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Improved Estimation of Population Variance Utilizing Known Auxiliary Parameters

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Abstract **Original Research Article**

Even similar things, whether created artificially or naturally, can vary. We should therefore try to estimate this variation. For improved population variance estimate, we propose a Searls ratio type estimator in the current research employing data on the tri-mean and the third quartile of the auxiliary variable. Up to the first-degree approximation, the suggested estimator's bias and mean squared error (MSE) are determined. The characterising scalar's ideal value is discovered, and given this ideal value, the least MSE is discovered. The mean squared errors of the suggested estimator and the competing estimators are contrasted conceptually and experimentally. Given that it has the lowest MSE of the above competing estimators, the recommended estimator has been shown to be the most effective.
Keywords: Population Variance, Estimator, Main and Auxiliary variables, Bias, MSE, PRE.

1. INTRODUCTION

One of the key indicators of dispersion is population variance, which is important for making day-to-day business decisions. The variance is obvious and occurs naturally. The literature has a very strong foundation for the accurate estimation of the parameters. It is advantageous for big populations to reduce errors since doing so will ultimately result in time and planning and decision-making cost savings. Making accurate estimates is essential for timely policymaking. The sample variance, which has the desirable characteristics of a good estimator, is mostly used to estimate variance. The sample variance of this approach could be quite considerable, which is one of its major downsides. Finding an estimator with a sample distribution that is tightly distributed around the population variance is therefore necessary. As a result, the auxiliary data is necessary to achieve this goal.

The auxiliary variable, denoted by X, which has a strong association with the study variable, denoted by Y, provides additional information. When Y and X have a strong positive correlation and the regression line of one passes through the origin, the ratio estimators are employed to estimate the enhanced population variance. When Y and X have a strong negative correlation and the regression line of one crosses through the origin, product type estimators are utilized to improve population variance estimation. In either scenario, the known auxiliary variable is used in conjunction with regression type estimators to improve population variance estimation of the primary variable.

Using the auxiliary data, Singh and Singh (2001) proposed a ratio-type estimator for a enhanced estimation of the population variance. Later, Singh and Singh (2003) provided an improved regression approach for estimating population variance in a two-phase sample design. A useful family of chain estimators was also proposed by Jhajj *et al.*, (2005) for the elevated estimation of the population variance under the sub-sampling method. Furthermore, Shabbir and Gupta (2007) focused on the development of auxiliary parameter-based variance estimation. Then, Kadilar and Cingi (2007) proposed various enhancements to the simple random sampling scheme's variance estimation. Using the understanding of the kurtosis of an auxiliary variable in sample surveys, Singh *et al.*, (2008) proposed a virtually impartial ratio and product type estimator of the finite population variance. A correction remark on the improved estimation of population variance using auxiliary parameters was reported by Grover (2010). Additionally, Singh and Solanki (2012) proposed a novel method utilising auxiliary data for variance estimate in simple random sampling.

Yadav and Kadilar (2014), on the other hand, suggested a two-parameter increased variance estimator using auxiliary parameters. An improved family of estimators for estimating population variance using auxiliary variable quartiles was proposed by Singh and Pal (2016). Yadav *et al.*, (2017) suggested an improved variance estimator using the auxiliary variable's known tri-mean and interquartile range. Using the well-known tri-mean and third quartile of the auxiliary variable, Yadav *et al.*, (2019) have proposed an increased estimator of the population variance. When outliers were present, Naz *et al.*, (2020) offered ratio-type estimators of population variance and employed unconventional dispersion measures of the auxiliary variable, which had a high correlation with the primary variable under discussion. Olayiwola *et al.*, (2021) worked on a new exponential ratio estimator of population variance and shown improvement over many existing estimators of population variance. Bhushan *et al.*, (2022) suggested some new modified classes of population variance utilizing the known auxiliary parameters.

Sharma *et al.*, (2022) and Searls (1964) served as inspiration for this investigation. To improve the population variance estimation of the key variable in this study, we propose a Searls type estimator and use a known population tri-mean and third quartile. Bias in sampling is examined up to an approximation of order one, and mean squared error (MSE) is as well. The remaining portions of the essay have been divided into sections. Review of population variance estimators for the research variable using auxiliary variable parameters that are known can be found in Section 2. The suggested estimators and their sample characteristics up to the first order approximation are described in Section 3. The efficiency comparison of the proposed estimator with the competing estimators and the requirements for the proposed estimator's superiority over competing estimators are explained in Section 4. The empirical research presented in Section 5 is the one in which the biases and MSEs for the actual natural population were computed. The conclusions drawn from the numerical study's findings are discussed in Section 6. The conclusion of the results of the study is presented in Section 7 and the paper ends with the references.

