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Quantum Information Transformation from One Particle onto another Particle by Using Hybrid and Hyper Entangled States

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Abstract **Original Research Article**

In this paper, we propose two protocols for the transfer of quantum information carried by one particle onto another particle by using the path-spin hybrid and hyper entangled states as the quantum resources. It has been shown that the double slit arrangement with a spin flipper at one of its openings can be used for the generation of path-spin hybrid and hyper entangled states. By using such path-spin entangled states as the quantum resource, the quantum information encoded in one particle is transferred to another particle via spin and path measurements.

Keywords: Hybrid entangled state; hyper entangled state; double slit; spin flipper, quantum information transformation.

1. INTRODUCTION

Quantum entanglement that is inherent in quantum mechanics is an indispensable ingredient for information processing in the field of quantum computing and quantum information theory [1]. Quantum entanglement concept was first realized in terms of position-momentum variables in the famous thought experiment [2]. By maturing the idea of quantum entanglement with the help of theory and experiment, the idea was extended to both continuous variables systems [3] and discrete variables systems [4]. With further development of the quantum entanglement idea, many interesting applications such as quantum computation [5], dense coding [6], teleportation [7] and quantum cryptography [8] have been developed with the help of spin entangled states. The ability to preserve and manipulate entangled states is the distinguishing feature of quantum computers, responsible both for their power and for the difficulty of building those [9]. Although the vast majority of quantum states are entangled, preparation and isolation of useful entangled states such as a maximally entangled Einstein-Podolsky-Rosen (EPR) pair is not trivial due to the absorption and radiation of the particles or noise in the quantum channel [10]. Also from the experimental point of view, a successful information processing protocol requires a complete Bell-state analysis (BSA) [11]. However, with only linear optical elements, a complete BSA is impossible and only 50 % optimal success probability can be achieved [12-14]. To overcome the problems related to preserve and manipulate entangled states, many entanglement purification or entanglement concentration schemes [15-19] have been proposed on one hand, and on the other hand, searching for entanglement between different degrees of freedom called hybrid and hyper entanglement has become an interesting and ongoing research area to overcome the problems related to complete Bell-state discrimination [20-26]. For example, the generation of entanglement between polarization and the linear momentum of a single photon [27] and the polarization and the angular momentum of a single photon was experimentally demonstrated [28]. The idea of creating entanglement between the path and the spin degrees of freedom for a single spin-1/2 particle was proposed [29] and such path-spin hybrid entangled states for single neutrons was experimentally realized [30]. Compared with spatially separated spin entangled systems, hybrid and hyper entangled systems are supposed to reduce the physical resources for quantum information tasks involving plenty of quantum systems [31, 32]. Besides, it is unnecessary to preserve entanglement as an information transmission channel between distant parties, which enables information processing schemes to be insensitive to decoherence [33]. In a study, an entanglement between path-spin degrees of freedom of a single spin-1/2 particle using a beam-splitter and a spin-flipper, is shown to be swapped onto and entanglement between spin-spin degrees of freedom [34].

In the current study, we propose two protocols to transfer information carried by particles onto other particles using the path-spin hybrid and hyper entangled states as the quantum channels. The mathematics of the proposed protocols are very similar to the original quantum teleportation definition [7], but the quantum channels used are different. In the proposed protocols, entangled pairs of particles are not needed to execute the information transformation task, which simplifies the process in a great extent. The physical arrangement that produces the quantum channel is a slit with two openings together with a spin flipper and a path detector.

