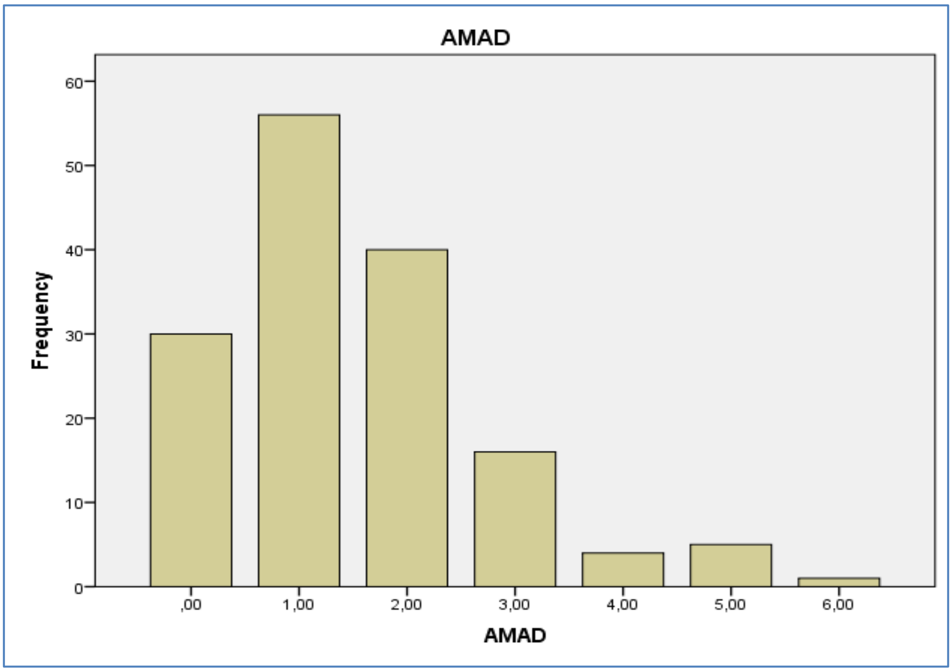

Fig-3: Atlético Madrid's goal numbers graph (AMAD: Atlético Madrid)

Poisson distribution,

$$f(x;\lambda) = P(X = x) = \frac{e^{-\lambda}\lambda^{x}}{x!}, \quad x = 0,1,2,\ldots$$

λ Parameter, the only parameter of the distribution, is the average of the distribution at the same time. The mean and variance of the Poisson distribution are equal. In this case, for $\lambda = 1.934$ which belongs to Real Madrid, the probabilities of scoring x=0,1,2,3,4,5,6 and 7 goals can be calculated.

$$P(X = 0) = \frac{e^{-1.934}(1.934^{0})}{0!} = 0.144538$$

$$P(X = 1) = \frac{e^{-1.934}(1.934^{1})}{1!} = 0.279568$$

$$P(X=2)=\frac{e^{-1.934}(1.934^2)}{2!}=0.270371$$

$$P(X=3)=\frac{e^{-1.934}(1.934^3)}{3!}=0.174318$$

$$P(X=4)=\frac{e^{-1.934}(1.934^4)}{4!}=0.084292$$

$$P(X=5)=\frac{e^{-1.934}(1.934^5)}{5!}=0.032608$$

$$P(X=6)=\frac{e^{-1.934}(1.934^6)}{6!}=0.010512$$

$$P(X=7)=\frac{e^{-1.934}(1.934^7)}{7!}=0.002905$$

The probabilities of these calculations are multiplied by 152 and the expected values (E_{NS}) are found. From here χ_o^2 statistics is calculated as follows with

$$\chi_o^2=\frac{(O_i-E_i)^2}{E_i}$$

formula. It was calculated as

$$\chi_o^2=\frac{(26-21.9698162)^2}{21.9698162}+\frac{(34-42.4942602)^2}{42,4942602}+\frac{(47-41.0964327)^2}{41.0964327}$$
$$+\frac{(25-26.4963908)^2}{26.4963908}+\frac{(13-12.8124026)^2}{12.8124026}+\frac{(4-4.95637801)^2}{4.95637801}$$
$$+\frac{(2-1.59778015)^2}{1.59778015}+\frac{(1-0.44149199)^2}{0.44149199}=4.365$$

$\chi_o^2=4.365$. Since the number of categories is k=8 and the number of parameters is m=1, the degree of freedom is sd=8-1-1=6. The Chi-Square critical value is 12.6% at the 5% significance level. Briefly, as $\chi_o^2=4.365<\chi_{6;0.05}^2=12.6$, H_0 hypothesis is accepted. It was observed that the number of goals was Poisson distributed. The necessary calculations are presented in Table 4.