2. LITERATURE REVIEW

Let the finite population U is made up of N different and recognizable units $U_1, U_2, ..., U_N$ and the 'Simple Random Sampling Without Replacement' (SRSWOR) method is used to collect a sample of size n units from this population, assuming that Y and X has a strong correlation between them. Let (Y_i, X_i) be the observation on the i[th] unit of the population, $i = 1, 2, ..., N$.

The most suitable estimator for population variance S_y^2 is the sample variance s_y^2, given by,

$$t_0 = s_y^2 = \frac{1}{n-1}\sum_{i=1}^{n}(y_i - \bar{y})^2$$

The variance of t_0 for an approximation of degree one is,

$$V(t_0) = \gamma S_y^4 (\lambda_{40} - 1) \text{ .... (1)}$$

Where,

$$\gamma = \frac{1}{n} - \frac{1}{N}, \quad S_y^2 = \frac{1}{N-1}\sum_{i=1}^{N}(y_i - \bar{Y})^2, \quad \bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i, \quad \bar{Y} = \frac{1}{N}\sum_{i=1}^{N} y_i, \quad \lambda_{rs} = \frac{\mu_{rs}}{\mu_{20}^{r/2}\mu_{02}^{s/2}},$$

$$\mu_{rs} = \frac{1}{N-1}\sum_{i=1}^{N}(Y_i - \bar{Y})^r (X_i - \bar{X})^s$$

Isaki (1983) utilized the known positively correlated auxiliary information and suggested the following usual ratio estimator of S_y^2 as,

$$t_r = s_y^2 \left[\frac{S_x^2}{s_x^2}\right]$$

It is a biased estimator and its MSE up to the first order of approximation is,

$$MSE(t_r) = \gamma S_y^4 [(\lambda_{40} - 1) + (\lambda_{04} - 1) - 2(\lambda_{22} - 1)] \text{ .... (2)}$$

Upadhyaya and Singh (1999) used the known coefficient of kurtosis of X and introduced an estimator of S_y^2 as,

$$t_1 = s_y^2 \left[\frac{S_x^2 + \beta_2}{s_x^2 + \beta_2} \right]$$

The MSE of t_1 for an approximation of order one is,

$$MSE\ (t_1) = \gamma\, S_y^4\, [(\lambda_{40} - 1) + R_1^2 (\lambda_{04} - 1) - 2R_1 (\lambda_{22} - 1)] \text{ ……………… (3)}$$

Where,

$$R_1 = \frac{S_x^2}{S_x^2 + \beta_2} \text{ and } S_x^2 = \frac{1}{N-1} \sum_{i=1}^{N} (x_i - \bar{X})^2$$

Kadilar and Cingi (2006) suggested three estimators of S_y^2 utilizing S_x^2, β_2 and C_x as,

$$t_2 = s_y^2 \left[\frac{S_x^2 + C_x}{s_x^2 + C_x} \right],\ t_3 = s_y^2 \left[\frac{S_x^2 \beta_2 + C_x}{s_x^2 \beta_2 + C_x} \right],\ t_4 = s_y^2 \left[\frac{S_x^2 C_x + \beta_2}{s_x^2 C_x + \beta_2} \right]$$

The MSEs of t_i ($i = 2, 3, 4$) for an approximation of order one is,

$$MSE\ (t_i) = \gamma\, S_y^4\, [(\lambda_{40} - 1) + R_i^2 (\lambda_{04} - 1) - 2R_i (\lambda_{22} - 1)] \text{ ……………….. (4)}$$

Where,

$$R_2 = \frac{S_x^2}{S_x^2 + C_x},\ R_3 = \frac{S_x^2 \beta_2}{S_x^2 \beta_2 + C_x},\ R_4 = \frac{S_x^2 C_x}{S_x^2 C_x + \beta_2} \text{ and } C_x = \frac{S_x}{\bar{X}}$$