2. Generation of hybrid and hyper entangled states via double slit arrangement:

Generation of Hybrid Entangled state

Let us consider a two slit arrangement as shown in Fig-1. The incident particle goes through either slit x or slit y. If the particle goes through slit x, a spin flipper (SF) positioned at x flips the spin of the particle i.e., from $|1\rangle$ to $|0\rangle$ or vice versa. If it goes through the slit y, however, the spin of the particle remains the same. Thus, the position of the particle will get entangled with its spin itself and the output will be an entangled state of the position and the spin of the particle. Here, $|0\rangle$ and $|1\rangle$ represent up and down spin polarized states respectively. That is

$$|\uparrow\rangle_S = |0\rangle_S = \begin{pmatrix}1\\0\end{pmatrix}, \quad |\downarrow\rangle_S = |1\rangle_S = \begin{pmatrix}0\\1\end{pmatrix}$$

When spin-1/2 particle, which is initially in the spin polarized state $|0\rangle_S$, incident on the double slit arrangement, the quantum state of the outgoing particle will be $|\phi\rangle = \frac{1}{\sqrt{2}}(|x1\rangle + |y0\rangle)$. Such hybrid entangled state can be used as the quantum resource for quantum information transfer from one particle to another. Thus we see that when spin-1/2 particle incident on a double slit arrangement shown in Fig-1, generates hybrid entangled states between its spin and path. Now, our goal is to use such path-spin hybrid entangled state as a resource for the transfer of quantum information from one particle to another particle.

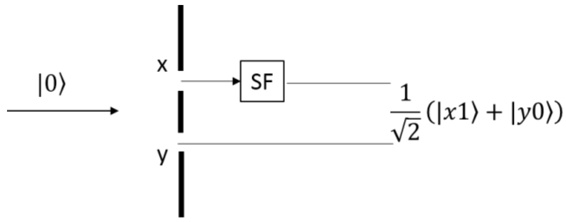

Figure 1: Generation of hybrid entangled state. SF represents spin flipper

Generation of hyper entangled states

If particle B is positioned at one of the slits (say x) and particle A is allowed to incident on the double slit, then, the spin of particle B gets entangled with the path of particle A. In the schematic diagram shown in Fig.-2, we position particle B at slit x. We also have a path detector (PD) at slit y to detect the path of the particle A. If particle A goes through slit x, it activate spin flipper which flips the spin of particle B. If it goes through slit y the spin of particle B remains the same. At the same time due to the PD, the path of the incident particle A is measured. But PD does not do anything to the spin of particle A. Thus, the spin of particle B gets entangled with the path of particle A. Such kind of entanglement is known as hyper entanglement and the entangled state is known as hyper entangled state.

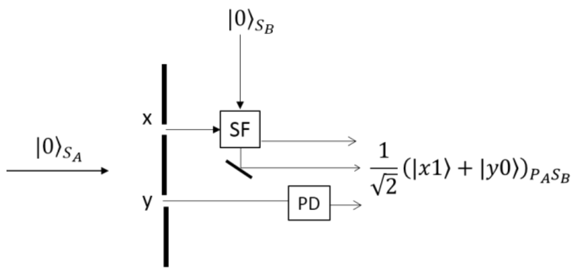

Figure 2: Generation of hyper entangled state. SF and PD represent spin flipper and path detector respectively. SF is activated only when particle A incident on it and then it flips the spin of particle B. Here, P_A and S_B stand for path and spin of particles A and B respectively

3. Transfer of quantum information from one particle to another particle*:*

Let us consider the quantum information encoded in the quantum state of a spin-1/2 particle A is given by

$|\psi\rangle = (\alpha|0\rangle + \beta|1\rangle)_{S_A}$……………… (1)

In which coefficients α & β are arbitrary and unknown satisfying the normalization condition $|\alpha|^2 + |\beta|^2 = 1$. We want to transfer this quantum information onto another spin-1/2 particle B.

We propose two different protocols using hybrid entangled states and hyper entangled state as the quantum resource.

