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For All Symmetric and Asymmetric Perturbation Modes, Self-Gravitating Stability of a Fluid Cylinder Embedded in a Confined Liquid Pervaded by Magnetic Field

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Abstract **Review Article**

For all symmetric and asymmetric perturbation modes, the self-gravitating stability of a fluid cylinder contained in a limited liquid penetrated by magnetic field has been explored. The (MHD) basic equations are solved after the problem is formulated. The results of an analytical study of a broad eigen-value relation are confirmed quantitatively. The stability of a fluid cylinder is investigated under the influence of self-gravitating, inertial, and electromagnetic forces. In ax symmetric states, the electromagnetic force has both stabilizing and destabilizing effects. The model is entirely stable for all wavelength values when the magnetic field strength is quite high. As we have superposed gas-oil layer combination fluids, this phenomenon is of interest both academically and during geological drilling in the earth's crust. When the model's destabilizing activity is decreased and inhibited, the model's stability behavior emerges.

Keywords: Hydromagnetic; Stability; Selfgravitating.

INTRODUCTION

The purpose of this study is to look into the hydro-magnetic (MHD) stability of an annular fluid jet. Chandrasekhar [4] demonstrated the MHD stability of a complete fluid cylinder permeated by a homogeneous magnetic field. Kendall conducted tests to determine the stability of an annular fluid jet. Furthermore, he drew attention to the need to investigate the model's stability in general for its critical astrophysical applications. For the first time, Chandrasekhar [4] gives the classical of the capillary instability of a gas cylinder submerged in a liquid for ax symmetric perturbation. Drazin and Reid [7], Hasan [9], Elazab *et al*. [8], and Hasan [9]. For all ax symmetric and non-ax symmetric modes, the dispersion relation was valid. For all kinds of disturbance, Cheng examined the instability of a gas jet in an incompressible liquid. However, we must point out that Cheng's results [5] are not to be taken lightly. Kindall [10] used sophisticated equipment to conduct tests on the model's disintegration. H.M. Barakat The magnetohydrodynamic (MHD) stability of an oscillating fluid cylinder in the presence of a magnetic field is investigated [3]. The ax symmetric magneto-hydrodynamic (MHD) self-gravitating stability of fluid cylinder is discussed by Barakat.H. M[2]. Modes of Mehring C and Sirignano[11] Ax symmetric capillary waves on thin annular liquid sheets are discussed. The goal of this research is to determine the self-gravitating stability of a fluid cylinder contained in a confined liquid with a magnetic field for all symmetric and asymmetric perturbation modes.

The problem's formulation

Consider a gas cylinder with radius enclosed by a bounded liquid with radius and a cylindrical shape. A homogeneous magnetic field pervades both of the fluids. The gas density is assumed to be, whereas the bounded liquid density is assumed to be. The gas and liquid are assumed to be incompressible, non-viscous, completely conducting, and flowing at velocity = (0, 0, U). This self-gravitating medium surrounds a bounded gas-core liquid jet. The self-gravitating, electromagnetic field and the pressure gradient force operate on each of the two fluids. Only self-gravitating force acts on the surrounding medium. These equations are suitable for the task at hand. Are Provided by

$$\rho^{(i)}\left[\frac{\partial u^{(i)}}{\partial t}+\left(u^{(i)}.\nabla\right)u^{(i)}\right]=\mu\left(\nabla\wedge H^{(i)}\right)\wedge H^{(i)}+\rho^{(i)}\nabla V^{(i)}-\nabla P^{(i)} \qquad (1)$$

$$\nabla^2 V^{(i)}=-4\pi G\rho^{(i)} \qquad (2)$$

$$\nabla.u^{(i)}=0\ ,\qquad \nabla.H^{(i)}=0 \qquad (3)$$

$$\frac{\partial H^{(i)}}{\partial t}=\left(H^{(i)}.\nabla\right)\ u^{(i)}-\left(u^{(i)}.\nabla\right)H^{(i)} \qquad (4)$$

$$\nabla=\left(\frac{\partial}{\partial r},\frac{1}{r}\frac{\partial}{\partial\varphi},\frac{\partial}{\partial z}\right),\qquad (i)=(1),(2) \qquad (5)$$

We have a gas-core bounded liquid in the medium surrounding it.