Subramani & Kumarpandiyan (2012a) utilized the known median M_d of X and proposed the following estimator of S_y^2 as,

$$t_5 = s_y^2 \left[\frac{S_x^2 + M_d}{s_x^2 + M_d} \right]$$

The MSE of t_5 for an approximation of order one is,

$$MSE\ (t_5) = \gamma\, S_y^4\, [(\lambda_{40} - 1) + R_5^2 (\lambda_{04} - 1) - 2R_5 (\lambda_{22} - 1)] \text{ ……………… (5)}$$

Where,

$$R_1 = \frac{S_x^2}{S_x^2 + M_d}$$

Subramani & Kumarpandiyan (2012b) utilized the known quartiles of X and their functions and suggested the following five estimators of S_y^2 as,

$$t_6 = s_y^2 \left[\frac{S_x^2 + Q_1}{s_x^2 + Q_1} \right],\ t_7 = s_y^2 \left[\frac{S_x^2 + Q_3}{s_x^2 + Q_3} \right],\ t_8 = s_y^2 \left[\frac{S_x^2 + Q_r}{s_x^2 + Q_r} \right],\ t_9 = s_y^2 \left[\frac{S_x^2 + Q_d}{s_x^2 + Q_d} \right],\ t_{10} = s_y^2 \left[\frac{S_x^2 + Q_a}{s_x^2 + Q_a} \right]$$

The MSEs of t_i ($i = 6, 7, ..., 10$) for an approximation of order one is,

$$MSE\ (t_i) = \gamma\, S_y^4\, [(\lambda_{40} - 1) + R_i^2 (\lambda_{04} - 1) - 2R_i (\lambda_{22} - 1)] \text{ …………………. (6)}$$

Where,

$$R_6 = \frac{S_x^2}{S_x^2 + Q_1},\ R_7 = \frac{S_x^2}{S_x^2 + Q_3},\ R_8 = \frac{S_x^2}{S_x^2 + Q_r},\ R_9 = \frac{S_x^2}{S_x^2 + Q_d},\ R_{10} = \frac{S_x^2}{S_x^2 + Q_a} \text{ and } Q_r = Q_3 - Q_1,$$

$$Q_d = \frac{Q_3 - Q_1}{2},\ Q_a = \frac{Q_3 + Q_1}{2}.$$

Subramani & Kumarpandiyan (2013) suggested a new estimator of S_y^2 using known S_x^2, M_d and C_x as,

$$t_{11} = s_y^2 \left[\frac{S_x^2 C_x + M_d}{s_x^2 C_x + M_d} \right]$$

The MSE of t_5 for an approximation of order one is,

$$MSE\ (t_{11}) = \gamma S_y^4 [(\lambda_{40} - 1) + R_{11}^2 (\lambda_{04} - 1) - 2R_{11}(\lambda_{22} - 1)] \quad \text{……………… (7)}$$

Where,

$$R_{11} = \frac{S_x^2 C_x}{S_x^2 C_x + M_d}$$

Khan & Shabbir (2013) utilized the known third quartile Q_3 of X and correlation coefficient between Y and X and suggested an estimator of S_y^2 as,

$$t_{12} = s_y^2 \left[\frac{S_x^2 \rho + Q_3}{s_x^2 \rho + Q_3} \right]$$

The MSE of t_{12} for an approximation of order one is,

$$MSE\ (t_{12}) = \gamma S_y^4 [(\lambda_{40} - 1) + R_{12}^2 (\lambda_{04} - 1) - 2R_{12}(\lambda_{22} - 1)] \quad \text{…………………. (8)}$$

Where,

$$R_{12} = \frac{S_x^2 \rho}{S_x^2 \rho + Q_3}$$

Maqbool and Javaid (2017) utilized known S_x^2, TM and Q_a of X and suggested the following estimator of S_y^2 as,

$$t_{13} = s_y^2 \left[\frac{S_x^2 + (TM + Q_a)}{s_x^2 + (TM + Q_a)} \right]$$

The MSE of t_{13} for an approximation of order one is,

$$MSE\ (t_{13}) = \gamma S_y^4 [(\lambda_{40} - 1) + R_{13}^2 (\lambda_{04} - 1) - 2R_{13}(\lambda_{22} - 1)] \quad \text{……………… (9)}$$