Table-4: Expected frequency and Chi-square test statistics calculation (Real Madrid)

n*Oi	**Oi**	**p_i**	**Ei=152* p_i**	**Oi-Ei**	**$(Oi-Ei)^2$**	**$(Oi-Ei)^2/Ei$**
0	26	0.144538	21.9698162	4.030	16.242	0.739
34	34	0.279568	42.4942602	-8.494	72.152	1.698
94	47	0.270371	41.0964327	5.904	34.852	0.848
75	25	0.174318	26.4963908	-1.496	2.239	0.085
52	13	0.084292	12.8124026	0.188	0.035	0.003
20	4	0.032608	4.95637801	-0.956	0.915	0.185
12	2	0.010512	1.59778015	0.402	0.162	0.101
7	1	0.002905	0.44149199	0.559	0.312	0.707
294						**4.365**

p_i: Probability, n=152

$$\bar{X}=\frac{n*O_i}{152}=\frac{0+34+94+75+52+20+12+7}{152}=\frac{294}{152}=1.93421$$

Since the number of goals is under the Poisson distribution, the probabilities of scoring 1, 2, …,n or scoring more goals and scoring fewer goals by Real Madrid per match can be calculated. For example, if the probability of scoring a maximum of 2 goals is calculated,

$$P(X \leq 2) = P(X = 0) + P(X = 1) + P(X = 2) = \frac{e^{-1.934}(1.934^0)}{0!} + \frac{e^{-1.934}(1.934^1)}{1!} + \frac{e^{-1.934}(1.934^2)}{2!} = 0.424$$

The probability of scoring a maximum of 3 goals,

$$P(X \leq 3) = P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3)$$
$$= \frac{e^{-1.934}(1.934^0)}{0!} + \frac{e^{-1.934}(1.934^1)}{1!} + \frac{e^{-1.934}(1.934^2)}{2!} + \frac{e^{-1.934}(1.934^3)}{3!} = 0.869$$

is calculated as

The probability of scoring a minimum of 2 goals, that is, the probability of scoring 2 or more goals,

$$P(X \geq 2) = P(X = 2) + P(X = 3) + \cdots += 1 - P(X < 2) = 1 - [P(X = 0) + P(X = 1)]$$
$$= 1 - \frac{e^{-1.934}(1.934^0)}{0!} - \frac{e^{-1.934}(1.934^1)}{1!} = 0.576$$

and

the probability of scoring 3 or more goals (minimum 3 goals) is calculated as follows.

$$P(X \geq 3) = 1 - P(X < 3) = 1 - P(X = 0) - P(X = 1) - P(X = 2)$$

$$= 1 - \frac{e^{-1.934}(1.934^0)}{0!} - \frac{e^{-1.934}(1.934^1)}{1!} - \frac{e^{-1.934}(1.934^2)}{2!} = 0.306$$

And it is found as

Similarly, the estimation of the maximum and the minimum number of goals between 1 and 5 goals is given in Table 5.

Table-5: The probabilities of the minimum and maximum goals that Real Madrid can score

	p_i		p_i
P(X≤1)	0.424106	P(X≥1)	0.575894
P(X≤2)	0.694477	P(X≥2)	0.305523
P(X≤3)	0.868796	P(X≥3)	0.131204
P(X≤4)	0.953088	P(X≥4)	0.046912
P(X≤5)	0.985695	P(X≥5)	0.014305

According to the results in Table 5, the probabilities of scoring 2 and 3 goals by Real Madrid are 30.6% and 13.1%, respectively. The probabilities of scoring a maximum of 3 and 4 goals are 86.9% and 95.3%, respectively.