$$\nabla^2 V^{(i)}=0 \qquad (6)$$

The gas medium has a superscript (1), while the liquid medium has a superscript (2) Where (is the magnetic field intensity, (the self-gravitating potential, and the self-gravitating constant, and (and (are the fluid velocity vector and kinetic pressure, respectively. For the unperturbed state, the system of basic equations (1)-(6) is solved. As a result, the kinetic pressures of the magnetic, surface, and fluids are related.

$$\text{by}P_0^{(1)}=\rho^{(1)}\,V_0^{(1)}-\frac{\mu}{2}(\alpha H_0)^2+C_1 \qquad (7)$$

$$P_0^{(2)}=\rho^{(2)}\,V_0^{(2)}-\frac{\mu}{2}(\alpha H_0)^2+C_2 \qquad (8)$$

Applying the balance of the pressure across the fluid interface $r=a$, and $r=qa$, we find

$$C_1=\frac{\mu}{2}(\alpha H_0)^2-\rho^{(2)}\left(2\pi Ga^2\left(\rho^{(2)}-\rho^{(1)}\right)\ln\ q+\pi Ga^2\left[(\rho^{(1)})^2+(\rho^{(2)})^2(q^2-1)\right]\right. \qquad (9)$$

$$C_2=\frac{\mu}{2}(\alpha H_0)^2+\pi Ga^2\rho^{(2)}\left(\rho^{(1)}\right)(1+\ln q^2)\text{-}\rho^{(2)}(1-q^2+\ln q^2)] \qquad (10)$$

Perturbation analysis

For small departure from the unperturbed state, every physical quantity $Q(r,\varphi,z,t)$ could be expressed as.

$$Q(r,\varphi,z,t)=Q_0(r)+\varepsilon_0(t)Q_1(r,\varphi,z)+\cdots \qquad (11)$$

Where Q stands for $u^{(i)},P^{(i)},H^{(i)},V^{(i)}$, the amplitude of perturbation $\varepsilon(t)$ at time t is

$$\varepsilon(t)=\varepsilon_0\exp(\sigma\,t) \qquad (12)$$

Where σ is the growth rate of the instability or rather the oscillation frequency if ($\sigma=iw\ with\ i=\sqrt{-1}$ **) is imaginary. The perturbed radii distances f the gas cylinder is given by**

$$r=a+\varepsilon_0 a_1\ \text{Where}\quad a_1=a\exp\left(\sigma t+i(km+m\varphi)\right) \qquad (13)$$

Where (k) is the longitudinal wave number and (m an integer) is the transverse wave number. The linearzed perturbation equation deduced from the fundamental equations (1)- (7) are given by

$$\rho^{(i)}\left[\frac{\partial u_1^{(i)}}{\partial t}+\left(u_0^{(i)}.\nabla\right)u_0^{(i)}\right]-\mu\left(H_0^{(i)}.\nabla\right)H_0^{(i)}=-\nabla P_1^{(i)}+\rho^{(i)}\nabla V_1^{(i)}-\mu\nabla\left(H_0^{(i)}.H_1^{(i)}\right) \qquad (14)$$

$$\nabla^2=V_1^{(i)}=0\,,\qquad \nabla.H_1^{(I)}=0\quad,\quad \nabla.u_1^{(I)}=0 \qquad (15)$$

$$\frac{\partial H_1^{(i)}}{\partial t}=\nabla\wedge(u_0^{(i)}\wedge H_1^{(i)})+\nabla\wedge\left(u_1^{(i)}\wedge H_0^{(i)}\right) \qquad (16)$$

$$\text{And}\qquad \nabla^2 V_1^{(3)}=0 \qquad (17)$$

Based on the linear perturbation technique and stability theory, every perturbed quantity $Q_1(r,\varphi,z,t)$ could be expressed as $\exp\big(\sigma t + i(km + m\varphi)\big)$ times an amplitude function of r,

$$\mathbf{Q_1}(\mathbf{r},\boldsymbol{\varphi},\mathbf{z},\mathbf{t}) = \mathbf{q_1}(\mathbf{r})\mathbf{exp}\big(\boldsymbol{\sigma t} + \boldsymbol{i}(\boldsymbol{km} + \boldsymbol{m\varphi})\big) \qquad (\mathbf{18})$$