Where,

$$R_{13} = \frac{S_x^2}{S_x^2 + (TM + Q_a)} \text{ and } TM = \frac{Q_1 + 2Q_2 + Q_3}{4}$$

Khalil *et al.*, (2018) suggested the following three estimators of S_y^2 using the known auxiliary parameters as,

$$t_{14} = s_y^2 \left[\frac{S_x^2 + C_x S_x}{s_x^2 + C_x S_x} \right],\ t_{15} = s_y^2 \left[\frac{S_x^2 + C_x \overline{X}}{s_x^2 + C_x \overline{X}} \right],\ t_{16} = s_y^2 \left[\frac{S_x^2 + C_x M_d}{s_x^2 + C_x M_d} \right]$$

The MSEs of t_i ($i = 14,15,16$) for an approximation of order one is,

$$MSE\ (t_i) = \gamma S_y^4 [(\lambda_{40} - 1) + R_i^2(\lambda_{04} - 1) - 2R_i(\lambda_{22} - 1)] \quad \text{………….. (10)}$$

Where,

$$R_{14} = \frac{S_x^2}{S_x^2 + C_x S_x},\ R_{15} = \frac{S_x^2}{S_x^2 + C_x \bar{X}},\ R_{16} = \frac{S_x^2}{S_x^2 + C_x M_d}$$

Yadav *et al.*, (2019) worked on an improved estimator of S_y^2 using some known auxiliary parameters as,

$$t_{17} = s_y^2 \left[\frac{S_x^2 + (TM + Q_3)}{s_x^2 + (TM + Q_3)} \right]$$

The MSE of t_{17} for an approximation of order one is,

$$MSE\ (t_{17}) = \gamma S_y^4 [(\lambda_{40} - 1) + R_{17}^2(\lambda_{04} - 1) - 2R_{17}(\lambda_{22} - 1)] \quad \text{…………………. (11)}$$

where,

$$R_{17} = \frac{S_x^2}{S_x^2 + (TM + Q_3)}$$

Sharma *et al.*, (2022) suggested the following estimator of S_y^2 utilizing the known S_x^2, TM and Q_3 of X as,

$$t_{18} = \kappa s_y^2 \left[\frac{S_x^2 + (TM + Q_3)}{s_x^2 + (TM + Q_3)} \right]$$

Where, κ is a scalar, to be obtained such that the MSE of t_{18} is minimum.

The optimum value of κ which minimizes the MSE of t_{18} is,

$$\kappa = \frac{A}{B} \quad \text{………………… (12)}$$

Where,

$A = 1 + R_{17}^2 \gamma (\lambda_{04} - 1) - R_{17} \gamma (\lambda_{22} - 1)$ and

$B = 1 + \gamma (\lambda_{40} - 1) + 3R_{17}^2 \gamma (\lambda_{04} - 1) - 4R_{17} \gamma (\lambda_{22} - 1)$

The minimum value of MSE of t_{18} for the optimum value of κ is,

$$MSE_{\min}(t_{18}) = S_y^4 \left[1 - \frac{A^2}{B} \right] \quad \text{……………………….. (13)}$$

3. PROPOSED ESTIMATOR

Motivated by Searls (1964) and Sharma *et al.*, (2022), we suggest a class of S_y^2 using aome auxiliary parameters as,

$$t_p = \kappa_1 s_y^2 + \kappa_2 s_y^2 \left[\frac{S_x^2 + (TM + Q_3)}{s_x^2 + (TM + Q_3)} \right] \quad \text{…………………… (14)}$$

Where, κ_1 and κ_2 are the characterizing scalars, to be obtained that the MSE of t_p is minimum.

We make the following assumptions in order to study the bias and MSE of the introduced estimator:
$s_y^2 = S_y^2(1+\varepsilon_0)$ and $s_x^2 = S_x^2(1+\varepsilon_1)$ such that $E(\varepsilon_i)=0$ for $(i=0,1)$ and $E(\varepsilon_0^2)=\gamma(\lambda_{40}-1)$, $E(\varepsilon_1^2)=\gamma(\lambda_{04}-1)$, $E(\varepsilon_0\varepsilon_1)=\gamma(\lambda_{22}-1)$.