For $\lambda = 2.368$ which belongs to Barcelona, the probabilities of scoring x=0,1,2,3,4,5,6 and 8 goals can be calculated.

$$P(X = 0) = \frac{e^{-2.368}(2.368^0)}{0!} = 0.093628$$

$$P(X = 1) = \frac{e^{-2.368}(2.368^1)}{1!} = 0.221752$$

$$P(X = 2) = \frac{e^{-2.368}(2.368^2)}{2!} = 0.262601$$

$$P(X = 3) = \frac{e^{-2.368}(2.368^3)}{3!} = 0.207316$$

$$P(X = 4) = \frac{e^{-2.368}(2.368^4)}{4!} = 0.122753$$

$$P(X=5)=\frac{e^{-2.368}(2.368^5)}{5!}=0.058146$$

$$P(X=6)=\frac{e^{-2.368}(2.368^6)}{6!}=0.022952$$

$$P(X=8)=\frac{e^{-2.368}(2.368^7)}{8!}=0.007766$$

These values are multiplied by n=152 and the expected values are calculated. The χ_o^2 conformance test was calculated as above, and it was determined that the number of goals was following the Poisson distribution. The necessary calculations are presented in Table 6.

Tablo-6: Expected frequency and Chi-square test statistics calculation (Barcelona)

n*Oi	Oi	pi	Ei=152* pi	Oi-Ei	(Oi-Ei)^2	(Oi-Ei)^2/Ei
0	13	0.093628	14.2315242	-1.232	1.517	0.10657
35	35	0.221752	33.7062409	1.294	1.674	0.049659
94	47	0.262601	39.9152844	7.085	50.193	1.257493
63	21	0.207316	31.5120659	-10.512	110.504	3.506705
72	18	0.122753	18.6584597	-0.658	0.434	0.023237
70	14	0.058146	8.83821753	5.162	26.644	3.014635
18	3	0.022952	3.48877	-0.489	0.239	0.068476
8	1	0.007766	1.18041088	-0.180	0.033	0.027574
360						**8.054348**

Here n=152.

$$\bar{X}=\frac{n*O_i}{152}=\frac{0+35+94+63+72+70+18+8}{152}=\frac{360}{152}=2.368421$$

$$\chi_o^2=\frac{(13-14.2315242)^2}{14.2315242}+\frac{(35-33.7062409)^2}{33.7062409}+\frac{(47-39.9152844)^2}{39.9152844}$$
$$+\frac{(21-31.5120659)^2}{31.5120659}+\frac{(18-18.6584597)^2}{18.6584597}+\frac{(14-8.83821753)^2}{8.83821753}$$
$$+\frac{(3-3.48877)^2}{3.48877}+\frac{(1-1.18041088)^2}{1.18041088}=8.054348$$

it was calculated as,

$\chi_o^2=8.054$. Since the number of categories is k=8 and the number of parameters is m=1, the degree of freedom is sd=8-1-1=6. The Chi-Square critical value is 12.6% at the 5% significance level. Briefly, as

$\chi_o^2=8.054<\chi_{6;0.05}^2=12.6$ H_0 hypothesis is accepted. It was observed that the number of goals was Poisson distributed. Since the number of goals is per the Poisson distribution, the probabilities of scoring 1, 2, …,n or scoring more goals and scoring fewer goals by Barcelona per match can be calculated (Table 7).

Table-7: The probabilities of the minimum and maximum goals that Real Madrid can score

	p_i		p_i
P(X≤1)	0.31538	P(X≥1)	0.68462
P(X≤2)	0.577981	P(X≥2)	0.422019
P(X≤3)	0.785297	P(X≥3)	0.214703
P(X≤4)	0.90805	P(X≥4)	0.09195
P(X≤5)	0.966196	P(X≥5)	0.033804

When Table 7 is examined, the probability of scoring at least 2 goals by Barcelona is 42.2%, and the probability of scoring at least 3 goals by Barcelona is 21.5%. The probability of scoring a maximum of 4 goals by Barcelona is 90.8%.

For $\lambda=1.52$ which belongs to Atletico Madrid, the probabilities of scoring x=0,1,2,3,4,5 and 6 goals can be calculated as follows.

$$P(X=0)=\frac{e^{-1.52}(1.52^0)}{0!}=0.218769$$

$$P(X=1)=\frac{e^{-1.52}(1.52^1)}{1!}=0.332472$$

$$P(X=2)=\frac{e^{-1.52}(1.52^2)}{2!}=0.252635$$

$$P(X=3)=\frac{e^{-1.52}(1.52^3)}{3!}=0.12798$$

$$P(X=4)=\frac{e^{-1.52}(1.52^4)}{4!}=0.048624$$

$$P(X=5)=\frac{e^{-1.52}(1.52^5)}{5!}=0.014779$$

$$P(X=6)=\frac{e^{-1.52}(1.52^6)}{6!}=0.003743$$

These values are multiplied by n=152 and the expected values are calculated. The χ_o^2 conformance test was calculated as above, and it was determined that the number of goals was following the Poisson distribution. The necessary calculations are shown in Table 8.