By an apple to expansion (18), the relevant perturbation equation (14)-(17) are solved; finally, the non-singular solution is given by

$$\boldsymbol{u}_1^{(i)} = \frac{-(\boldsymbol{\sigma} + \boldsymbol{iku})}{((\boldsymbol{\sigma} + \boldsymbol{iku})^2 + (\boldsymbol{\Omega}_A^{(i)})^2)}\nabla\boldsymbol{\Pi}_1^{(i)} \qquad (\mathbf{19})$$

$$\boldsymbol{H}_1^{(i)} = \frac{\boldsymbol{\alpha H_0}}{(\boldsymbol{\sigma} + \boldsymbol{iku})}\frac{\partial\big(\boldsymbol{u}_1^{(i)}\big)}{\partial \boldsymbol{z}} \qquad (\mathbf{20})$$

$$\boldsymbol{\Pi}_1^{(1)} = \boldsymbol{A_1}\, \boldsymbol{I_m}(\boldsymbol{kr})\boldsymbol{a_1} \qquad (\mathbf{21})$$

$$\boldsymbol{\Pi}_1^{(2)} = \big(\boldsymbol{A_2}\, \boldsymbol{I_m}(\boldsymbol{kr}) + \boldsymbol{B_1}\, \boldsymbol{K_2}(\boldsymbol{kr})\big)\boldsymbol{a_1} \qquad (\mathbf{22})$$

$$\mathbf{V}_1^{(1)} = \boldsymbol{B_2}\, \boldsymbol{I_m}(\boldsymbol{kr})\boldsymbol{a_1} \qquad (\mathbf{23})$$

$$\mathbf{V}_1^{(2)} = \big(\boldsymbol{C_1}\, \boldsymbol{I_m}(\boldsymbol{kr}) + \boldsymbol{C_2}\boldsymbol{K_2}(\boldsymbol{kr})\big)\boldsymbol{a_1} \qquad (\mathbf{24})$$

$$\mathbf{V}_1^{(3)} = \boldsymbol{D}\, \boldsymbol{K_m}(\boldsymbol{kr})\boldsymbol{a_1} \qquad (\mathbf{25})$$

Where $I_m(kr)$ and $K_m(kr)$ are the modified Bessel function of first and second kind of order m, a while $A_1, A_2, B_1, B_2, C_1, C_2,\ and\ D$ are constant of integration to be determined by the appropriate boundary condition and $(\Omega_A^{(i)})$ is the Alfven wave frequency defined in terms of H_0 by

$$(\boldsymbol{\Omega}_A^{(i)}) = \sqrt{\frac{\boldsymbol{\mu H_0^2 k^2}}{\boldsymbol{\rho^{(i)}}}} \qquad (\mathbf{26})$$

Boundary condition

The solutions of equation for the unperturbed system and perturbed system must satisfy certain boundary condition across the fluid interface at ($r = a,\ and\ \ r = qa$). These boundary conditions are given as follows:

1) The normal component of the velocity vector $u_1^{(1)}$ must be compatible with the velocity of the gas-liquid interface particles across the surface at $(r = a)$ i.e.,

$$u_{1r}^{(1)} = \frac{\partial \boldsymbol{r}}{\partial \boldsymbol{t}}, \qquad (\mathbf{27})$$

2) The radial component $u_{1r}^{(1)}$of the gas velocity vector $u^{(1)}$ must equal that of the liquid $u_{1r}^{(2)}$ at $r = a$ i.e.,

$$u_{1r}^{(1)} = u_{1r}^{(2)} \qquad (\mathbf{28})$$

3) The normal component $(u_{1r}^{(2)})$ of the velocity vector $(u^{(2)})$ of the liquid region vanishes across the liquid-tenuous medium at $(r = qa)$ i.e.,

$$u_{1r}^{(2)} = 0 \qquad (\mathbf{29})$$

4) The self-gravitating potential of the gas and the liquid and their derivatives are continuous across the gas-liquid interface at ($r = a$) i.e.,

$$\boldsymbol{V}_1^{(1)} + \boldsymbol{R_1}\frac{\partial \boldsymbol{V}_0^{(1)}}{\partial \boldsymbol{r}} = \boldsymbol{V}_1^{(2)} + \boldsymbol{R_1}\frac{\partial \boldsymbol{V}_0^{(2)}}{\partial \boldsymbol{r}} \qquad (\mathbf{30})$$