The suggested estimator in (14) may be expressed in terms of ε_i's as,

$$t_p = \kappa_1 S_y^2(1+e_0)+\kappa_2 S_y^2(1+e_0)(1+R_{17}e_1)^{-1}$$

By extending the term in the equation above, simplifying it, and bringing the terms up to about order one, we have,

$$t_p = S_y^2[\kappa_1(1+e_0)+\kappa_1(1+e_0-R_{17}e_1-R_{17}e_0e_1+R_{17}^2e_1^2)]$$

S_y^2 being subtracted from both sides of the equation above gives us,

$$t_p - S_y^2 = S_y^2[\kappa_1(1+e_0)+\kappa_1(1+e_0-R_{17}e_1-R_{17}e_0e_1+R_{17}^2e_1^2)-1] \quad \text{………………. (15)}$$

When calculating the bias of the suggested estimator, we start with expectation of (15) and put various values of expectations as,

$$B(t_p) = S_y^2[\kappa_1+\kappa_1\{1-R_{17}\gamma(\lambda_{22}-1)+R_{17}^2\gamma(\lambda_{04}-1)\}-1] \quad \text{…………………. (16)}$$

By obtaining the expectation and entering the values of multiple expectations as squares of equation (3), we can obtain the MSE of t_p,

$$MSE(t_p) = S_y^4\begin{bmatrix}1+\kappa_1^2\{1+\gamma(\lambda_{40}-1)\}+\kappa_2^2\{1+\gamma(\lambda_{40}-1)+3R_{17}^2\gamma(\lambda_{04}-1)-4R_{17}\gamma(\lambda_{22}-1)\}\\ -2\kappa_1-2\kappa_2\{1-R_{17}\gamma(\lambda_{22}-1)+R_{17}^2\gamma(\lambda_{04}-1)\}\\ +2\kappa_1\kappa_2\{1+\gamma(\lambda_{40}-1)-2R_{17}\gamma(\lambda_{22}-1)+R_{17}^2\gamma(\lambda_{04}-1)\}\end{bmatrix}$$

$$MSE(t_p) = S_y^4[1+\kappa_1^2A+\kappa_2^2B-2\kappa_1-2\kappa_2C+2\kappa_1\kappa_2D] \quad \text{………………… (17)}$$

Where,

$$A = \{1+\gamma(\lambda_{40}-1)\}$$

$$B = \{1+\gamma(\lambda_{40}-1)+3R_{17}^2\gamma(\lambda_{04}-1)-4R_{17}\gamma(\lambda_{22}-1)\}$$

$$C = \{1-R_{17}\gamma(\lambda_{22}-1)+R_{17}^2\gamma(\lambda_{04}-1)\}$$

$$D = \{1+\gamma(\lambda_{40}-1)-2R_{17}\gamma(\lambda_{22}-1)+R_{17}^2\gamma(\lambda_{04}-1)\}$$

The optimum values of κ_1 and κ_2, which minimizes the MSE of t_p are respectively given as,

$$\kappa_{1(opt)} = \frac{DC-B}{D^2-AB} \text{ and } \kappa_{2(opt)} = \frac{D-AC}{D^2-AB}$$

The least value of $MSE(t_p)$ for the optimal values of κ_1 and κ_2 is,

$$MSE_{\min}(t_p) = S_y^4\left[1-\frac{\begin{Bmatrix}C(D-AC)(D^2-AB)+2(DC-B)(D^2-AB)\\ -2(DC-B)(D-AC)-A(DC-B)^2-B(D-AC)^2\end{Bmatrix}}{(D^2-AB)^2}\right]$$

$$MSE_{\min}(t_p) = S_y^4\left[1-\frac{L}{M^2}\right] \quad \text{…………….. (19)}$$

Where,

$$L=\begin{Bmatrix} C(D-AC)(D^2-AB)+2(DC-B)(D^2-AB) \\ -2(DC-B)(D-AC)-A(DC-B)^2-B(D-AC)^2 \end{Bmatrix}$$

$$M=(D^2-AB)$$

4. EFFICIENCY COMPARISON

The efficiency criteria over the competing estimators are produced under this section, where t_p is theoretically contrasted with the current competing estimators.