Protocol-I: In this protocol, the following steps are involved-

(i) The particle B is allowed to pass through the double slit arrangement shown in Fig-1. The path and spin of the particle B gets entangled. If particle B is initially in the spin polarized state $|0\rangle_{S_B}$, then, the output state will be

$|\phi\rangle = \frac{1}{\sqrt{2}}(|x1\rangle + |y0\rangle)_{P_B S_B}$………………… (2)

Here, P_B and S_B stand for path and spin of particle B, respectively. This hybrid entangled state of the particle 'B' can itself be used as the quantum resource to transfer the information encoded in particle A to the particle B. The combined state of the particles A and B is

$|\chi\rangle = |\psi\rangle_{S_A} \otimes \frac{1}{\sqrt{2}}(|x1\rangle + |y0\rangle)_{P_B S_B} = \frac{1}{\sqrt{2}}(\alpha|0x1\rangle + \alpha|0y0\rangle + \beta|1x1\rangle + \beta|1y0\rangle)_{S_A\, P_B S_B}$………. (3)

(ii) The CNOT gate operation is performed with spin of particle A as control qubit and spin of particle B as target qubit.

$|\chi\rangle = \frac{1}{\sqrt{2}}(\alpha|0x0\rangle + \alpha|0y1\rangle + \beta|1x0\rangle + \beta|1y1\rangle)_{S_A\, P_B S_B}$……………. (4)

(iii) The hadamard operation is performed on the spin state of particle A.

After this step, the combined state of particles A and B becomes

$$|\chi\rangle = \frac{1}{2}\begin{bmatrix} |0\rangle_{S_A}\{|x\rangle_{P_B}(\alpha|1\rangle + \beta|0\rangle)_{S_B} + |y\rangle_{P_B}(\alpha|0\rangle + \beta|1\rangle)_{S_B}\} \\ +|1\rangle_{S_A}\{|x\rangle_{P_B}(\alpha|1\rangle - \beta|0\rangle)_{S_B} + |y\rangle_{P_B}(\alpha|0\rangle - \beta|1\rangle)_{S_B}\} \end{bmatrix}$$
…………….. (5)

(iii) The measurement is performed on the spin of particle A and the path of particle B.

(iv) Depending upon the spin and path measurements of particles A and B respectively, a suitable unitary transformation should be done on the spin state of particle B to restore the information in the particle B.

In this way, the quantum information is destroyed from particle A and copied to particle B. The suitable unitary transformations corresponding to each measurement are given in Table-1.

Table-1: Unitary transformation corresponding to the measurement on the spin of particle A and the path of particle B

Measurement on S_A and P_B	Required Unitary Transformation on S_B	Final State of Particle B
$|0\rangle_{S_A}|x\rangle_{P_B}$	σ_x	$(\alpha|0\rangle + \beta|1\rangle)_{S_B}$
$|0\rangle_{S_A}|y\rangle_{P_B}$	I	$(\alpha|0\rangle + \beta|1\rangle)_{S_B}$
$|1\rangle_{S_A}|x\rangle_{P_B}$	$\sigma_z\sigma_x$	$(\alpha|0\rangle + \beta|1\rangle)_{S_B}$
$|1\rangle_{S_A}|y\rangle_{P_B}$	σ_z	$(\alpha|0\rangle + \beta|1\rangle)_{S_B}$

Protocol-II: In this protocol, the following steps are involved:

(i) The particle B is sent to the spin flipper (SF) positioned at one of the slits (say x) and particle A is sent to the double slit. If particle A goes through slit x, it activate spin flipper which flips the spin of particle B. If particle A goes through slit y it gives us a sign in path detector (PD) that particle A goes through slit y. But PD does not do anything to the spin of particle A. Thus, the spin of particle B is entangled with the path of particle A (see Fig-2).

If particle B is initially in the spin polarized state $|0\rangle_{S_B}$, then, the combined state of particle A and particle B will be

$|\eta\rangle = (\alpha|0\rangle + \beta|1\rangle)_{S_A} \otimes \frac{1}{\sqrt{2}}(|x1\rangle + |y0\rangle)_{P_A S_B}$………….. (6)

(ii) Single CNOT gate operation is performed with the spin of particle A as control qubit and the spin of particle B as target qubit.