And

$$\frac{\partial \boldsymbol{V}_1^{(1)}}{\partial \boldsymbol{r}} + \boldsymbol{R_1}\frac{\partial^2 \boldsymbol{V}_0^{(1)}}{\partial \boldsymbol{r}^2} = \frac{\partial \boldsymbol{V}_1^{(2)}}{\partial \boldsymbol{r}} + \boldsymbol{R_1}\frac{\partial^2 \boldsymbol{V}_0^{(2)}}{\partial \boldsymbol{r}^2} \qquad (\mathbf{31})$$

5) The self-gravitating potential of the liquid and tenuous medium and their derivatives must be continuous at $(r = qa)$

6) The normal component of the magnetic field must be continuous across the gas-liquid interface at $(r = a)$

Upon applying the foregoing boundary-condition at $(r = a)$ and $(r = qa)$, the constant of integration are identified as follows:

$$\boldsymbol{A_1} = \frac{-((\boldsymbol{\sigma}+\boldsymbol{iku})^2+(\boldsymbol{\Omega}_A^{(i)})^2)\boldsymbol{a}^2}{\boldsymbol{x}\grave{\boldsymbol{I}}_m(\boldsymbol{x})} \qquad \textit{(32)}$$

$$\boldsymbol{A_2} = \frac{-\boldsymbol{a}^2((\boldsymbol{\sigma}+\boldsymbol{iku})^2+(\boldsymbol{\Omega}_A^{(i)})^2)\grave{\boldsymbol{K}}_m(\boldsymbol{y})}{\boldsymbol{x}[\grave{\boldsymbol{I}}_m(\boldsymbol{x})\grave{\boldsymbol{K}}_m(\boldsymbol{y})-\grave{\boldsymbol{I}}_m(\boldsymbol{y})\grave{\boldsymbol{K}}_m(\boldsymbol{x})]} \qquad (\mathbf{33})$$

$$\boldsymbol{B_1} = \frac{\boldsymbol{a}^2((\boldsymbol{\sigma}+\boldsymbol{iku})^2+(\boldsymbol{\Omega}_A^{(i)})^2)\grave{\boldsymbol{I}}_m(\boldsymbol{y})}{\boldsymbol{x}[\grave{\boldsymbol{I}}_m(\boldsymbol{x})\grave{\boldsymbol{K}}_m(\boldsymbol{y})-\grave{\boldsymbol{I}}_m(\boldsymbol{y})\grave{\boldsymbol{K}}_m(\boldsymbol{x})]} \qquad (\mathbf{34})$$

$$B_2 = 4\Pi G[(\rho^{(1)} - \rho^{(2)})a^2 K_m(x) + (qa)^2 \rho^{(2)} K_m(y)] \quad (35)$$
$$C_1 = 4\Pi G \rho^{(2)} (qa)^2 K_m(y) \quad (36)$$
$$C_2 = 4\Pi G(\rho^{(1)} - \rho^{(2)})a^2 I_m(x) \quad (37)$$
$$D = 4\Pi G[(\rho^{(1)} - \rho^{(2)})a^2 I_m(x) + (qa)^2 \rho^{(2)} I_m(y)] \quad (38)$$

Where ($x = ka$ and $y = qx$) are the dimensionless longitudinal wave-numbers.

By resorting to the foregoing solution of equations for the unperturbed and perturbed state and by applying the compatibility condition, that the normal component of the total stress must be continuous across the gas-liquid interface at ($r = a$), the following stability criterion can be derived state

$$(\sigma + iku)^2 = F[Q + S] \quad (39)$$
$$F = \frac{x\grave{I}_m(x)[\grave{I}_m(x))\grave{K}_m(y) - \grave{K}_m(x)\grave{I}_m(y)]}{I_m(x)[\grave{I}_m(x)\grave{K}_m(y) - \grave{K}_m(x)\grave{I}_m(y)] - (\frac{\rho^{(2)}}{\rho^{(1)}})\grave{I}_m(x)[I_m(x)\grave{K}_m(y) - K_m(x)\grave{I}_m(y)]} \quad (40)$$
$$Q = 4\Pi G\rho^{(1)}(1 - (\frac{\rho^{(2)}}{\rho^{(1)}})[(1 - \left(\frac{\rho^{(2)}}{\rho^{(1)}}\right) I_m(x)K_m(x) + q^2 \left(\frac{\rho^{(2)}}{\rho^{(1)}}\right) I_m(x)K_m(x) - \frac{1}{2}] \quad (41)$$
$$S = \frac{-\mu H_0^2}{\rho^{(1)}a^2}\left(\frac{xI_m(x)}{\grave{I}_m(x)}\right) + \frac{\mu H_0^2}{\rho^{(2)}a^2}\left(\frac{\rho^{(2)}}{\rho^{(1)}}\right)\frac{x(I_m(x)\grave{K}_m(y) - K_m(x)\grave{I}_m(y)}{\left(\grave{I}_m(x)\grave{K}_m(y) - \grave{K}_m(x)\grave{I}_m(y)\right)} \quad (42)$$