Performance-wise, the estimator t_p outperforms the sample variance for the condition if,

$$V(t_0)-MSE_{\min}(t_p)=S_y^4\left[1-\frac{L}{M^2}-\gamma(\lambda_{40}-1)\right]>0 \text{ or, } \frac{L}{M^2}+\gamma(\lambda_{40}-1)<1 \quad \text{……………… (20)}$$

The suggested estimator t_p outperforms the estimate from Isaki (1983) under the condition if,

$$MSE(t_R)-MSE_{\min}(t_p)=S_y^2\left[1-\frac{L}{M^2}-\gamma\{(\lambda_{40}-1)+(\lambda_{04}-1)-2(\lambda_{22}-1)\}\right]>0$$

$$\text{or, } \frac{L}{M^2}+\gamma\{(\lambda_{40}-1)+(\lambda_{04}-1)-2(\lambda_{22}-1)\}<1 \quad \text{……………….. (21)}$$

The suggested estimator t_p is better than the mentioned competing estimators t_i ($i=1,2,...,17$) under the conditions if,

$$MSE(t_i)-MSE_{\min}(t_p)=S_y^2\left[1-\frac{L}{M^2}-\gamma\{(\lambda_{40}-1)+R_i^2(\lambda_{04}-1)-2R_i(\lambda_{22}-1)\}\right]>0,\ (i=1,2,...,17)$$

or,

$$\frac{L}{M^2}+\gamma\{(\lambda_{40}-1)+R_i^2(\lambda_{04}-1)-2R_i(\lambda_{22}-1)\}<1 \quad \text{……………….. (22)}$$

The suggested estimator t_p outperforms the Sharma *et al.*, (2022) estimator under the condition if,

$$MSE_{\min}(t_{18})-MSE_{\min}(t_p)=\frac{L}{M^2}-\frac{A^2}{B}>0$$

5. NUMERICAL STUDY

The efficiency criteria of t_p over competing estimators are confirmed in this section. We used the population listed in Sharma *et al.*, (2022) for the investigation under consideration. The biases and MSEs of the suggested and competing estimators have been numerically calculated. Table 1 lists this population's characteristics.

Table 1: Parameters of the population in Sharma *et al.*, (2022)

$N=80,\ n=20,\ \bar{Y}=51.8264,\ \bar{X}=11.2646,\ \rho=0.9413,\ S_y=18.3549,\ C_y=0.3542,$
$S_x=8.4563,\ C_x=0.7507,\ \lambda_{04}=2.8664,\ \lambda_{40}=2.2667,\ \lambda_{22}=2.2209,\ Q_1=5.1500,$
$Q_3=16.975,\ Q_r=11.825,\ Q_d=5.9125,\ Q_a=11.0625,\ T_m=9.318,\ M_d=7.575$

The biases and MSEs of t_p and the estimators in competition along with the percentage relative efficiency (PRE) of t_p over competing estimators of S_y^2 are presented in Table 2 given below.

Table 2: The MSE of various estimators and the PRE with respect to t_0

Estimator	MSE	PRE
Sample variance t_0	5,393.89	100.00
Isaki (1983) estimator t_r	3,925.16	137.42
Upadhyaya and Singh (1999) estimator t_1	3,658.41	147.44
Kadilar and Cingi (2006) estimator t_2	3,850.16	140.10
Kadilar and Cingi (2006) estimator t_3	3,898.56	138.36
Kadilar and Cingi (2006) estimator t_4	3,580.83	150.63
Subramani & Kumarpandiyan (2012a) estimator t_5	4,157.95	129.72
Subramani & Kumarpandiyan (2012b) estimator t_6	3,480.55	154.97
Subramani & Kumarpandiyan (2012b) estimator t_7	2,908.65	185.44
Subramani & Kumarpandiyan (2012b) estimator t_8	3,098.41	174.09
Subramani & Kumarpandiyan (2012b) estimator t_9	3,427.19	157.39
Subramani & Kumarpandiyan (2012b) estimator t_{10}	3,133.33	172.15
Subramani & Kumarpandiyan (2013) estimator t_{11}	2,467.88	218.56
Khan & Shabbir (2013) estimator t_{12}	2,878.56	187.38
Maqbool and Javaid (2017) estimator t_{13}	2,820.06	191.27
Khalil *et al.*, (2018) estimator t_{14}	2,547.21	211.76
Khalil *et al.*, (2018) estimator t_{15}	2,450.18	220.14
Khalil *et al.*, (2018) estimator t_{16}	2,580.75	209.00
Yadav *et al.*, (2019) estimator t_{17}	2,040.12	264.39
Sharma *et al.*, (2022) estimator t_{18}	1,986.22	271.57
Proposed estimator t_p	1832.42	294.36

The MSE of various estimators and PRE with respect to t_0 are shown below in Figures 1 and 2 respectively.