Table-8: Expected frequency and Chi-square test statistics calculation (Atletico Madrid).

n*Oi	Oi	pi	Ei=152* pi	Oi-Ei	(Oi-Ei)^2	(Oi-Ei)^2/Ei
0	30	0.218769	33.253	-3.253	10.582	0.318218
56	56	0.332472	50.536	5.464	29.858	0.590832
80	40	0.252635	38.401	1.599	2.558	0.066623
48	16	0.12798	19.453	-3.453	11.922	0.61289
16	4	0.048624	7.391	-3.391	11.498	1.555669
25	5	0.014779	2.246	2.754	7.582	3.375235
6	1	0.003743	0.569	0.431	0.186	0.32648
231						**6.845946**

$$\chi_o^2=\frac{(30-33.253)^2}{33.253}+\frac{(56-50.536)^2}{50.536}+\frac{(40-38.401)^2}{38.401}$$

$$+\frac{(16-19.453)^2}{19.453}+\frac{(4-7.391)^2}{7.391}+\frac{(5-2.246)^2}{2.246}$$

$$+\frac{(1-0.569)^2}{0.569}=6.845946$$

It was calculated as

$\chi_o^2=6.846$. Since the number of categories is k=7 and the number of parameters is m=1, the degree of freedom is sd=7-1-1=5. The Chi-Square critical value is 12.6% at the 5% significance level. Briefly, as

$\chi_o^2=6.846<\chi^2_{5;0.05}=11.1$ H_0 hypothesis is accepted. Therefore, it was observed that the number of goals is Poisson distributed. therefore, the probability of scoring 1, 2, …,n, or more and fewer goals by Atletico Madrid per match can also be calculated (Table 9).

Table-9: The probabilities of the minimum and maximum goals that Atletico Madrid can score

	p_i		p_i
$P(X \leq 1)$	0.551154	$P(X \geq 1)$	0.448846
$P(X \leq 2)$	0.80381	$P(X \geq 2)$	0.19619
$P(X \leq 3)$	0.931822	$P(X \geq 3)$	0.068178
$P(X \leq 4)$	0.980467	$P(X \geq 4)$	0.019533
$P(X \leq 5)$	0.995255	$P(X \geq 5)$	0.004745

As we can see in Table 9, the probability of scoring at least 2 goals by Atletico Madrid is 19.6%, and the probability of scoring a maximum of 3 goals is 93.2%.

4. DISCUSSION

Unlike the method used in this study, in a previous study, the success of the prediction accuracy was investigated by using the results of football matches played during the 2013-2014 and 2014-2015 seasons of 16 football leagues from Europe and various machine learning algorithms (Naive Bayes, BayesNet, Multilayer Perceptron, Logit Boost Decision Table, ZeroR and C4.5). There was a maximum difference of 4% between the algorithms with the best and worst results. It has been reported that the classification algorithms used are not very suitable for the football dataset, since the highest result is at the level of 50-52% [25]. In another study, match results have been predicted Turkey by performing football analytics with Bayesian networks for Turkish football leagues. The retrospective and prospective performances of the model have been in the range of 60%-70% accurate predictions [26]. In the study of Çelik (2021), [27], the number of scorings in the range of 0-15, 16-30, 31-45, 46-60, 61-75 and 76-90 minutes by Beşiktaş, Fenerbahçe, Galatasaray and Trabzonspor in the Turkish Super League between 2013-2014 and 2020-2021 seasons have been estimated using the method of inhomogeneous Poisson processes.

5. CONCLUSION

In this study, the probability distribution of the number of goals of the 3 teams that have won the most of the championships in the Spanish La Liga, one of the most difficult and well-established football leagues in the world, was investigated. The number of goals in weekly matches (152 matches) in the last 4 years was taken into account. As a result of the research, it was determined by performing the Chi-Square Conformance test that the number of goals was Poisson distributed. As a result of the Poisson distribution, the average number of goals scored by Real Madrid, Barcelona and Atletico teams was 1,934, 2,368 and 1.52, respectively. The variance of the number of goals scored by these teams was determined to be 2.034, 2.446 and 1.549, respectively. Therefore, average and variance values are very close to each other. The probabilities of scoring at least 3 goals by Real Madrid, Barcelona and Atlético Madrid were calculated as 0.131, 0.215 and 0.068, respectively. It was revealed that the Poisson distribution provided good results in the estimation of the number of goals and the probability of goals in football matches.

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