DISCUSSION

Equation (39) is the desired relation of the present model of a gas cylinder volume embedded into a bounded liquid subjected to a self-gravitating, pressure-gradient and magneto-dynamic force. It relates the growth rate σ with the modified Bessel function $I_0(x)$ and $K_0(x)$ and their derivative, the wave number m and x, the amplitude U of the streaming velocity , Ω the oscillation frequency of the oscillating streaming, α is the parameter of the magnetic field in the gas cylinder and with the parameters T, ρ, a, μ and $H_0$of the problem. One has to mention here that the relation (39) contain $(\frac{-\mu H_0^2}{\rho^{(1)}a^2})$ and $(\frac{\mu H_0^2}{\rho^{(2)}a^2})$ as a unit of $(time)^{-2}$ .The denisities ($\rho^{(1)}$, $\rho^{(2)}$)of the gas and liquid region, the gas-liquid radii ratio q , the magnetic field intensity and permeability H_0 and μ, the self gravitating constant G for the gas radius a. The dispersion relation (39) is the linear combination of a of a dispersion relation of the same model being acted upon by the self-gravitating force only. In the latter work, Chandrasekhar utilized the technique of presenting the solenoidal vectors in terms of poloidal and toroidal quantities which are valid only for the m=0. This linear combination is also true if the acting force is the capillary and electromagnetic force whether the model is full fluid cylinder (Radwan and Elazab [12].

Stability Discussion

Before we discuss the ordinary stability, marginal stability and instability of the system under consideration, it is desirable to study the behaviors of the Bessel functions and also those of the compound functions contained in the relation (39).

In view of the recurrence relations (see Abramowitz and Stegun [1])

$$2\,\grave{I}_m(x) = I_{m-1}(x) + I_{m+1}(x), \quad (43)$$

$$2\,\grave{K}_m(x) = -K_{m-1}(x) - K_{m+1}(x) \quad (44)$$

Because $I_m(x)$ is monotonic increasing and positive definite $I_m(x) > 0$ for all modes of perturbation $m \geq 0$ and nonzero values of $x \neq 0$, while $K_m(x)$ is monotonically decreasing but never negative, i.e., $K_m(x) > 0$ we may show that

$$\grave{I}_m(x) > 0, \qquad \grave{K}_m(x) < 0 \quad (45)$$

Also for $m \geq 1$ for all values of $x \neq 0$, we $have$

$$2I_m(x)K_m(x) < 1 \quad (46)$$

In order to discuss the stability and instability states region of the present models, the dispersion relation (39). One has to refer here that if we suppose that ($m = 0$) the dispersion relation yields

$$(\sigma + iku)^2 = \left(\frac{x\grave{I}_0(x)\ [\grave{I}_0(x))\grave{K}_0(y) - \grave{K}_0(x)\grave{I}_0(y)]}{I_0(x)[\grave{I}_0(x)\grave{K}_0(y) - \grave{K}_0(x)\grave{I}_0(y)] - \left(\frac{\rho^{(2)}}{\rho^{(1)}}\right)\grave{I}_0(x)[I_0(x)\grave{K}_0(y) - K_0(x)\grave{I}_0(y)]} \right)$$
$$.\left(4\Pi G\rho^{(1)}(1 - \frac{\rho^{(2)}}{\rho^{(1)}}).\left[1 - \left(\frac{\rho^{(2)}}{\rho^{(1)}}\right)I_0(x)K_0(x) + q^2\left(\frac{\rho^{(2)}}{\rho^{(1)}}\right)I_0(x)K_0(x) - \frac{1}{2}\right] + \left[\frac{-\mu H_0^2}{\rho^{(1)}a^2}\left(\frac{xI_0(x)}{\grave{I}_0(x)}\right) + \frac{\mu H_0^2}{\rho^{(2)}a^2}\left(\frac{\rho^{(2)}}{\rho^{(1)}}\right)\frac{x(I_0(x)\grave{K}_0(y) - K_0(x)\grave{I}_0(y)}{\left(\grave{I}_0(x)\grave{K}_0(y) - \grave{K}_0(x)\grave{I}_0(y)\right)}\right]\right) \quad (47)$$