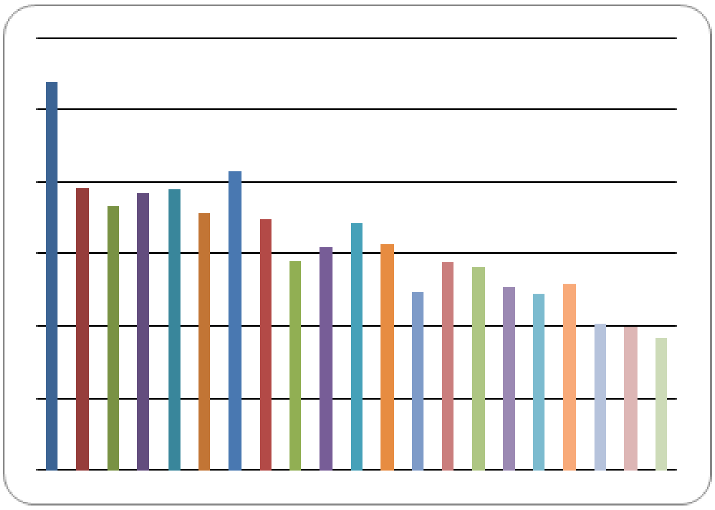

Figure-1: MSE of various estimators

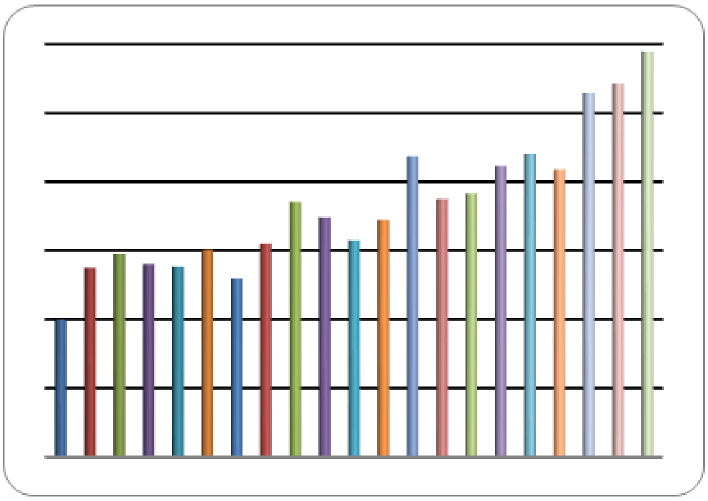

Figure-2: PRE with respect to t_0

7. RESULTS AND DISCUSSION

Table 2 demonstrates that, among the competing estimators of S_y^2, the suggested estimator t_p has the smallest MSE. The MSE of t_p is 1832.42, whereas the MSEs of the competing estimators are located in the range [2,040.12 5,393.89]. The PREs of the various estimators range between [100.00 271.57] and [294.36], whereas that of t_p is [100.00 271.57]. As a result, we can see that Sharma *et al.*, (2022) estimator performs best among the competing estimators that also employ the sample variance estimator, and the recommended estimator performs better than Sharma *et al.*, (2022) estimator of population variance.

8. CONCLUSION

In the paper, we proposed an estimator using Searls' technique for the SRSWOR Scheme for a better estimation of S_y^2. We arrived at the bias and MSE of t_p expressions up to the first degree approximation. With the competing estimators of S, the theoretical and empirical comparison of t_p is done. For the natural population listed in Sharma *et al.* (2022), the MSEs and PREs for the proposed and competing estimators have been calculated. The outcomes supported the claim that, among the competing S_y^2 estimators listed above, the suggested estimator had the least MSE. This accomplishes the goal of identifying estimators that are superior than the available competing estimators. Because of this, it is anticipated that using t_p for an increased estimation of S_y^2 under a simple random sampling technique will be advantageous in a variety of application areas. As a result, the study's goal of finding a more effective estimator has been achieved, and it may be used in many policies for business decisions in actual sectors of application like life insurance, automotive, banking, marketing, etc.

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