(iii) A hadamard gate operation is performed on the spin state of the particle A.

After 3rd step, the combined state of particle A and particle B will become

$$|\eta\rangle = \frac{1}{2}\begin{bmatrix} |0\rangle_{S_A}\{|x\rangle_{P_A}(\alpha|1\rangle + \beta|0\rangle)_{S_B} + |y\rangle_{P_A}(\alpha|0\rangle + \beta|1\rangle)_{S_B}\} \\ +|1\rangle_{S_A}\{|x\rangle_{P_A}(\alpha|1\rangle - \beta|0\rangle)_{S_B} + |y\rangle_{P_A}(\alpha|0\rangle - \beta|1\rangle)_{S_B}\} \end{bmatrix}$$
………… (7)

(iv) The spin and path of particle A are measured.

(v) Depending upon the path and spin measurements of particle A, a suitable unitary transformation is performed on the spin state of particle B to restore the original information in the particle B.

The suitable unitary transformations to be done on the spin state of particle B, corresponding to the spin and path measurement results of particle A are given in Table-2. Thus the information encoded in particle A is transferred to the particle B.

Table-2: Unitary transformation to be done on the spin of the particle B corresponding to the measurement on the spin and path of particle A

Spin and Path measurement results	Required Unitary Transformation on S_B	Final State of Particle B
$\vert 0\rangle_{S_A}\vert x\rangle_{P_A}$	σ_x	$(\alpha\vert 0\rangle + \beta\vert 1\rangle)_{S_B}$
$\vert 0\rangle_{S_A}\vert y\rangle_{P_A}$	I	$(\alpha\vert 0\rangle + \beta\vert 1\rangle)_{S_B}$
$\vert 1\rangle_{S_A}\vert x\rangle_{P_A}$	$\sigma_z\sigma_x$	$(\alpha\vert 0\rangle + \beta\vert 1\rangle)_{S_B}$
$\vert 1\rangle_{S_A}\vert y\rangle_{P_A}$	σ_z	$(\alpha\vert 0\rangle + \beta\vert 1\rangle)_{S_B}$

4. CONCLUSION AND DISCUSSION

In the present study, by utilizing hybrid and hyper entangled states as the quantum resource, we have proposed two protocols for the transfer of quantum information encoded in the quantum states of a particle to another particle. It has been shown that path-spin entangled state can be generated by using double slit and a spin flipper at one of its opening. Here, the spin flippers used in both protocols are different. In protocol-I, the spin flipper flips the spin of the particle passing through it and generates hybrid entangled state of its path and spin. On the other hand, in protocol-II, the particle coming from the double slit activates the spin flipper that flips the spin of another particle passing through it and resulting in the hyper entanglement between path of the particle falling on the double slit and the spin of the other particle thus, generating path-spin hyper entangled states.

In the proposed protocols, if the particle B is not sent to some other place, then, the present protocols are similar to the copying of information from one place (hard disc/pen drive) to another place (hard disc/pen drive) in classical world. But in contrast to the classical case, the quantum information is destroyed at one place (particle A) and copied to another place (particle B).

In both protocols, before path and spin measurements, if the particle B is sent to a distant place (say, receiver), then the information can be sent to a distant place. The sender sends the particle B to the receiver before the spin and path measurements are performed. After sending the particle B to the receiver, spin and path measurements are performed. In protocol-I, sender measures the spin of particle A and conveys the measurement results to the receiver via classical communication. Next, receiver measures the path of particle B and performs suitable unitary operations on the spin state of particle B to restore to original information. In protocol-II, sender performs spin and path measurements and conveys the measurement results to the receiver via classical communication. Depending upon the spin and path measurement results, receiver performs suitable unitary operation on particle B to restore the original information. Thus, the proposed protocols can be used in transferring the information from one particle to another by using hybrid and hyper entangled states.

Declarations: The authors have no conflicts of interest to declare that are relevant to the content of this article.

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