And we use the relation

$$\grave{K}_0(x) = -K_1(x)\,, \qquad \grave{I}_0(x) = I_1(x) \quad (48)$$

With the Wronskian relation

$$W_m\big(I_m(x), K_m(x)\big) = I_m(x)\grave{K}_m(x) - K_m(x)\grave{I}_m(x) = -x^{-1} \quad (49)$$

The dispersion relation (39) reduce to

$$(\sigma + iku)^2 = \left(\frac{xI_1(x)[K_1(x)I_1(y) - K_1(y)I_1(x)]}{I_0(x)[K_1(x)I_1(y) - K_1(y)I_1(x)] - \left(\frac{\rho^{(2)}}{\rho^{(1)}}\right)I_1(x)[K_0(x)I_1(y) - K_1(y)I_0(x)]} \right)$$
$$.\left(4\Pi G\rho^{(1)}\left(1 - \frac{\rho^{(2)}}{\rho^{(1)}}\right)\left[1 - \left(\frac{\rho^{(2)}}{\rho^{(1)}}\right)I_0(x)K_0(x) + q^2\left(\frac{\rho^{(2)}}{\rho^{(1)}}\right)I_0(x)K_0(x) - \frac{1}{2}\right] + \frac{-\mu H_0^2}{\rho^{(1)}a^2}\left(\frac{xI_0(x)}{I_1(x)}\right) + \frac{\mu H_0^2}{\rho^{(2)}a^2}\left(\frac{\rho^{(2)}}{\rho^{(1)}}\right)\frac{x[K_0(x)I_1(y) - K_1(y)I_0(x)]}{[K_1(x)I_1(y) - K_1(y)I_1(x)]} \right) \quad (50)$$

Limiting Cases

Some previous reported works could be obtained as limiting cases for the present general dispersion relation (41). A lot of approximation $(\rho^{(2)} = 0,\ H_o = 0,\ u = 0\ and\ m = 0)$ are required for the criterion (52), to obtain.

$$\sigma^2 = 4\Pi G\rho^{(1)}\left(\frac{xI_1(x)}{I_0(x)}\right)\left[I_0(x)K_0(x) - \frac{1}{2}\right] \quad (51)$$

The relation (52) was derived for the first time by Chandrsakhar and Fermi [1] using a different technique from that which is used here.

If we assume that $(\rho^{(2)} = 0,\ H_o = 0,\ u = 0\ and\ m \geq 0)$

$$\sigma^2 = 4\Pi G\rho^{(1)}\left(\frac{x\grave{I}_m(x)}{I_m(x)}\right)\left[I_m(x)K_m(x) - \frac{1}{2}\right] \quad (52)$$

This corroborates the relation derived by Chandrasekhar [3] using the normal-mode analysis .If we suppose that the fluids are not conducting and $(H_0 = 0)$ we obtain from (39)

$$(\sigma + iku)^2 = \mathrm{F\,.\,M} \quad (53)$$

Where F and M are defined by equations (40), (41). Equation (53) is the dispersion relations for the present, model under the effect only the self-gravitating force in the gas and liquid. If we suppose that t $(G = 0)$ we obtain from (39).

$$(\sigma + iku)^2 = \mathrm{F\,.\,S} \quad (54)$$

Where F and S are defined by equations (40), (43). Equation (54) is the dispersion relations for the present, model under the effect only the electromagnetic force due to the existence of the uniform magnetic field in the gas and liquid.

CONCLUSION

1) The self-gravitating force destabilizes only for a restricted range of wave numbers in the symmetric mode (m=0), but it stabilizes for all other perturbation
2) For all short and long wavelengths in all symmetric and asymmetric modes of disturbance, the electromagnetic force due to the pervading uniform

magnetic field in the gas and the liquid wave has a considerable stabilizing influence

3) The liquid-gas radii ratio has a stabilizing tendency, which holds true for all and values
4) In both symmetric and asymmetric types of perturbation, the liquid-gas densities ratio has a considerable destabilizing influence for all wavelengths**.**

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